Classification of groups generated by 3-state automata over a 2-letter alphabet

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1 Introduction

Automaton groups were formally introduced in the beginning of 1960's [Glu61, Hoř63] but it took a while to realize their importance, utility, and, at the same time, complexity. Among the publications from the first decade of the study of automaton groups let us distinguish [Zar64, Zar65] and the book [GP72].

The first substantial results came only in the 1970's and in the beginning of the 1980's when it was shown in [Ale72, Sus79, Gri80, GS83b] that automaton groups provide examples of finitely generated infinite torsion groups, thus making a contribution to one of the most famous problems in algebra — the General Burnside Problem (more information on all three versions of the problem can be found in [Adi79, Gol68, Gup89, Kos90, Zel91, GL02]). The methods used to study the properties of the examples from [Ale72, Sus79, Gri80] are very different. The methods used in [Ale72] are typical for the theory of finite automata (in fact the provided proof was incorrect; the first correct proof appears in [Mer83] as a combination of the results from [Gri80] and [Mer83], as well as in the third edition of the book [KM82] and in [KAP85]). The exposition in [Sus79] is based on Kalujnin's tableaux coming from his theory of iterated wreath products of cyclic groups of prime order p. The approach in [Gri80] is based on the ideas of self-similarity and contraction. These ideas are apparent both in the proof of the infiniteness and the torsion property of the group. The self-similarity is apparent from the fact that the set of all states of the automaton is used as a generating set for the group (now it is common to call such groups self-similar). The contraction property here means that the length of the elements contracts by a factor bounded away from 1 when one passes to sections. A principal tool introduced in the beginning of the 1980's was the language of actions on rooted

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trees suggested by Gupta and Sidki in [GS83b], which helped tremendously in bringing geometric insight to the subject.

A new indication of the importance of automaton groups came when it was shown that some of them provided the first examples of groups of intermediate growth [Gri83, Gri84, Gri85]. This not only answered the question of J.Milnor [Mil68] about existence of such groups, but also answered a number of other questions in and around group theory, including M. Day's problem [Day57] on existence of amenable but not elementary amenable groups. Basically, even to this day, all known examples of groups of intermediate growth and non-elementary amenable groups are based on automaton groups.

Investigations in the last two decades [Gri84, Gri85, GS83b, GS83a, Lys85, Neu86, Sid87a, Sid87b, Gri89, Roz93, Gri98, Gri99, Gri00, BG00a, BG00b, GŻ01, Nek05, GŠ06] show that many automaton groups possess numerous interesting, and sometimes unusual, properties. This includes just infiniteness (the groups constructed in [Gri84, Gri85] as well as in [GS83a] answer a question from [CM82] on new examples of infinite groups with finite quotients), finiteness of width, or more generally polynomial growth of the dimension of the successive quotients in the lower central series [BG00b] (answering a question of E. Zelmanov on classification of groups of finite width), branch properties [Gri84, Neu86, Gri00] (answering some questions of S. Pride and M. Edjvet [Pri80, EP84]), finiteness of the index of maximal subgroups and presence or absence of the congruence property [Per00, Per02] (related to topics in profinite groups), existence of groups with exponential but not uniformly exponential growth [Wil04b, Wil04a, Bar03, Nek07b] (providing an answer to a question of M. Gromov), subgroup separability and conjugacy separability [GW00], further examples of amenable groups but not amenable (or even sub-exponentially amenable) groups [GŻ02a, BV05, GNŠ06a], amenability of groups generated by bounded automata [BKN], and so on. The word problem can be solved in contracting self-similar groups by using an extremely effective *branch algorithm* [Gri84, Sav03]. The conjugacy problem can also be solved in many cases [WZ97, Roz98, Leo98, GW00] (in fact we do not know of an example of an automaton group with unsolvable conjugacy problem). In some instances, it is even known that the membership problem is solvable [GW03].

In addition to the formulation of many algebraic properties of groups generated by finite automata, a number of links and applications were discovered during the last decade. This includes asymptotic and spectral properties of the Cayley graphs and Schreier graphs associated to the action on the rooted tree with respect to the set of generators given by the set of states of the automaton. For instance, it is shown in [GŻ01] that the discrete Laplacian on the Cayley graph of the Lamplighter group $\mathbb{Z} \ltimes (\mathbb{Z}/2\mathbb{Z})^{\mathbb{Z}}$ has pure point spectrum. This fact was used to answer a question of M. Atiyah on L^2-Betti numbers of closed manifolds [GLSŻ00]. The methods developed in the study of the spectral properties of Schreier graphs of self-similar groups can be used to construct Laplacians on fractal sets and to study their spectral properties (see [GN07, NT08].

A new and fruitful direction, bringing further applications of self-similar groups, was established by the introduction of the notions of iterated mon-

odromy groups and limit spaces by V. Nekrashevych. The theory established a link between contracting self-similar groups and the geometry of Julia sets of expanding maps. An example of an application of self-similar groups to holomorphic dynamics is given by the solution (by L. Bartholdi and V. Nekrashevych in [BN06]) of the "twisted rabbit" problem of J. Hubbard. The book [Nek05] provides a comprehensive introduction to this theory.

In many situations automaton groups serve as renorm groups. For instance this happens in the study of classical fractals, in the study of the behavior of dynamical systems [Oli98], and in combinatorics — for example in Hanoi Towers games on k pegs, $k \geq 3$, as observed by Z. Šunić (see [GŠ06]).

There is interest of computer scientists and logicians in automaton groups, since they may be relevant in the solution of important complexity problems (see [RS] for ideas in this direction). Self-similar groups of intermediate growth are mentioned by Wolfram in [Wol02] as examples of "multiway systems" with complex behavior.

Among the major problems in many areas of mathematics are the classification problems. If the objects are given combinatorially then it is naturally to try to classify them first by complexity and then within each complexity class.

A natural complexity parameter in our situation is the pair (m, n) where m is the number of states of the automaton generating the group and n is the cardinality of the alphabet.

There are 64 invertible 2-state automata acting on a 2-letter alphabet, but there are only six non-isomorphic $(2, 2)$-automaton groups, namely, the trivial group, $\mathbb{Z}/2\mathbb{Z}$, $\mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$, $\mathbb{Z}$, the infinite dihedral group D_∞, and the lamplighter group $\mathbb{Z} \wr \mathbb{Z}/2\mathbb{Z}$ [GNS00, GŻ01] (more details are given in Theorem 7 below). A classification of semigroups generated by 2-state automata (not necessary invertible) over a 2-letter alphabet is provided by I. Reznikov and V. Sushchanskiĭ [RS02a]. Some examples from this class, including an automaton generating a semigroup of intermediate growth, were studied in the subsequent papers [RS02c, RS02b, BRS06].

It is not known how many pairwise non-isomorphic groups exists for any class (m, n) when either $m > 2$ or $n > 2$. Unfortunately, the number of automata that has to be treated grows super-exponentially with either of the two arguments (there are $m^{mn}(n!)^m$ invertible (m, n)-automata).

Nevertheless, a reasonable task is to consider the problem of classification for small values of m and n and try to classify the $(3, 2)$-automaton groups and $(2, 3)$-automaton groups.

Our research group (with some contribution by Y. Vorobets and M. Vorobets) has been working on the problem of classification of $(3, 2)$-automaton groups for the last four yeas and most of the obtained results are presented in this article. Our research goals moved in three main directions:

1. Search for new interesting groups and an attempt to use them to solve known problems. An example of such a group is the Basilica group (see automaton [852]). It is the first example of an amenable group (shown in [BV05]) that is not sub-exponentially amenable group (shown in [GŻ02a]).

2. Recognition of already known groups as self-similar groups, and use of the

self-similar structure in finding new results and applications for such groups. As examples we can mention the free group of rank 3 (see automaton [2240]), the free product of three copies of $\mathbb{Z}/2\mathbb{Z}$ (see automaton [846]), Baumslag-Solitar groups $BS(1, \pm 3)$ (see automata [870] and [2294]), the Klein bottle group (see automaton [2212]), and the group of orientation preserving automorphisms of the 2-dimensional integer lattice (see automaton [2229]).

3. Understanding of typical phenomena that occur for various classes of automaton groups, formulation and proofs of reasonable conjectures about the structure of self-similar groups.

The main general results on the class of groups generated by $(3, 2)$-automata are as follows.

Theorem 1. *There are at most 122 non-isomorphic groups generated by* $(3, 2)$*-automata.*

The numbers in brackets in the next two theorems are references to the numbers of the corresponding automata (more on this encoding will be said later). Here and thereafter, C_n denotes the cyclic group of order n.

Theorem 2. *There are* 6 *finite groups in the class: the trivial group* $\{1\}$ [1]*,* C_2 [1090]*,* $C_2 \times C_2$ [730]*,* D_4 [847]*,* $C_2 \times C_2 \times C_2$ [802] *and* $D_4 \times C_2$ [748]*.*

Theorem 3. *There are* 6 *abelian groups in the class: the trivial group* $\{1\}$ [1]*,* C_2 [1090]*,* $C_2 \times C_2$ [730]*,* $C_2 \times C_2 \times C_2$ [802]*,* $\mathbb{Z}$ [731] *and* $\mathbb{Z}^2$ [771]*.*

Theorem 4. *The only nonabelian free group in the class is the free group of rank* 3 *generated by the Aleshin-Vorobets-Vorobets automaton* [2240]*.*

Theorem 5. *There are no infinite torsion groups in the class.*

The short list of general results does not give full justice to the work that has been done. Namely, in most individual cases we have provided a lot of results and detailed information for the group in question. The variety is rather extreme and it is not surprising at all that one cannot formulate too many general results.

More work and, likely, some new invariants are required to further distinguish the 122 groups that are listed in this paper as potentially non-isomorphic. In some cases one could try to use the rigidity of actions on rooted trees (see [LN02]), since in many cases it is easier to distinguish actions than groups. In the contracting case one could use, for instance, the geometry of the Schreier graphs and limit spaces to distinguish the actions.

Next natural step would be to consider the case of $(2, 3)$-automaton groups or 2-generated self-similar groups of binary tree automorphisms defined by recursions in which every section is either trivial, a generator, or an inverse of a generator. The cases $(4, 2)$ and $(5, 2)$ also seem to be attractive, as there are many remarkable groups in these classes.

Another possible direction is to study more carefully only certain classes of automata (such as the classical linear automata, bounded and polynomially

growing automata in the sense of Sidki [Sid00], etc.) and the properties of the corresponding automaton groups.

Many computations used in our work were performed by the package `AutomGrp` for `GAP` system, developed by Y. Muntyan and D. Savchuk [MS08]. The package is not specific to $(3,2)$-automaton groups (in fact, many functions are implemented also for groups of tree automorphisms that are not necessarily generated by automata).

2 Regular rooted trees, automorphisms, and self-similarity

Let X be an alphabet on d ($d \geq 2$) letters. Most often we set $X = \{0, 1, \ldots, d-1\}$. The set of finite words over X, denote by X^*, has the structure of a *regular rooted d-ary tree*, which we also denote by X^*. The empty word $\emptyset$ is the *root* of the tree and every vertex v has d children, namely the words vx, for x in X. The words of length n constitute *level n* in the tree.

The group of tree automorphisms of X^* is denoted by $\mathrm{Aut}(X^*)$. Tree automorphisms are precisely the permutations of the vertices that fix the root and preserve the levels of the tree. Every automorphism f of X^* can be decomposed as

$$f = \alpha_f(f_0, \ldots, f_{d-1}) \tag{1}$$

where f_x, for x in X, are automorphisms of X^* and α_f is a permutation of the set X. The permutation α_f is called the *root permutation* of f and the automorphisms f_x (denoted also by $f|_x$), x in X, are called *sections* of f. The permutation α_f describes the action of f on the first letter of every word, while the automorphism f_x, for x in X, describes the action of f on the tail of the words in the subtree xX^*, consisting of the words in X^* that start with x. Thus the equality (1) describes the action of f through decomposition into two steps. In the first step the d-tuple $(f_0, \ldots, f_{d-1})$ acts on the d subtrees hanging below the root, and then the permutation α_f, permutes these d subtrees. Thus we have

$$f(xw) = \alpha_f(x) f_x(w), \tag{2}$$

for x in X and w in X^*. Second level sections of f are defined as the sections of the sections of f, i.e., $f_{xy} = (f_x)_y$, for $x, y \in X$, and more generally, for a word u in X^* and a letter x in X the section of f at ux is defined as $f_{ux} = (f_u)_x$, while the section of f at the root is f itself.

The group $\mathrm{Aut}(X^*)$ decomposes algebraically as

$$\mathrm{Aut}(X^*) = \mathsf{Sym}(X) \ltimes \mathrm{Aut}(X^*)^X = \mathsf{Sym}(X) \wr \mathrm{Aut}(X^*), \tag{3}$$

where $\wr$ is the *permutational wreath product* in which the active group $\mathsf{Sym}(X)$ permutes the coordinates of $\mathrm{Aut}(X^*)^X = (\mathrm{Aut}(X^*), \ldots, \mathrm{Aut}(X^*))$. For arbitrary automorphisms f and g in $\mathrm{Aut}(X^*)$ we have

$$\alpha_f(f_0, \ldots, f_{d-1})\alpha_g(g_0, \ldots, g_{d-1}) = \alpha_f\alpha_g(f_{g(0)}g_0, \ldots, f_{g(d-1)}g_{d-1}).$$

For future use we note the following formula regarding the sections of a composition of tree automorphisms. For tree automorphisms f and g and a vertex u in X^*,

$$(fg)_u = f_{g(u)} g_u. \tag{4}$$

The group of tree automorphisms $\mathrm{Aut}(X^*)$ is a pro-finite group. Namely, $\mathrm{Aut}(X^*)$ has the structure of an infinitely iterated wreath product

$$\mathrm{Aut}(X^*) = \mathsf{Sym}(X) \wr (\mathsf{Sym}(X) \wr (\mathsf{Sym}(X) \wr \dots))$$

of the finite group $\mathsf{Sym}(X^*)$ (this follows from (3)). This product is the inverse limit of the sequence of finitely iterated wreath products of the form $\mathsf{Sym}(X) \wr (\mathsf{Sym}(X) \wr (\mathsf{Sym}(X) \wr \cdots \wr \mathsf{Sym}(X)))$. Every subgroup of $\mathrm{Aut}(X^*)$ is residually finite. A canonical sequence of normal subgroups of finite index intersecting trivially is the sequence of level stabilizers. The n-th *level stabilizer* of a group G of tree automorphisms is the subgroup $\mathrm{Stab}_G(n)$ of $\mathrm{Aut}(X^*)$ that consists of all tree automorphisms in G that fix the vertices in the tree X^* up to and including level n.

The *boundary* of the tree X^* is the set X^ω of right infinite words over X (infinite geodesic rays in X^* connecting the root to "infinity"). The boundary has a natural structure of a metric space in which two infinite words are close if they agree on long finite prefixes. More precisely, for two distinct rays ξ and ζ, define the distance to be $d(\xi,\zeta) = 1/2^{|\xi \wedge \zeta|}$, where $|\xi \wedge \zeta|$ denotes the length of the longest common prefix $\xi \wedge \zeta$ of ξ and ζ. The induced topology on X^ω is the Tychonoff product topology (with X discrete), and X^ω is a Cantor set. The group of isometries $\mathrm{Isom}(X^\omega)$ and the group of tree automorphisms $\mathrm{Aut}(X^*)$ are canonically isomorphic. Namely, the action of the automorphism group $\mathrm{Aut}(X^*)$ can be extended to an isometric action on X^ω, simply by declaring that (1) and (2) are valid for right infinite words.

We now turn to the concept of self-similarity. The tree X^* is a highly self-similar object (the subtree uX^* consisting of words with prefix u is canonically isomorphic to the whole tree) and we are interested in groups of tree automorphisms in which this self-similarity structure is reflected.

Definition 1. A group G of tree automorphisms is *self-similar* if, every section of every automorphism in G is an element of G.

Equivalently, self-similarity can be expressed as follows. A group G of tree automorphisms is self-similar if, for every g in G and a letter x in X, there exists a letter y in X and an element h in G such that

$$g(xw) = yh(w),$$

for all words w over X.

Self-replicating groups constitute a special class of self-similar groups. Examples from this class are very common in applications. A self-similar group G is *self-replicating* if, for every vertex u in X^*, the homomorphism $\varphi_u : \mathrm{Stab}_G(u) \to G$ from the stabilizer of the vertex u in G to G, given by $\varphi(g) = g_u$, is surjective.

At the end of the section, let us mention the class of *branch groups*. Branch groups were introduced [Gri00] where it is shown that they constitute one of the three classes of just-infinite groups (infinite groups with no proper, infinite, homomorphic images). If a class of groups $\mathcal{C}$ is closed under homomorphic images and if it contains infinite, finitely generated examples then it contains just-infinite examples (this is because every infinite, finitely generated group has a just-infinite image). Such examples are minimal infinite examples in $\mathcal{C}$. We note that, for example, the group of intermediate growth constructed in [Gri80] is a branch automaton group that is a just-infinite 2-group. i.e., it is an infinite, finitely generated, torsion group that has no proper infinite quotients. The Hanoi Towers group [GŠ07] is a branch group that is not just infinite [GNŠ06b]. The iterated monodromy group $IMG(z^2+i)$ [GSŠ07] is a branch groups, while $\mathcal{B} = IMG(z^2-1)$ is not a branch group, but only weakly branch. More generally, it is shown in [BN07] that the iterated monodromy groups of post-critically finite quadratic maps are branch groups in the pre-periodic case and weakly branch groups in the periodic case (the case refers to the type of post-critical behavior).

We now define regular (weakly) branch groups. A level transitive group $G \leq \mathrm{Aut}(X^*)$ of k-ary tree automorphisms is a *regular branch group* over K if K is a normal subgroup of finite index in G such that $K \times \cdots \times K$ is geometrically contained in K. By definition, the subgroup K has the property that $K \times \cdots \times K$ is geometrically contained in K, denoted by $K \times \cdots \times K \preceq K$, if

$$K \times \cdots \times K \leq \psi(K \cap \mathrm{Stab}_G(1))$$

where ψ is the homomorphism $\psi : \mathrm{Stab}_G(1) \to \mathrm{Aut}(X^*) \times \cdots \times \mathrm{Aut}(X^*)$ given by $\psi(g) = (g_0, g_1, \ldots, g_{k-1})$. If instead of asking for K to have finite index in G we only require that K is nontrivial, we say that G is *regular weakly branch group* over K. Note that if G is level transitive and K is normal in G, in order to show that G is regular (weakly) branch group over K, it is sufficient to show that $K \times 1 \times \cdots \times 1 \preceq K$ (i.e. $K \times 1 \times \cdots \times 1 \leq \psi(K \cap \mathrm{Stab}_G(1))$). More on the class of branch group can be found in [Gri00] and [BGŠ03].

3 Automaton groups

The full group of tree automorphisms $Aut(X^*)$ is self-similar, since the section of every tree automorphism is just another tree automorphism. However, this group is rather large (uncountable). For various reasons, one may be interested in ways to define (construct) finitely generated self-similar groups. Automaton groups constitute a special class of finitely generated self-similar groups. We provide two ways of thinking about automaton groups. One is through finite wreath recursions and the other through finite automata.

Every finite system of recursive relations of the form

$$\begin{cases} s^{(1)} & = & \alpha_1\left(s_0^{(1)}, s_1^{(1)}, \ldots, s_{d-1}^{(1)}\right), \\ \ldots & & \\ s^{(k)} & = & \alpha_k\left(s_0^{(k)}, s_1^{(k)}, \ldots, s_{d-1}^{(k)}\right), \end{cases} \tag{5}$$

where each symbol $s_j^{(i)}$, $i = 1, \dots, k$, $j = 0, \dots, d-1$, is a symbol in the set of symbols $\{s^{(1)}, \dots, s^{(k)}\}$ and $\alpha_1, \dots, \alpha_k$ are permutations in $\mathsf{Sym}(X)$, has a unique solution in $\mathrm{Aut}(X^*)$ (in the sense that the above recursive relations represent the decompositions of the tree automorphisms $s^{(1)}, \dots, s^{(k)}$). Thus, the action of the automorphism defined by the symbol $s^{(i)}$ is given recursively by $s^{(i)}(xw) = \alpha_i(x) s_x^{(i)}(w)$.

The group G generated by the automorphisms $s^{(1)}, \dots, s^{(k)}$ is a finitely generated self-similar group of automorphisms of X^*. This follows since sections of products are products of sections (see (4)) and all sections of the generators of G are generators of G.

When a self-similar group is defined by a system of the form (5), we say that it is defined by a *wreath recursion*. We switch now the point of view from wreath recursions to invertible automata.

Definition 2. A *finite automaton* A is a 4-tuple $\mathcal{A} = (S, X, \pi, \tau)$ where S is a finite set of *states*, X is a finite *alphabet* of cardinality $d \geq 2$, $\pi : S \times X \to X$ is a map, called *output map*, and $\tau : S \times X \to S$ is a map, called *transition map*. If in addition, for each state s in S, the restriction $\pi_q : X \to X$ given by $\pi_s(x) = \pi(s, x)$ is a permutation in $\mathsf{Sym}(X)$, the automaton $\mathcal{A}$ is invertible.

In fact, we will be only concerned with finite invertible automata and, in the rest of the text, we will use the word automaton for such automata.

Each state s of the automaton $\mathcal{A}$ defines a tree automorphism of X^*, which we also denote by s. By definition, the root permutation of the automorphism s (defined by the state s) is the permutation π_s and the section of s at x is $\tau(s, x)$. Therefore

$$s(xw) = \pi_s(x) \tau(s, x)(w) \tag{6}$$

for every state s in S, letter x in X and word w over X.

Definition 3. Given an automaton $\mathcal{A} = (S, X, \pi, \tau)$, the group of tree automorphisms generated by the states of $\mathcal{A}$ is denoted by $G(\mathcal{A})$ and called the *automaton group* defined by $\mathcal{A}$.

The generating set S of the automaton group $G(\mathcal{A})$ generated by the automaton $\mathcal{A} = (S, X, \pi, \tau)$ is called the *standard* generating set of $G(\mathcal{A})$ and plays a distinguished role.

Directed graphs provide convenient representation of automata. The vertices of the graph, called *Moore diagram* of the automaton $\mathcal{A} = (S, X, \pi, \tau)$, are the states in S. Each state s is labeled by the root permutation $\alpha_s = \pi_s$ and, for each pair $(s, x) \in S \times X$, an edge labeled by x connects s to $s_x = \tau(s, x)$. Several examples are presented in Figure 1. The states of the 5-state automaton in the left half of the figure generate the group $\mathcal{G}$ of intermediate growth mentioned in the introduction (σ denotes the permutation exchanging 0 and 1, and 1 denotes the trivial vertex permutation). The top of the three 2-state automata on the right in Figure 1 is the so called *binary adding machine*, which generates the infinite cyclic group $\mathbb{Z}$. The other two automata both generate the Lamplighter group $L_2 = \mathbb{Z} \wr \mathbb{Z}/2\mathbb{Z} = \mathbb{Z} \ltimes (\bigoplus \mathbb{Z}/2\mathbb{Z})^{\mathbb{Z}}$ (see [GNS00]).

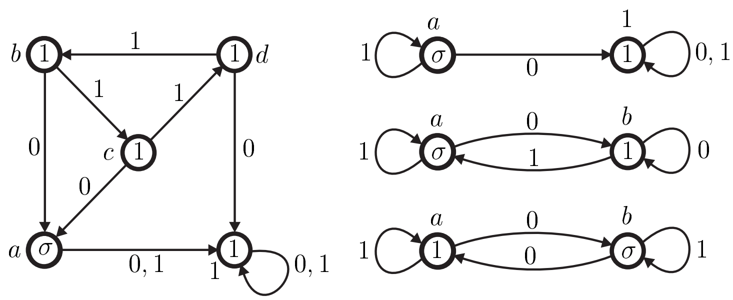

Figure 1: An automaton generating $\mathcal{G}$, the binary adding machine, and two Lamplighter automata

The corresponding wreath recursions for the adding machine and for the two automata generating the Lamplighter group are given by

$$
\begin{array}{lll}
a = \sigma(1, a) & \quad a = \sigma(b, a) & \quad a = (b, a) \\
1 = (1, 1) & \quad b = (b, a), & \quad b = \sigma(a, b)
\end{array}
\tag{7}
$$

respectively.

The class of *polynomially growing automata* was introduced by Sidki in [Sid00]. Sidki proved in [Sid04] that no group generated by such an automaton contains free subgroups of rank 2. As we already indicated in the introduction, for the subclass of so called bounded automata the corresponding groups are amenable [BKN]. Recall that an automaton $\mathcal{A}$ is called *bounded* if, for every state s of $\mathcal{A}$, the function $f_s(n)$ counting the number of active sections of s at level n is bounded (a state is *active* if its vertex permutation is nontrivial).

There are other classes of automata (and corresponding automaton groups) that deserve special attention. We end the section by mentioning several such classes.

The class of *linear automata* consists of automata in which both the set of states S and the alphabet X have a structure of a vector space (over a finite field) and both the output and the transition function are linear maps (see [GP72] and [Eil76]).

The class of *bi-invertible automata* consists of automata in which both the automaton and its dual are invertible. Some of the automata in our classification are bi-invertible, most notably the Aleshin-Vorobets-Vorobets automaton [2240] generating the free group F_3 of rank 3 and the Bellaterra automaton [846] generating the free product $C_2 * C_2 * C_2$. In fact, both of these have even stronger property of being *fully invertible*. Namely, not only the automaton and its dual are invertible, but also the dual of the inverse automaton is invertible.

Another important class is the class of automata satisfying the *open set* condition. Every automaton in this class contains a *trivial state* (a state defining

the trivial tree automorphism) and this state can be reached from any other state.

One may also study automata that are *strongly connected* (i.e. automata for which the corresponding Moore diagrams are strongly connected as directed graphs), automata in which no path contains more than one active state (such as the automaton defining $\mathcal{G}$ in Figure 1), and so on.

4 Schreier graphs

Let G be a group generated by a finite set S and let G act on a set Y. We denote by $\Gamma = \Gamma(G, S, Y)$ the *Schreier graph* of the action of G on Y. The vertices of Γ are the elements of Y. For every pair (s, y) in $S \times Y$ an edge labeled by s connects y to $s(y)$. An *orbital Schreier graph* of the action is the Schreier graph $\Gamma(G, S, y)$ of the action of G on the G-orbit of y, for some y in Y.

Let G be a group of tree automorphisms of X^* generated by a finite set S. The levels X^n, $n \geq 0$, are invariant under the action of G and we can consider the sequence of finite Schreier graphs $\Gamma_n(G, S) = \Gamma(G, S, X^n)$, $n \geq 0$. Let $\xi = x_1x_2x_3 \ldots \in X^\omega$ be an infinite ray. Then the pointed Schreier graphs $(\Gamma_n(G, S), x_1x_2 \ldots x_n)$ converge in the local topology (see [Gri84] or [GŻ99]) to the pointed orbital Schreier graph $(\Gamma(G, S, \xi), \xi)$.

Schreier graphs may be sometimes used to compute the spectrum of some operators related to the group. For a group of tree automorphisms G generated by a finite symmetric set S there is a natural unitary representation in the space of bounded linear operators $\mathcal{H} = B(L_2(X^\omega))$, given by $\pi_g(f)(x) = f(g^{-1}x)$ (the measure on the boundary X^ω is just the product measure associated to the uniform measure on X). Consider the spectrum of the operator

$$M = \frac{1}{|S|} \sum_{s \in S} \pi_s$$

corresponding to this unitary representation. The spectrum of M for a self-similar group G is approximated by the spectra of the finite dimensional operators induced by the action of G on the levels of the tree (see [BG00a]. Denote by $\mathcal{H}_n$ the subspace of $\mathcal{H} = B(L_2(X^\omega))$ spanned by the characteristic functions f_v, $v \in X^n$, of the cylindrical sets corresponding to the $|X|^n$ vertices on level n. The subspace $\mathcal{H}_n$ is invariant under the action of G and $\mathcal{H}_n \subset \mathcal{H}_{n+1}$. Denote by $\pi_g^{(n)}$ the restriction of π_g on $\mathcal{H}_n$. Then, for $n \geq 0$, the operator

$$M_n = \frac{1}{|S|} \sum_{s \in S} \pi_s^{(n)}$$

is finite dimensional. Moreover,

$$sp(M) = \overline{\bigcup_{n \geq 0} sp(M_n)},$$

i.e., the spectra of the operators M_n converge to the spectrum of M.

The table of information provided in Section 8 includes, in each case, the histogram of the spectrum of the operator M_9.

If P is the stabilizer of a point on the boundary X^ω, then one can consider the quasi-regular representation $\rho_{G/P}$ of G in $\ell^2(G/P)$.

Theorem 6 ([BG00a]). *If G is amenable or the Schreier graph G/P (the Schreier graph of the action of G on the cosets of P) is amenable then the spectrum of M and the spectrum of the quasi-regular representation $\rho_{G/P}$ coincide.*

In case the parabolic subgroup P is "small", the last result may be used to compute the spectrum of the Markov operator on the Cayley graph of the group. This approach was successfully used, for instance, to compute the spectrum of the Lamplighter group in [GŻ01] (see also [KSS06]).

5 Contracting groups, limit spaces, and iterated monodromy groups

Definition 4. A group G generated by an automaton over alphabet X is *contracting* if there exists a finite subset $\mathcal{N} \subset G$ such that for every $g \in G$ there exists n (generally depending on g) such that section g_v belongs to $\mathcal{N}$ for all words $v \in X^*$ of length at least n. The smallest set $\mathcal{N}$ with this property is called the *nucleus* of the group G.

The above definition makes sense for arbitrary self-similar groups — not necessarily automaton groups and, moreover, not necessarily finitely generated groups. In the case of an automaton group the contracting property may be equivalently stated as follows. An automaton group $G = G(\mathcal{A})$ is contracting if there exist constants κ, C, and N, with $0 \leq \kappa < 1$, such that $|g_v| \leq \kappa|g| + C$, for all vertices v of length at least N and $g \in G$ (the length is measured with respect to the standard generating set S consisting of the states of $\mathcal{A}$). The contraction property is a key ingredient in many inductive arguments and algorithms involving the decomposition $g = \alpha_g(g_0, \dots, g_{d-1})$. Indeed, the contraction property implies that, for all sufficiently long elements g, all sections of g at vertices on level at least N are strictly shorter than g.

Contracting groups have rich geometric structure. Each contracting group is the iterated monodromy group of its *limit dynamical system*. This system is an (orbispace) self-covering of the *limit space* of the group. The limit space is a limit of the graphs of the action of G on the levels X^n of the tree X^* and is defined in the following way.

Definition 5. Let G be a contracting group over X. Denote by $X^{-\omega}$ the space of all left-infinite sequences $\dots x_2 x_1$ of elements of X with the direct product (Tykhonoff) topology. We say that two sequences $\dots x_2 x_1$ and $\dots y_2 y_1$ are

asymptotically equivalent if there exists a sequences $g_k \in G$, assuming a finite set of values, and such that

$$g_k(x_k \dots x_1) = y_k \dots y_1$$

for all $k \geq 1$. The quotient of the space $X^{-\omega}$ by this equivalence relation is called the *limit space* of G.

The following proposition, proved in [Nek05] (Proposition 3.6.4) is a convenient way to compute the asymptotic equivalence.

Proposition 1. *Let a contracting group G be generated by a finite automaton A. Then the asymptotic equivalence is the equivalence relation generated by the set of pairs $(\dots x_2x_1, \dots y_2y_1)$ for which there exists a sequence g_k of states of A such that $g_k(x_k) = y_k$ and $g_k|_{x_k} = g_{k-1}$.*

The limit dynamical system is the map induced by the shift $\dots x_2x_1 \mapsto \dots x_3x_2$. The limit space is a compact metrizable topological space of finite topological dimension (see [Nek05], Theorem 3.6.3). If the group is self-replicating, then the limit space is locally connected and path connected.

The main tool of finding the limit space of a contracting group is realization of the group as the iterated monodromy group of an expanding partial orbispace self-covering. An exposition of the theory of such self-coverings is given in [Nek05]. In particular, if G is the iterated monodromy group of a post-critically finite complex rational function, then the limit space of G is homeomorphic to the Julia set of the function (see Theorems 5.5.3 and 6.4.4 of [Nek05]).

The limit space does not change when we pass from X to X^n in the natural way (we will change then the limit dynamical system to its nth iterate). It also does not change if we post-conjugate the wreath recursion by an element of the wreath product $Symm(X) \ltimes G^X$, i.e., conjugate the group G by an element of the form $\gamma = \pi(g_0\gamma, g_1\gamma)$, where $\pi \in Symm(X)$ and $g_0, g_1 \in G$.

The limit space can be also visualized using its subdivision into *tiles*. This method is especially effective, when the group is generated by bounded automata.

Definition 6. Let G be a contracting group. A *tile* $\mathcal{T}_G$ of G is the quotient of the space $X^{-\omega}$ by the equivalence relation, which identifies two sequences $\dots x_2x_1$ and $\dots y_2y_1$ if there exists a sequence $g_k \in G$ assuming a finite number of values and such that

$$g_k(x_k \dots x_1) = y_k \dots y_1, \quad g_k|_{x_k \dots x_1} = 1$$

for all k.

Again, an analog of Proposition 1 is true: the equivalence relation from Definition 6 is generated by the identifications $\dots x_2x_1 = \dots y_2y_1$ of sequences for which there exists a sequence $g_k, k = 0, 1, 2, \dots$ of elements of the nucleus such that $g_k(x_k) = y_k$, $g_k|_{x_k} = g_{k-1}$ and $g_0 = 1$.

Suppose that G satisfies the *open set condition*, i.e., the trivial state can be reached from any other state of the generating automaton. Then the *boundary* of the tile $\mathcal{T}_G$ is the image in $\mathcal{T}_G$ of the set of sequences $\ldots x_2 x_1$ such that there exists a sequence $g_k \in G$ assuming a finite number of values and such that $g_k|_{x_k \ldots x_1} \neq 1$. If G is generated by a finite symmetric set S, then it is sufficient to look for the sequence g_k inside S.

The limit space of G is obtained from the tile by some identifications of the points of the boundary. If the group G is generated by bounded automata, then its boundary consists of a finite number of points and it is not hard to identify them (i.e., to identify the sequences encoding them).

For $v \in X^n$ denote by $\mathcal{T}_G v$ the image of the cylindrical set $X^{-\omega}v$ in $\mathcal{T}_G$. It is easy to see that the map $\ldots x_2 x_1 \mapsto \ldots x_2 x_1 v$ induces a homeomorphism of $\mathcal{T}_G$ with $\mathcal{T}_G v$ and that

$$\mathcal{T}_G = \bigcup_{v \in X^n} \mathcal{T}_G v.$$

It is proved in [Nek05] that two pieces $\mathcal{T}_G v_1$ and $\mathcal{T}_G v_2$ intersect if and only if $g(v_1) = v_2$ for an element g of the nucleus of G and that they intersect only along images of the boundary of $\mathcal{T}_G$.

This suggests the following procedure of visualizing the limit space in the case of bounded automata. Identify the points of the boundary of the tile. We get a finite list B of points, represented by a finite list W of infinite sequences (some points may be represented by several sequences). Draw the tile as a graph with $|B|$ "boundary points" (vertices) and identify the boundary points with the points of B labeled by sequences W. Take now $|X|$ copies of this tile, corresponding to different letters of X. Append the corresponding letters $x \in X$ to the ends of the labels $w \in W$ of the boundary points of each of the copy of the tile. Some of the obtained labels will be related by the equivalence relation of Definition 6, i.e., represent the same points of the tile $\mathcal{T}_G$. Glue the corresponding points together. Some of the obtained labels will belong to W. These points will be the new boundary points. In this way we get a new graph with labeled boundary points. Repeat now the procedure several times, rescaling the graph in such a way that the original first order graphs become small. We will get in this way a graph resembling the tile $\mathcal{T}_G$ (see Chapter V in [Bon07] for more details). Making the necessary identifications of its boundary we get an approximation of the limit space of G. More details on this inductive approximation procedure can be found in [Nek05] Section 3.10.

The limit space of a finitely generated contracting self-similar group G can also be viewed as a hyperbolic boundary in the following way. For a given finite generating system S of G define the *self-similarity graph* $\Sigma(G, S)$ as the graph with set of vertices X^* in which two vertices $v_1, v_2 \in X^*$ are connected by an edge if and only if either $v_i = xv_j$, for some $x \in X$ (vertical edges), or $s(v_i) = v_j$ for some $s \in S$ (horizontal edges). In case of a contracting group, the self-similarity graph $\Sigma(G, S)$ is Gromov-hyperbolic and its hyperbolic boundary is homeomorphic to the limit space $\mathcal{J}_G$.

The iterated monodromy group (IMG) construction is dual to the limit space

construction. It may be defined for partial self-coverings of orbispaces, but we will only provide the definition in case of topological spaces, since we do not need the more general construction in this text (all iterated monodromy groups that appear later are related to partial self-coverings of the Riemann sphere).

Let $\mathcal{M}$ be a path connected and locally path connected topological space and let $\mathcal{M}_1$ be an open path connected subset of $\mathcal{M}$. Let $f : \mathcal{M}_1 \to \mathcal{M}$ be a d-fold covering. Denote by f^n the n-fold iteration of the map f. Then $f^n : \mathcal{M}_n \to \mathcal{M}$, where $\mathcal{M}_n = f^{-n}(\mathcal{M})$, is a d^n-fold covering.

Fix a base point $t \in \mathcal{M}$ and let T_t be the disjoint union of the sets $f^{-n}(t), n \geq 0$ (formally speaking, these sets may not be disjoint in $\mathcal{M}$). The set of pre-images T_t has a natural structure of a rooted d-ary tree. The base point t is the root, the vertices in f^{-n} constitute level n and every vertex z in $f^{-n}(t)$ is connected by an edge to $f(z)$ in $f^{-n+1}(t)$, for $n \geq 1$. The fundamental group $\pi_1(\mathcal{M}, t)$ acts naturally, through the monodromy action, on every level $f^{-n}(t)$ and, in fact, acts by automorphisms on T_t.

Definition 7. The *iterated monodromy group* $IMG(f)$ of the covering f is the quotient of the fundamental group $\pi_1(\mathcal{M}, t)$ by the kernel of its action on the tree of pre-images T_t.

6 Classification guide

Every 3-state automaton $\mathcal{A}$ with set of states $S = \{\mathbf{0}, \mathbf{1}, \mathbf{2}\}$ acting on the 2-letter alphabet $X = \{0, 1\}$ is assigned a unique number as follows. Given the wreath recursion

$$\begin{cases} \mathbf{0} = \sigma^{a_{11}}(a_{12}, a_{13}), \\ \mathbf{1} = \sigma^{a_{21}}(a_{22}, a_{23}), \\ \mathbf{2} = \sigma^{a_{31}}(a_{32}, a_{33}), \end{cases}$$

defining the automaton $\mathcal{A}$, where $a_{ij} \in \{\mathbf{0}, \mathbf{1}, \mathbf{2}\}$ for $j \neq 1$ and $a_{i1} \in \{0, 1\}$, $i = 1, 2, 3$, assign the number

$$\begin{aligned} &\mathrm{Number}(\mathcal{A}) = \\ &\qquad a_{12} + 3a_{13} + 9a_{22} + 27a_{23} + 81a_{32} + \\ &\qquad 243a_{33} + 729(a_{11} + 2a_{21} + 4a_{31}) + 1 \end{aligned}$$

to $\mathcal{A}$. With this agreement every $(3, 2)$-automaton is assigned a unique number in the range from 1 to 5832. The numbering of the automata is induced by the lexicographic ordering of all automata in the class. Each of the automata numbered 1 through 729 generates the trivial group, since all vertex permutations are trivial in this case. Each of the automata numbered 5104 through 5832 generates the cyclic group C_2 of order 2, since both states represent the automorphism that acts by changing all letters in every word over X. Therefore the nontrivial part of the classification is concerned with the automata numbered by 730 through 5103.

Denote by $\mathcal{A}_n$ the automaton numbered by n and by G_n the corresponding group of tree automorphisms. Sometimes we may use just the number to refer to the corresponding automaton or group.

The following three operations on automata do not change the isomorphism class of the group generated by the corresponding automaton (and do not change the action on the tree in essential way):

(i) passing to inverses of all generators,

(ii) permuting the states of the automaton,

(iii) permuting the alphabet letters.

Definition 8. Two automata $\mathcal{A}$ and $\mathcal{B}$ that can be obtained from one another by using a composition of the operations (i)–(iii), are called *symmetric.*

For instance, the two automata in the lower right part of Figure 1 are symmetric. The wreath recursion for the automaton obtained by permuting both the names of the states and the alphabet letters of the first of these two automata is

$$\begin{aligned} a &= \ \ (b, a) \\ b &= \sigma(b, a) \end{aligned}$$

and this wreath recursion describes exactly the inverses of the tree automorphism defining the second of the two automata.

Additional identifications can be made after automata minimization is applied.

Definition 9. If the minimization of an automaton $\mathcal{A}$ is symmetric to the minimization of an automaton $\mathcal{B}$, we say that the automata $\mathcal{A}$ and $\mathcal{B}$ are *minimally symmetric* and write $\mathcal{A} \sim \mathcal{B}$.

There are 194 classes of $(3, 2)$-automata that are pairwise not minimally symmetric. Of these, 10 are minimally symmetric to automata with fewer than 3 states and, as such, are subject of Theorem 7 ([GNS00], see below).

At present, it is known that there are no more than 122 non-isomorphic $(3, 2)$-automaton groups. Some information on these groups is given in Section 8.

The proofs of some particular properties of the 194 classes of non-equivalent automata (and in particular, all known isomorphisms) can be found in Section 9. The few general results that hold in the whole class were already mentioned in the introduction.

The table in Section 7 may be used to determine the equivalence and the group isomorphism class for each automaton. Every class is numbered by the smallest number of an automaton in the class. For instance, an entry such as $x \sim y \cong z$ means that the automata with the smallest number in the equivalence and the (known) isomorphism class of x are y and z, respectively. While the equivalence classes are easy to determine the isomorphism class is not. Therefore, there may still be some additional isomorphisms between some of the classes (which would eventually cause changes in the z numbers and consolidation of some of the current isomorphism classes).

If one is interested in some particular $(3, 2)$-automaton $\mathcal{A}$, we recommend the following procedure:

- Use the table in Section 7 to find numbers for the representatives of the equivalence and the isomorphism class of $\mathcal{A}$. Minimizing the automaton and finding the symmetry is a straightforward task, which is not presented here.

- Use Section 8 to find information on the group generated by $\mathcal{A}$ (more precisely, the isomorphic group generated by the chosen representative in the class).

- Use Section 9 to find the proof of the isomorphism and some known properties.

7 Table of equivalence classes (and known isomorphisms)

For explanation of the entries see Section 6.

1 through 729 $\sim 1 \simeq 1$,

730	$\sim$	730	$\cong$	730	769	$\sim$	767	$\cong$	731	808	$\sim$	804	$\cong$	731	847	$\sim$	847	$\cong$	847
731	$\sim$	731	$\cong$	731	770	$\sim$	770	$\cong$	730	809	$\sim$	807	$\cong$	771	848	$\sim$	848	$\cong$	750
732	$\sim$	731	$\cong$	731	771	$\sim$	771	$\cong$	771	810	$\sim$	810	$\cong$	802	849	$\sim$	849	$\cong$	849
733	$\sim$	731	$\cong$	731	772	$\sim$	768	$\cong$	731	811	$\sim$	748	$\cong$	748	850	$\sim$	848	$\cong$	750
734	$\sim$	734	$\cong$	730	773	$\sim$	771	$\cong$	771	812	$\sim$	750	$\cong$	750	851	$\sim$	851	$\cong$	847
735	$\sim$	734	$\cong$	730	774	$\sim$	774	$\cong$	730	813	$\sim$	749	$\cong$	749	852	$\sim$	852	$\cong$	852
736	$\sim$	731	$\cong$	731	775	$\sim$	775	$\cong$	775	814	$\sim$	750	$\cong$	750	853	$\sim$	849	$\cong$	849
737	$\sim$	734	$\cong$	730	776	$\sim$	776	$\cong$	776	815	$\sim$	756	$\cong$	748	854	$\sim$	852	$\cong$	852
738	$\sim$	734	$\cong$	730	777	$\sim$	777	$\cong$	777	816	$\sim$	753	$\cong$	753	855	$\sim$	855	$\cong$	847
739	$\sim$	739	$\cong$	739	778	$\sim$	776	$\cong$	776	817	$\sim$	749	$\cong$	749	856	$\sim$	856	$\cong$	856
740	$\sim$	740	$\cong$	740	779	$\sim$	779	$\cong$	779	818	$\sim$	753	$\cong$	753	857	$\sim$	857	$\cong$	857
741	$\sim$	741	$\cong$	741	780	$\sim$	780	$\cong$	780	819	$\sim$	752	$\cong$	752	858	$\sim$	858	$\cong$	858
742	$\sim$	740	$\cong$	740	781	$\sim$	777	$\cong$	777	820	$\sim$	820	$\cong$	820	859	$\sim$	857	$\cong$	857
743	$\sim$	743	$\cong$	739	782	$\sim$	780	$\cong$	780	821	$\sim$	821	$\cong$	821	860	$\sim$	860	$\cong$	860
744	$\sim$	744	$\cong$	744	783	$\sim$	783	$\cong$	775	822	$\sim$	821	$\cong$	821	861	$\sim$	861	$\cong$	861
745	$\sim$	741	$\cong$	741	784	$\sim$	748	$\cong$	748	823	$\sim$	821	$\cong$	821	862	$\sim$	858	$\cong$	858
746	$\sim$	744	$\cong$	744	785	$\sim$	749	$\cong$	749	824	$\sim$	824	$\cong$	820	863	$\sim$	861	$\cong$	861
747	$\sim$	747	$\cong$	739	786	$\sim$	750	$\cong$	750	825	$\sim$	824	$\cong$	820	864	$\sim$	864	$\cong$	864
748	$\sim$	748	$\cong$	748	787	$\sim$	749	$\cong$	749	826	$\sim$	821	$\cong$	821	865	$\sim$	865	$\cong$	820
749	$\sim$	749	$\cong$	749	788	$\sim$	752	$\cong$	752	827	$\sim$	824	$\cong$	820	866	$\sim$	866	$\cong$	866
750	$\sim$	750	$\cong$	750	789	$\sim$	753	$\cong$	753	828	$\sim$	824	$\cong$	820	867	$\sim$	866	$\cong$	866
751	$\sim$	749	$\cong$	749	790	$\sim$	750	$\cong$	750	829	$\sim$	820	$\cong$	820	868	$\sim$	866	$\cong$	866
752	$\sim$	752	$\cong$	752	791	$\sim$	753	$\cong$	753	830	$\sim$	821	$\cong$	821	869	$\sim$	869	$\cong$	869
753	$\sim$	753	$\cong$	753	792	$\sim$	756	$\cong$	748	831	$\sim$	821	$\cong$	821	870	$\sim$	870	$\cong$	870
754	$\sim$	750	$\cong$	750	793	$\sim$	775	$\cong$	775	832	$\sim$	821	$\cong$	821	871	$\sim$	866	$\cong$	866
755	$\sim$	753	$\cong$	753	794	$\sim$	776	$\cong$	776	833	$\sim$	824	$\cong$	820	872	$\sim$	870	$\cong$	870
756	$\sim$	756	$\cong$	748	795	$\sim$	777	$\cong$	777	834	$\sim$	824	$\cong$	820	873	$\sim$	869	$\cong$	869
757	$\sim$	739	$\cong$	739	796	$\sim$	776	$\cong$	776	835	$\sim$	821	$\cong$	821	874	$\sim$	874	$\cong$	874
758	$\sim$	740	$\cong$	740	797	$\sim$	779	$\cong$	779	836	$\sim$	824	$\cong$	820	875	$\sim$	875	$\cong$	875
759	$\sim$	741	$\cong$	741	798	$\sim$	780	$\cong$	780	837	$\sim$	824	$\cong$	820	876	$\sim$	876	$\cong$	876
760	$\sim$	740	$\cong$	740	799	$\sim$	777	$\cong$	777	838	$\sim$	838	$\cong$	838	877	$\sim$	875	$\cong$	875
761	$\sim$	743	$\cong$	739	800	$\sim$	780	$\cong$	780	839	$\sim$	839	$\cong$	821	878	$\sim$	878	$\cong$	878
762	$\sim$	744	$\cong$	744	801	$\sim$	783	$\cong$	775	840	$\sim$	840	$\cong$	840	879	$\sim$	879	$\cong$	879
763	$\sim$	741	$\cong$	741	802	$\sim$	802	$\cong$	802	841	$\sim$	839	$\cong$	821	880	$\sim$	876	$\cong$	876
764	$\sim$	744	$\cong$	744	803	$\sim$	803	$\cong$	771	842	$\sim$	842	$\cong$	838	881	$\sim$	879	$\cong$	879
765	$\sim$	747	$\cong$	739	804	$\sim$	804	$\cong$	731	843	$\sim$	843	$\cong$	843	882	$\sim$	882	$\cong$	882
766	$\sim$	766	$\cong$	730	805	$\sim$	803	$\cong$	771	844	$\sim$	840	$\cong$	840	883	$\sim$	883	$\cong$	883
767	$\sim$	767	$\cong$	731	806	$\sim$	806	$\cong$	802	845	$\sim$	843	$\cong$	843	884	$\sim$	884	$\cong$	884
768	$\sim$	768	$\cong$	731	807	$\sim$	807	$\cong$	771	846	$\sim$	846	$\cong$	846	885	$\sim$	885	$\cong$	885

886	$\sim$	884	$\cong$	884	932	$\sim$	932	$\cong$	820	978	$\sim$	753	$\cong$	753	1024	$\sim$	876	$\cong$	876
887	$\sim$	887	$\cong$	887	933	$\sim$	933	$\cong$	849	979	$\sim$	749	$\cong$	749	1025	$\sim$	879	$\cong$	879
888	$\sim$	888	$\cong$	888	934	$\sim$	930	$\cong$	821	980	$\sim$	753	$\cong$	753	1026	$\sim$	882	$\cong$	882
889	$\sim$	885	$\cong$	885	935	$\sim$	933	$\cong$	849	981	$\sim$	752	$\cong$	752	1027	$\sim$	820	$\cong$	820
890	$\sim$	888	$\cong$	888	936	$\sim$	936	$\cong$	820	982	$\sim$	838	$\cong$	838	1028	$\sim$	821	$\cong$	821
891	$\sim$	891	$\cong$	891	937	$\sim$	937	$\cong$	937	983	$\sim$	839	$\cong$	821	1029	$\sim$	821	$\cong$	821
892	$\sim$	739	$\cong$	739	938	$\sim$	938	$\cong$	938	984	$\sim$	840	$\cong$	840	1030	$\sim$	821	$\cong$	821
893	$\sim$	741	$\cong$	741	939	$\sim$	939	$\cong$	939	985	$\sim$	839	$\cong$	821	1031	$\sim$	824	$\cong$	820
894	$\sim$	740	$\cong$	740	940	$\sim$	938	$\cong$	938	986	$\sim$	842	$\cong$	838	1032	$\sim$	824	$\cong$	820
895	$\sim$	741	$\cong$	741	941	$\sim$	941	$\cong$	941	987	$\sim$	843	$\cong$	843	1033	$\sim$	821	$\cong$	821
896	$\sim$	747	$\cong$	739	942	$\sim$	942	$\cong$	942	988	$\sim$	840	$\cong$	840	1034	$\sim$	824	$\cong$	820
897	$\sim$	744	$\cong$	744	943	$\sim$	939	$\cong$	939	989	$\sim$	843	$\cong$	843	1035	$\sim$	824	$\cong$	820
898	$\sim$	740	$\cong$	740	944	$\sim$	942	$\cong$	942	990	$\sim$	846	$\cong$	846	1036	$\sim$	856	$\cong$	856
899	$\sim$	744	$\cong$	744	945	$\sim$	945	$\cong$	941	991	$\sim$	865	$\cong$	820	1037	$\sim$	857	$\cong$	857
900	$\sim$	743	$\cong$	739	946	$\sim$	838	$\cong$	838	992	$\sim$	866	$\cong$	866	1038	$\sim$	858	$\cong$	858
901	$\sim$	820	$\cong$	820	947	$\sim$	840	$\cong$	840	993	$\sim$	866	$\cong$	866	1039	$\sim$	857	$\cong$	857
902	$\sim$	821	$\cong$	821	948	$\sim$	839	$\cong$	821	994	$\sim$	866	$\cong$	866	1040	$\sim$	860	$\cong$	860
903	$\sim$	821	$\cong$	821	949	$\sim$	840	$\cong$	840	995	$\sim$	869	$\cong$	869	1041	$\sim$	861	$\cong$	861
904	$\sim$	821	$\cong$	821	950	$\sim$	846	$\cong$	846	996	$\sim$	870	$\cong$	870	1042	$\sim$	858	$\cong$	858
905	$\sim$	824	$\cong$	820	951	$\sim$	843	$\cong$	843	997	$\sim$	866	$\cong$	866	1043	$\sim$	861	$\cong$	861
906	$\sim$	824	$\cong$	820	952	$\sim$	839	$\cong$	821	998	$\sim$	870	$\cong$	870	1044	$\sim$	864	$\cong$	864
907	$\sim$	821	$\cong$	821	953	$\sim$	843	$\cong$	843	999	$\sim$	869	$\cong$	869	1045	$\sim$	883	$\cong$	883
908	$\sim$	824	$\cong$	820	954	$\sim$	842	$\cong$	838	1000	$\sim$	820	$\cong$	820	1046	$\sim$	884	$\cong$	884
909	$\sim$	824	$\cong$	820	955	$\sim$	955	$\cong$	937	1001	$\sim$	821	$\cong$	821	1047	$\sim$	885	$\cong$	885
910	$\sim$	820	$\cong$	820	956	$\sim$	956	$\cong$	956	1002	$\sim$	821	$\cong$	821	1048	$\sim$	884	$\cong$	884
911	$\sim$	821	$\cong$	821	957	$\sim$	957	$\cong$	957	1003	$\sim$	821	$\cong$	821	1049	$\sim$	887	$\cong$	887
912	$\sim$	821	$\cong$	821	958	$\sim$	956	$\cong$	956	1004	$\sim$	824	$\cong$	820	1050	$\sim$	888	$\cong$	888
913	$\sim$	821	$\cong$	821	959	$\sim$	959	$\cong$	959	1005	$\sim$	824	$\cong$	820	1051	$\sim$	885	$\cong$	885
914	$\sim$	824	$\cong$	820	960	$\sim$	960	$\cong$	960	1006	$\sim$	821	$\cong$	821	1052	$\sim$	888	$\cong$	888
915	$\sim$	824	$\cong$	820	961	$\sim$	957	$\cong$	957	1007	$\sim$	824	$\cong$	820	1053	$\sim$	891	$\cong$	891
916	$\sim$	821	$\cong$	821	962	$\sim$	960	$\cong$	960	1008	$\sim$	824	$\cong$	820	1054	$\sim$	802	$\cong$	802
917	$\sim$	824	$\cong$	820	963	$\sim$	963	$\cong$	963	1009	$\sim$	847	$\cong$	847	1055	$\sim$	804	$\cong$	731
918	$\sim$	824	$\cong$	820	964	$\sim$	964	$\cong$	739	1010	$\sim$	848	$\cong$	750	1056	$\sim$	803	$\cong$	771
919	$\sim$	919	$\cong$	820	965	$\sim$	965	$\cong$	965	1011	$\sim$	849	$\cong$	849	1057	$\sim$	804	$\cong$	731
920	$\sim$	920	$\cong$	920	966	$\sim$	966	$\cong$	966	1012	$\sim$	848	$\cong$	750	1058	$\sim$	810	$\cong$	802
921	$\sim$	920	$\cong$	920	967	$\sim$	965	$\cong$	965	1013	$\sim$	851	$\cong$	847	1059	$\sim$	807	$\cong$	771
922	$\sim$	920	$\cong$	920	968	$\sim$	968	$\cong$	968	1014	$\sim$	852	$\cong$	852	1060	$\sim$	803	$\cong$	771
923	$\sim$	923	$\cong$	923	969	$\sim$	969	$\cong$	969	1015	$\sim$	849	$\cong$	849	1061	$\sim$	807	$\cong$	771
924	$\sim$	924	$\cong$	870	970	$\sim$	966	$\cong$	966	1016	$\sim$	852	$\cong$	852	1062	$\sim$	806	$\cong$	802
925	$\sim$	920	$\cong$	920	971	$\sim$	969	$\cong$	969	1017	$\sim$	855	$\cong$	847	1063	$\sim$	964	$\cong$	739
926	$\sim$	924	$\cong$	870	972	$\sim$	972	$\cong$	739	1018	$\sim$	874	$\cong$	874	1064	$\sim$	966	$\cong$	966
927	$\sim$	923	$\cong$	923	973	$\sim$	748	$\cong$	748	1019	$\sim$	875	$\cong$	875	1065	$\sim$	965	$\cong$	965
928	$\sim$	928	$\cong$	820	974	$\sim$	750	$\cong$	750	1020	$\sim$	876	$\cong$	876	1066	$\sim$	966	$\cong$	966
929	$\sim$	929	$\cong$	929	975	$\sim$	749	$\cong$	749	1021	$\sim$	875	$\cong$	875	1067	$\sim$	972	$\cong$	739
930	$\sim$	930	$\cong$	821	976	$\sim$	750	$\cong$	750	1022	$\sim$	878	$\cong$	878	1068	$\sim$	969	$\cong$	969
931	$\sim$	929	$\cong$	929	977	$\sim$	756	$\cong$	748	1023	$\sim$	879	$\cong$	879	1069	$\sim$	965	$\cong$	965

1070	$\sim$	969	$\cong$	969
1071	$\sim$	968	$\cong$	968
1072	$\sim$	883	$\cong$	883
1073	$\sim$	885	$\cong$	885
1074	$\sim$	884	$\cong$	884
1075	$\sim$	885	$\cong$	885
1076	$\sim$	891	$\cong$	891
1077	$\sim$	888	$\cong$	888
1078	$\sim$	884	$\cong$	884
1079	$\sim$	888	$\cong$	888
1080	$\sim$	887	$\cong$	887
1081	$\sim$	964	$\cong$	739
1082	$\sim$	966	$\cong$	966
1083	$\sim$	965	$\cong$	965
1084	$\sim$	966	$\cong$	966
1085	$\sim$	972	$\cong$	739
1086	$\sim$	969	$\cong$	969
1087	$\sim$	965	$\cong$	965
1088	$\sim$	969	$\cong$	969
1089	$\sim$	968	$\cong$	968
1090	$\sim$	1090	$\cong$	1090
1091	$\sim$	1091	$\cong$	731
1092	$\sim$	1091	$\cong$	731
1093	$\sim$	1091	$\cong$	731
1094	$\sim$	1094	$\cong$	1090
1095	$\sim$	1094	$\cong$	1090
1096	$\sim$	1091	$\cong$	731
1097	$\sim$	1094	$\cong$	1090
1098	$\sim$	1094	$\cong$	1090
1099	$\sim$	1090	$\cong$	1090
1100	$\sim$	1091	$\cong$	731
1101	$\sim$	1091	$\cong$	731
1102	$\sim$	1091	$\cong$	731
1103	$\sim$	1094	$\cong$	1090
1104	$\sim$	1094	$\cong$	1090
1105	$\sim$	1091	$\cong$	731
1106	$\sim$	1094	$\cong$	1090
1107	$\sim$	1094	$\cong$	1090
1108	$\sim$	883	$\cong$	883
1109	$\sim$	885	$\cong$	885
1110	$\sim$	884	$\cong$	884
1111	$\sim$	885	$\cong$	885
1112	$\sim$	891	$\cong$	891
1113	$\sim$	888	$\cong$	888
1114	$\sim$	884	$\cong$	884
1115	$\sim$	888	$\cong$	888
1116	$\sim$	887	$\cong$	887
1117	$\sim$	1090	$\cong$	1090
1118	$\sim$	1091	$\cong$	731
1119	$\sim$	1091	$\cong$	731
1120	$\sim$	1091	$\cong$	731
1121	$\sim$	1094	$\cong$	1090
1122	$\sim$	1094	$\cong$	1090
1123	$\sim$	1091	$\cong$	731
1124	$\sim$	1094	$\cong$	1090
1125	$\sim$	1094	$\cong$	1090
1126	$\sim$	1090	$\cong$	1090
1127	$\sim$	1091	$\cong$	731
1128	$\sim$	1091	$\cong$	731
1129	$\sim$	1091	$\cong$	731
1130	$\sim$	1094	$\cong$	1090
1131	$\sim$	1094	$\cong$	1090
1132	$\sim$	1091	$\cong$	731
1133	$\sim$	1094	$\cong$	1090
1134	$\sim$	1094	$\cong$	1090
1135	$\sim$	775	$\cong$	775
1136	$\sim$	777	$\cong$	777
1137	$\sim$	776	$\cong$	776
1138	$\sim$	777	$\cong$	777
1139	$\sim$	783	$\cong$	775
1140	$\sim$	780	$\cong$	780
1141	$\sim$	776	$\cong$	776
1142	$\sim$	780	$\cong$	780
1143	$\sim$	779	$\cong$	779
1144	$\sim$	955	$\cong$	937
1145	$\sim$	957	$\cong$	957
1146	$\sim$	956	$\cong$	956
1147	$\sim$	957	$\cong$	957
1148	$\sim$	963	$\cong$	963
1149	$\sim$	960	$\cong$	960
1150	$\sim$	956	$\cong$	956
1151	$\sim$	960	$\cong$	960
1152	$\sim$	959	$\cong$	959
1153	$\sim$	874	$\cong$	874
1154	$\sim$	876	$\cong$	876
1155	$\sim$	875	$\cong$	875
1156	$\sim$	876	$\cong$	876
1157	$\sim$	882	$\cong$	882
1158	$\sim$	879	$\cong$	879
1159	$\sim$	875	$\cong$	875
1160	$\sim$	879	$\cong$	879
1161	$\sim$	878	$\cong$	878
1162	$\sim$	937	$\cong$	937
1163	$\sim$	939	$\cong$	939
1164	$\sim$	938	$\cong$	938
1165	$\sim$	939	$\cong$	939
1166	$\sim$	945	$\cong$	941
1167	$\sim$	942	$\cong$	942
1168	$\sim$	938	$\cong$	938
1169	$\sim$	942	$\cong$	942
1170	$\sim$	941	$\cong$	941
1171	$\sim$	1090	$\cong$	1090
1172	$\sim$	1091	$\cong$	731
1173	$\sim$	1091	$\cong$	731
1174	$\sim$	1091	$\cong$	731
1175	$\sim$	1094	$\cong$	1090
1176	$\sim$	1094	$\cong$	1090
1177	$\sim$	1091	$\cong$	731
1178	$\sim$	1094	$\cong$	1090
1179	$\sim$	1094	$\cong$	1090
1180	$\sim$	1090	$\cong$	1090
1181	$\sim$	1091	$\cong$	731
1182	$\sim$	1091	$\cong$	731
1183	$\sim$	1091	$\cong$	731
1184	$\sim$	1094	$\cong$	1090
1185	$\sim$	1094	$\cong$	1090
1186	$\sim$	1091	$\cong$	731
1187	$\sim$	1094	$\cong$	1090
1188	$\sim$	1094	$\cong$	1090
1189	$\sim$	856	$\cong$	856
1190	$\sim$	858	$\cong$	858
1191	$\sim$	857	$\cong$	857
1192	$\sim$	858	$\cong$	858
1193	$\sim$	864	$\cong$	864
1194	$\sim$	861	$\cong$	861
1195	$\sim$	857	$\cong$	857
1196	$\sim$	861	$\cong$	861
1197	$\sim$	860	$\cong$	860
1198	$\sim$	1090	$\cong$	1090
1199	$\sim$	1091	$\cong$	731
1200	$\sim$	1091	$\cong$	731
1201	$\sim$	1091	$\cong$	731
1202	$\sim$	1094	$\cong$	1090
1203	$\sim$	1094	$\cong$	1090
1204	$\sim$	1091	$\cong$	731
1205	$\sim$	1094	$\cong$	1090
1206	$\sim$	1094	$\cong$	1090
1207	$\sim$	1090	$\cong$	1090
1208	$\sim$	1091	$\cong$	731
1209	$\sim$	1091	$\cong$	731
1210	$\sim$	1091	$\cong$	731
1211	$\sim$	1094	$\cong$	1090
1212	$\sim$	1094	$\cong$	1090
1213	$\sim$	1091	$\cong$	731
1214	$\sim$	1094	$\cong$	1090
1215	$\sim$	1094	$\cong$	1090
1216	$\sim$	739	$\cong$	739
1217	$\sim$	741	$\cong$	741
1218	$\sim$	740	$\cong$	740
1219	$\sim$	741	$\cong$	741
1220	$\sim$	747	$\cong$	739
1221	$\sim$	744	$\cong$	744
1222	$\sim$	740	$\cong$	740
1223	$\sim$	744	$\cong$	744
1224	$\sim$	743	$\cong$	739
1225	$\sim$	919	$\cong$	820
1226	$\sim$	920	$\cong$	920
1227	$\sim$	920	$\cong$	920
1228	$\sim$	920	$\cong$	920
1229	$\sim$	923	$\cong$	923
1230	$\sim$	924	$\cong$	870
1231	$\sim$	920	$\cong$	920
1232	$\sim$	924	$\cong$	870
1233	$\sim$	923	$\cong$	923
1234	$\sim$	838	$\cong$	838
1235	$\sim$	840	$\cong$	840
1236	$\sim$	839	$\cong$	821
1237	$\sim$	840	$\cong$	840
1238	$\sim$	846	$\cong$	846
1239	$\sim$	843	$\cong$	843
1240	$\sim$	839	$\cong$	821
1241	$\sim$	843	$\cong$	843
1242	$\sim$	842	$\cong$	838
1243	$\sim$	820	$\cong$	820
1244	$\sim$	821	$\cong$	821
1245	$\sim$	821	$\cong$	821
1246	$\sim$	821	$\cong$	821
1247	$\sim$	824	$\cong$	820
1248	$\sim$	824	$\cong$	820
1249	$\sim$	821	$\cong$	821
1250	$\sim$	824	$\cong$	820
1251	$\sim$	824	$\cong$	820
1252	$\sim$	928	$\cong$	820
1253	$\sim$	929	$\cong$	929

$1254 \sim 930 \cong 821$	$1300 \sim 777 \cong 777$	$1346 \sim 1094 \cong 1090$	$1392 \sim 933 \cong 849$
$1255 \sim 929 \cong 929$	$1301 \sim 783 \cong 775$	$1347 \sim 1094 \cong 1090$	$1393 \sim 929 \cong 929$
$1256 \sim 932 \cong 820$	$1302 \sim 780 \cong 780$	$1348 \sim 1091 \cong 731$	$1394 \sim 933 \cong 849$
$1257 \sim 933 \cong 849$	$1303 \sim 776 \cong 776$	$1349 \sim 1094 \cong 1090$	$1395 \sim 932 \cong 820$
$1258 \sim 930 \cong 821$	$1304 \sim 780 \cong 780$	$1350 \sim 1094 \cong 1090$	$1396 \sim 847 \cong 847$
$1259 \sim 933 \cong 849$	$1305 \sim 779 \cong 779$	$1351 \sim 874 \cong 874$	$1397 \sim 849 \cong 849$
$1260 \sim 936 \cong 820$	$1306 \sim 937 \cong 937$	$1352 \sim 876 \cong 876$	$1398 \sim 848 \cong 750$
$1261 \sim 955 \cong 937$	$1307 \sim 939 \cong 939$	$1353 \sim 875 \cong 875$	$1399 \sim 849 \cong 849$
$1262 \sim 956 \cong 956$	$1308 \sim 938 \cong 938$	$1354 \sim 876 \cong 876$	$1400 \sim 855 \cong 847$
$1263 \sim 957 \cong 957$	$1309 \sim 939 \cong 939$	$1355 \sim 882 \cong 882$	$1401 \sim 852 \cong 852$
$1264 \sim 956 \cong 956$	$1310 \sim 945 \cong 941$	$1356 \sim 879 \cong 879$	$1402 \sim 848 \cong 750$
$1265 \sim 959 \cong 959$	$1311 \sim 942 \cong 942$	$1357 \sim 875 \cong 875$	$1403 \sim 852 \cong 852$
$1266 \sim 960 \cong 960$	$1312 \sim 938 \cong 938$	$1358 \sim 879 \cong 879$	$1404 \sim 851 \cong 847$
$1267 \sim 957 \cong 957$	$1313 \sim 942 \cong 942$	$1359 \sim 878 \cong 878$	$1405 \sim 928 \cong 820$
$1268 \sim 960 \cong 960$	$1314 \sim 941 \cong 941$	$1360 \sim 1090 \cong 1090$	$1406 \sim 930 \cong 821$
$1269 \sim 963 \cong 963$	$1315 \sim 856 \cong 856$	$1361 \sim 1091 \cong 731$	$1407 \sim 929 \cong 929$
$1270 \sim 820 \cong 820$	$1316 \sim 858 \cong 858$	$1362 \sim 1091 \cong 731$	$1408 \sim 930 \cong 821$
$1271 \sim 821 \cong 821$	$1317 \sim 857 \cong 857$	$1363 \sim 1091 \cong 731$	$1409 \sim 936 \cong 820$
$1272 \sim 821 \cong 821$	$1318 \sim 858 \cong 858$	$1364 \sim 1094 \cong 1090$	$1410 \sim 933 \cong 849$
$1273 \sim 821 \cong 821$	$1319 \sim 864 \cong 864$	$1365 \sim 1094 \cong 1090$	$1411 \sim 929 \cong 929$
$1274 \sim 824 \cong 820$	$1320 \sim 861 \cong 861$	$1366 \sim 1091 \cong 731$	$1412 \sim 933 \cong 849$
$1275 \sim 824 \cong 820$	$1321 \sim 857 \cong 857$	$1367 \sim 1094 \cong 1090$	$1413 \sim 932 \cong 820$
$1276 \sim 821 \cong 821$	$1322 \sim 861 \cong 861$	$1368 \sim 1094 \cong 1090$	$1414 \sim 1090 \cong 1090$
$1277 \sim 824 \cong 820$	$1323 \sim 860 \cong 860$	$1369 \sim 1090 \cong 1090$	$1415 \sim 1091 \cong 731$
$1278 \sim 824 \cong 820$	$1324 \sim 955 \cong 937$	$1370 \sim 1091 \cong 731$	$1416 \sim 1091 \cong 731$
$1279 \sim 937 \cong 937$	$1325 \sim 957 \cong 957$	$1371 \sim 1091 \cong 731$	$1417 \sim 1091 \cong 731$
$1280 \sim 938 \cong 938$	$1326 \sim 956 \cong 956$	$1372 \sim 1091 \cong 731$	$1418 \sim 1094 \cong 1090$
$1281 \sim 939 \cong 939$	$1327 \sim 957 \cong 957$	$1373 \sim 1094 \cong 1090$	$1419 \sim 1094 \cong 1090$
$1282 \sim 938 \cong 938$	$1328 \sim 963 \cong 963$	$1374 \sim 1094 \cong 1090$	$1420 \sim 1091 \cong 731$
$1283 \sim 941 \cong 941$	$1329 \sim 960 \cong 960$	$1375 \sim 1091 \cong 731$	$1421 \sim 1094 \cong 1090$
$1284 \sim 942 \cong 942$	$1330 \sim 956 \cong 956$	$1376 \sim 1094 \cong 1090$	$1422 \sim 1094 \cong 1090$
$1285 \sim 939 \cong 939$	$1331 \sim 960 \cong 960$	$1377 \sim 1094 \cong 1090$	$1423 \sim 1090 \cong 1090$
$1286 \sim 942 \cong 942$	$1332 \sim 959 \cong 959$	$1378 \sim 766 \cong 730$	$1424 \sim 1091 \cong 731$
$1287 \sim 945 \cong 941$	$1333 \sim 1090 \cong 1090$	$1379 \sim 768 \cong 731$	$1425 \sim 1091 \cong 731$
$1288 \sim 964 \cong 739$	$1334 \sim 1091 \cong 731$	$1380 \sim 767 \cong 731$	$1426 \sim 1091 \cong 731$
$1289 \sim 965 \cong 965$	$1335 \sim 1091 \cong 731$	$1381 \sim 768 \cong 731$	$1427 \sim 1094 \cong 1090$
$1290 \sim 966 \cong 966$	$1336 \sim 1091 \cong 731$	$1382 \sim 774 \cong 730$	$1428 \sim 1094 \cong 1090$
$1291 \sim 965 \cong 965$	$1337 \sim 1094 \cong 1090$	$1383 \sim 771 \cong 771$	$1429 \sim 1091 \cong 731$
$1292 \sim 968 \cong 968$	$1338 \sim 1094 \cong 1090$	$1384 \sim 767 \cong 731$	$1430 \sim 1094 \cong 1090$
$1293 \sim 969 \cong 969$	$1339 \sim 1091 \cong 731$	$1385 \sim 771 \cong 771$	$1431 \sim 1094 \cong 1090$
$1294 \sim 966 \cong 966$	$1340 \sim 1094 \cong 1090$	$1386 \sim 770 \cong 730$	$1432 \sim 847 \cong 847$
$1295 \sim 969 \cong 969$	$1341 \sim 1094 \cong 1090$	$1387 \sim 928 \cong 820$	$1433 \sim 849 \cong 849$
$1296 \sim 972 \cong 739$	$1342 \sim 1090 \cong 1090$	$1388 \sim 930 \cong 821$	$1434 \sim 848 \cong 750$
$1297 \sim 775 \cong 775$	$1343 \sim 1091 \cong 731$	$1389 \sim 929 \cong 929$	$1435 \sim 849 \cong 849$
$1298 \sim 777 \cong 777$	$1344 \sim 1091 \cong 731$	$1390 \sim 930 \cong 821$	$1436 \sim 855 \cong 847$
$1299 \sim 776 \cong 776$	$1345 \sim 1091 \cong 731$	$1391 \sim 936 \cong 820$	$1437 \sim 852 \cong 852$

$1438 \sim 848 \cong 750$	$1484 \sim 888 \cong 888$	$1530 \sim 1091 \cong 731$	$1576 \sim 847 \cong 847$
$1439 \sim 852 \cong 852$	$1485 \sim 1094 \cong 1090$	$1531 \sim 1094 \cong 1090$	$1577 \sim 820 \cong 820$
$1440 \sim 851 \cong 847$	$1486 \sim 1091 \cong 731$	$1532 \sim 968 \cong 968$	$1578 \sim 874 \cong 874$
$1441 \sim 1090 \cong 1090$	$1487 \sim 966 \cong 966$	$1533 \sim 1094 \cong 1090$	$1579 \sim 838 \cong 838$
$1442 \sim 1091 \cong 731$	$1488 \sim 1091 \cong 731$	$1534 \sim 968 \cong 968$	$1580 \sim 748 \cong 748$
$1443 \sim 1091 \cong 731$	$1489 \sim 966 \cong 966$	$1535 \sim 806 \cong 802$	$1581 \sim 865 \cong 820$
$1444 \sim 1091 \cong 731$	$1490 \sim 804 \cong 731$	$1536 \sim 887 \cong 887$	$1582 \sim 856 \cong 856$
$1445 \sim 1094 \cong 1090$	$1491 \sim 885 \cong 885$	$1537 \sim 1094 \cong 1090$	$1583 \sim 820 \cong 820$
$1446 \sim 1094 \cong 1090$	$1492 \sim 1091 \cong 731$	$1538 \sim 887 \cong 887$	$1584 \sim 883 \cong 883$
$1447 \sim 1091 \cong 731$	$1493 \sim 885 \cong 885$	$1539 \sim 1094 \cong 1090$	$1585 \sim 849 \cong 849$
$1448 \sim 1094 \cong 1090$	$1494 \sim 1091 \cong 731$	$1540 \sim 851 \cong 847$	$1586 \sim 821 \cong 821$
$1449 \sim 1094 \cong 1090$	$1495 \sim 1090 \cong 1090$	$1541 \sim 824 \cong 820$	$1587 \sim 876 \cong 876$
$1450 \sim 1090 \cong 1090$	$1496 \sim 964 \cong 739$	$1542 \sim 878 \cong 878$	$1588 \sim 840 \cong 840$
$1451 \sim 1091 \cong 731$	$1497 \sim 1090 \cong 1090$	$1543 \sim 842 \cong 838$	$1589 \sim 749 \cong 749$
$1452 \sim 1091 \cong 731$	$1498 \sim 964 \cong 739$	$1544 \sim 756 \cong 748$	$1590 \sim 866 \cong 866$
$1453 \sim 1091 \cong 731$	$1499 \sim 802 \cong 802$	$1545 \sim 869 \cong 869$	$1591 \sim 858 \cong 858$
$1454 \sim 1094 \cong 1090$	$1500 \sim 883 \cong 883$	$1546 \sim 860 \cong 860$	$1592 \sim 821 \cong 821$
$1455 \sim 1094 \cong 1090$	$1501 \sim 1090 \cong 1090$	$1547 \sim 824 \cong 820$	$1593 \sim 885 \cong 885$
$1456 \sim 1091 \cong 731$	$1502 \sim 883 \cong 883$	$1548 \sim 887 \cong 887$	$1594 \sim 852 \cong 852$
$1457 \sim 1094 \cong 1090$	$1503 \sim 1090 \cong 1090$	$1549 \sim 848 \cong 750$	$1595 \sim 824 \cong 820$
$1458 \sim 1094 \cong 1090$	$1504 \sim 1091 \cong 731$	$1550 \sim 821 \cong 821$	$1596 \sim 879 \cong 879$
$1459 \sim 1094 \cong 1090$	$1505 \sim 965 \cong 965$	$1551 \sim 875 \cong 875$	$1597 \sim 843 \cong 843$
$1460 \sim 972 \cong 739$	$1506 \sim 1091 \cong 731$	$1552 \sim 839 \cong 821$	$1598 \sim 753 \cong 753$
$1461 \sim 1094 \cong 1090$	$1507 \sim 965 \cong 965$	$1553 \sim 750 \cong 750$	$1599 \sim 870 \cong 870$
$1462 \sim 972 \cong 739$	$1508 \sim 803 \cong 771$	$1554 \sim 866 \cong 866$	$1600 \sim 861 \cong 861$
$1463 \sim 810 \cong 802$	$1509 \sim 884 \cong 884$	$1555 \sim 857 \cong 857$	$1601 \sim 824 \cong 820$
$1464 \sim 891 \cong 891$	$1510 \sim 1091 \cong 731$	$1556 \sim 821 \cong 821$	$1602 \sim 888 \cong 888$
$1465 \sim 1094 \cong 1090$	$1511 \sim 884 \cong 884$	$1557 \sim 884 \cong 884$	$1603 \sim 849 \cong 849$
$1466 \sim 891 \cong 891$	$1512 \sim 1091 \cong 731$	$1558 \sim 852 \cong 852$	$1604 \sim 821 \cong 821$
$1467 \sim 1094 \cong 1090$	$1513 \sim 1094 \cong 1090$	$1559 \sim 824 \cong 820$	$1605 \sim 876 \cong 876$
$1468 \sim 1091 \cong 731$	$1514 \sim 969 \cong 969$	$1560 \sim 879 \cong 879$	$1606 \sim 840 \cong 840$
$1469 \sim 966 \cong 966$	$1515 \sim 1094 \cong 1090$	$1561 \sim 843 \cong 843$	$1607 \sim 749 \cong 749$
$1470 \sim 1091 \cong 731$	$1516 \sim 969 \cong 969$	$1562 \sim 753 \cong 753$	$1608 \sim 866 \cong 866$
$1471 \sim 966 \cong 966$	$1517 \sim 807 \cong 771$	$1563 \sim 870 \cong 870$	$1609 \sim 858 \cong 858$
$1472 \sim 804 \cong 731$	$1518 \sim 888 \cong 888$	$1564 \sim 861 \cong 861$	$1610 \sim 821 \cong 821$
$1473 \sim 885 \cong 885$	$1519 \sim 1094 \cong 1090$	$1565 \sim 824 \cong 820$	$1611 \sim 885 \cong 885$
$1474 \sim 1091 \cong 731$	$1520 \sim 888 \cong 888$	$1566 \sim 888 \cong 888$	$1612 \sim 855 \cong 847$
$1475 \sim 885 \cong 885$	$1521 \sim 1094 \cong 1090$	$1567 \sim 848 \cong 750$	$1613 \sim 824 \cong 820$
$1476 \sim 1091 \cong 731$	$1522 \sim 1091 \cong 731$	$1568 \sim 821 \cong 821$	$1614 \sim 882 \cong 882$
$1477 \sim 1094 \cong 1090$	$1523 \sim 965 \cong 965$	$1569 \sim 875 \cong 875$	$1615 \sim 846 \cong 846$
$1478 \sim 969 \cong 969$	$1524 \sim 1091 \cong 731$	$1570 \sim 839 \cong 821$	$1616 \sim 752 \cong 752$
$1479 \sim 1094 \cong 1090$	$1525 \sim 965 \cong 965$	$1571 \sim 750 \cong 750$	$1617 \sim 869 \cong 869$
$1480 \sim 969 \cong 969$	$1526 \sim 803 \cong 771$	$1572 \sim 866 \cong 866$	$1618 \sim 864 \cong 864$
$1481 \sim 807 \cong 771$	$1527 \sim 884 \cong 884$	$1573 \sim 857 \cong 857$	$1619 \sim 824 \cong 820$
$1482 \sim 888 \cong 888$	$1528 \sim 1091 \cong 731$	$1574 \sim 821 \cong 821$	$1620 \sim 891 \cong 891$
$1483 \sim 1094 \cong 1090$	$1529 \sim 884 \cong 884$	$1575 \sim 884 \cong 884$	$1621 \sim 1094 \cong 1090$

$1622 \sim 945 \cong 941$	$1668 \sim 1091 \cong 731$	$1714 \sim 821 \cong 821$	$1760 \sim 753 \cong 753$
$1623 \sim 1094 \cong 1090$	$1669 \sim 956 \cong 956$	$1715 \sim 750 \cong 750$	$1761 \sim 824 \cong 820$
$1624 \sim 963 \cong 963$	$1670 \sim 776 \cong 776$	$1716 \sim 821 \cong 821$	$1762 \sim 879 \cong 879$
$1625 \sim 783 \cong 775$	$1671 \sim 875 \cong 875$	$1717 \sim 875 \cong 875$	$1763 \sim 870 \cong 870$
$1626 \sim 882 \cong 882$	$1672 \sim 1091 \cong 731$	$1718 \sim 866 \cong 866$	$1764 \sim 888 \cong 888$
$1627 \sim 1094 \cong 1090$	$1673 \sim 857 \cong 857$	$1719 \sim 884 \cong 884$	$1765 \sim 849 \cong 849$
$1628 \sim 864 \cong 864$	$1674 \sim 1091 \cong 731$	$1720 \sim 852 \cong 852$	$1766 \sim 840 \cong 840$
$1629 \sim 1094 \cong 1090$	$1675 \sim 1094 \cong 1090$	$1721 \sim 843 \cong 843$	$1767 \sim 858 \cong 858$
$1630 \sim 1091 \cong 731$	$1676 \sim 942 \cong 942$	$1722 \sim 861 \cong 861$	$1768 \sim 821 \cong 821$
$1631 \sim 939 \cong 939$	$1677 \sim 1094 \cong 1090$	$1723 \sim 824 \cong 820$	$1769 \sim 749 \cong 749$
$1632 \sim 1091 \cong 731$	$1678 \sim 960 \cong 960$	$1724 \sim 753 \cong 753$	$1770 \sim 821 \cong 821$
$1633 \sim 957 \cong 957$	$1679 \sim 780 \cong 780$	$1725 \sim 824 \cong 820$	$1771 \sim 876 \cong 876$
$1634 \sim 777 \cong 777$	$1680 \sim 879 \cong 879$	$1726 \sim 879 \cong 879$	$1772 \sim 866 \cong 866$
$1635 \sim 876 \cong 876$	$1681 \sim 1094 \cong 1090$	$1727 \sim 870 \cong 870$	$1773 \sim 885 \cong 885$
$1636 \sim 1091 \cong 731$	$1682 \sim 861 \cong 861$	$1728 \sim 888 \cong 888$	$1774 \sim 855 \cong 847$
$1637 \sim 858 \cong 858$	$1683 \sim 1094 \cong 1090$	$1729 \sim 848 \cong 750$	$1775 \sim 846 \cong 846$
$1638 \sim 1091 \cong 731$	$1684 \sim 1091 \cong 731$	$1730 \sim 839 \cong 821$	$1776 \sim 864 \cong 864$
$1639 \sim 1094 \cong 1090$	$1685 \sim 938 \cong 938$	$1731 \sim 857 \cong 857$	$1777 \sim 824 \cong 820$
$1640 \sim 942 \cong 942$	$1686 \sim 1091 \cong 731$	$1732 \sim 821 \cong 821$	$1778 \sim 752 \cong 752$
$1641 \sim 1094 \cong 1090$	$1687 \sim 956 \cong 956$	$1733 \sim 750 \cong 750$	$1779 \sim 824 \cong 820$
$1642 \sim 960 \cong 960$	$1688 \sim 776 \cong 776$	$1734 \sim 821 \cong 821$	$1780 \sim 882 \cong 882$
$1643 \sim 780 \cong 780$	$1689 \sim 875 \cong 875$	$1735 \sim 875 \cong 875$	$1781 \sim 869 \cong 869$
$1644 \sim 879 \cong 879$	$1690 \sim 1091 \cong 731$	$1736 \sim 866 \cong 866$	$1782 \sim 891 \cong 891$
$1645 \sim 1094 \cong 1090$	$1691 \sim 857 \cong 857$	$1737 \sim 884 \cong 884$	$1783 \sim 770 \cong 730$
$1646 \sim 861 \cong 861$	$1692 \sim 1091 \cong 731$	$1738 \sim 847 \cong 847$	$1784 \sim 743 \cong 739$
$1647 \sim 1094 \cong 1090$	$1693 \sim 1094 \cong 1090$	$1739 \sim 838 \cong 838$	$1785 \sim 779 \cong 779$
$1648 \sim 1091 \cong 731$	$1694 \sim 941 \cong 941$	$1740 \sim 856 \cong 856$	$1786 \sim 743 \cong 739$
$1649 \sim 939 \cong 939$	$1695 \sim 1094 \cong 1090$	$1741 \sim 820 \cong 820$	$1787 \sim 734 \cong 730$
$1650 \sim 1091 \cong 731$	$1696 \sim 959 \cong 959$	$1742 \sim 748 \cong 748$	$1788 \sim 752 \cong 752$
$1651 \sim 957 \cong 957$	$1697 \sim 779 \cong 779$	$1743 \sim 820 \cong 820$	$1789 \sim 779 \cong 779$
$1652 \sim 777 \cong 777$	$1698 \sim 878 \cong 878$	$1744 \sim 874 \cong 874$	$1790 \sim 752 \cong 752$
$1653 \sim 876 \cong 876$	$1699 \sim 1094 \cong 1090$	$1745 \sim 865 \cong 820$	$1791 \sim 806 \cong 802$
$1654 \sim 1091 \cong 731$	$1700 \sim 860 \cong 860$	$1746 \sim 883 \cong 883$	$1792 \sim 767 \cong 731$
$1655 \sim 858 \cong 858$	$1701 \sim 1094 \cong 1090$	$1747 \sim 849 \cong 849$	$1793 \sim 740 \cong 740$
$1656 \sim 1091 \cong 731$	$1702 \sim 851 \cong 847$	$1748 \sim 840 \cong 840$	$1794 \sim 776 \cong 776$
$1657 \sim 1090 \cong 1090$	$1703 \sim 842 \cong 838$	$1749 \sim 858 \cong 858$	$1795 \sim 740 \cong 740$
$1658 \sim 937 \cong 937$	$1704 \sim 860 \cong 860$	$1750 \sim 821 \cong 821$	$1796 \sim 731 \cong 731$
$1659 \sim 1090 \cong 1090$	$1705 \sim 824 \cong 820$	$1751 \sim 749 \cong 749$	$1797 \sim 749 \cong 749$
$1660 \sim 955 \cong 937$	$1706 \sim 756 \cong 748$	$1752 \sim 821 \cong 821$	$1798 \sim 776 \cong 776$
$1661 \sim 775 \cong 775$	$1707 \sim 824 \cong 820$	$1753 \sim 876 \cong 876$	$1799 \sim 749 \cong 749$
$1662 \sim 874 \cong 874$	$1708 \sim 878 \cong 878$	$1754 \sim 866 \cong 866$	$1800 \sim 803 \cong 771$
$1663 \sim 1090 \cong 1090$	$1709 \sim 869 \cong 869$	$1755 \sim 885 \cong 885$	$1801 \sim 771 \cong 771$
$1664 \sim 856 \cong 856$	$1710 \sim 887 \cong 887$	$1756 \sim 852 \cong 852$	$1802 \sim 744 \cong 744$
$1665 \sim 1090 \cong 1090$	$1711 \sim 848 \cong 750$	$1757 \sim 843 \cong 843$	$1803 \sim 780 \cong 780$
$1666 \sim 1091 \cong 731$	$1712 \sim 839 \cong 821$	$1758 \sim 861 \cong 861$	$1804 \sim 744 \cong 744$
$1667 \sim 938 \cong 938$	$1713 \sim 857 \cong 857$	$1759 \sim 824 \cong 820$	$1805 \sim 734 \cong 730$

$1806 \sim 753 \cong 753$	$1852 \sim 777 \cong 777$	$1898 \sim 840 \cong 840$	$1944 \sim 972 \cong 739$
$1807 \sim 780 \cong 780$	$1853 \sim 750 \cong 750$	$1899 \sim 965 \cong 965$	$1945 \sim 1094 \cong 1090$
$1808 \sim 753 \cong 753$	$1854 \sim 804 \cong 731$	$1900 \sim 928 \cong 820$	$1946 \sim 963 \cong 963$
$1809 \sim 807 \cong 771$	$1855 \sim 774 \cong 730$	$1901 \sim 919 \cong 820$	$1947 \sim 1094 \cong 1090$
$1810 \sim 767 \cong 731$	$1856 \sim 747 \cong 739$	$1902 \sim 937 \cong 937$	$1948 \sim 945 \cong 941$
$1811 \sim 740 \cong 740$	$1857 \sim 783 \cong 775$	$1903 \sim 820 \cong 820$	$1949 \sim 783 \cong 775$
$1812 \sim 776 \cong 776$	$1858 \sim 747 \cong 739$	$1904 \sim 739 \cong 739$	$1950 \sim 864 \cong 864$
$1813 \sim 740 \cong 740$	$1859 \sim 734 \cong 730$	$1905 \sim 820 \cong 820$	$1951 \sim 1094 \cong 1090$
$1814 \sim 731 \cong 731$	$1860 \sim 756 \cong 748$	$1906 \sim 955 \cong 937$	$1952 \sim 882 \cong 882$
$1815 \sim 749 \cong 749$	$1861 \sim 783 \cong 775$	$1907 \sim 838 \cong 838$	$1953 \sim 1094 \cong 1090$
$1816 \sim 776 \cong 776$	$1862 \sim 756 \cong 748$	$1908 \sim 964 \cong 739$	$1954 \sim 1091 \cong 731$
$1817 \sim 749 \cong 749$	$1863 \sim 810 \cong 802$	$1909 \sim 930 \cong 821$	$1955 \sim 957 \cong 957$
$1818 \sim 803 \cong 771$	$1864 \sim 932 \cong 820$	$1910 \sim 920 \cong 920$	$1956 \sim 1091 \cong 731$
$1819 \sim 766 \cong 730$	$1865 \sim 923 \cong 923$	$1911 \sim 939 \cong 939$	$1957 \sim 939 \cong 939$
$1820 \sim 739 \cong 739$	$1866 \sim 941 \cong 941$	$1912 \sim 821 \cong 821$	$1958 \sim 777 \cong 777$
$1821 \sim 775 \cong 775$	$1867 \sim 824 \cong 820$	$1913 \sim 740 \cong 740$	$1959 \sim 858 \cong 858$
$1822 \sim 739 \cong 739$	$1868 \sim 747 \cong 739$	$1914 \sim 821 \cong 821$	$1960 \sim 1091 \cong 731$
$1823 \sim 730 \cong 730$	$1869 \sim 824 \cong 820$	$1915 \sim 957 \cong 957$	$1961 \sim 876 \cong 876$
$1824 \sim 748 \cong 748$	$1870 \sim 959 \cong 959$	$1916 \sim 839 \cong 821$	$1962 \sim 1091 \cong 731$
$1825 \sim 775 \cong 775$	$1871 \sim 846 \cong 846$	$1917 \sim 966 \cong 966$	$1963 \sim 1094 \cong 1090$
$1826 \sim 748 \cong 748$	$1872 \sim 968 \cong 968$	$1918 \sim 933 \cong 849$	$1964 \sim 960 \cong 960$
$1827 \sim 802 \cong 802$	$1873 \sim 929 \cong 929$	$1919 \sim 924 \cong 870$	$1965 \sim 1094 \cong 1090$
$1828 \sim 768 \cong 731$	$1874 \sim 920 \cong 920$	$1920 \sim 942 \cong 942$	$1966 \sim 942 \cong 942$
$1829 \sim 741 \cong 741$	$1875 \sim 938 \cong 938$	$1921 \sim 824 \cong 820$	$1967 \sim 780 \cong 780$
$1830 \sim 777 \cong 777$	$1876 \sim 821 \cong 821$	$1922 \sim 744 \cong 744$	$1968 \sim 861 \cong 861$
$1831 \sim 741 \cong 741$	$1877 \sim 741 \cong 741$	$1923 \sim 824 \cong 820$	$1969 \sim 1094 \cong 1090$
$1832 \sim 731 \cong 731$	$1878 \sim 821 \cong 821$	$1924 \sim 960 \cong 960$	$1970 \sim 879 \cong 879$
$1833 \sim 750 \cong 750$	$1879 \sim 956 \cong 956$	$1925 \sim 843 \cong 843$	$1971 \sim 1094 \cong 1090$
$1834 \sim 777 \cong 777$	$1880 \sim 840 \cong 840$	$1926 \sim 969 \cong 969$	$1972 \sim 1091 \cong 731$
$1835 \sim 750 \cong 750$	$1881 \sim 965 \cong 965$	$1927 \sim 930 \cong 821$	$1973 \sim 957 \cong 957$
$1836 \sim 804 \cong 731$	$1882 \sim 933 \cong 849$	$1928 \sim 920 \cong 920$	$1974 \sim 1091 \cong 731$
$1837 \sim 771 \cong 771$	$1883 \sim 924 \cong 870$	$1929 \sim 939 \cong 939$	$1975 \sim 939 \cong 939$
$1838 \sim 744 \cong 744$	$1884 \sim 942 \cong 942$	$1930 \sim 821 \cong 821$	$1976 \sim 777 \cong 777$
$1839 \sim 780 \cong 780$	$1885 \sim 824 \cong 820$	$1931 \sim 740 \cong 740$	$1977 \sim 858 \cong 858$
$1840 \sim 744 \cong 744$	$1886 \sim 744 \cong 744$	$1932 \sim 821 \cong 821$	$1978 \sim 1091 \cong 731$
$1841 \sim 734 \cong 730$	$1887 \sim 824 \cong 820$	$1933 \sim 957 \cong 957$	$1979 \sim 876 \cong 876$
$1842 \sim 753 \cong 753$	$1888 \sim 960 \cong 960$	$1934 \sim 839 \cong 821$	$1980 \sim 1091 \cong 731$
$1843 \sim 780 \cong 780$	$1889 \sim 843 \cong 843$	$1935 \sim 966 \cong 966$	$1981 \sim 1090 \cong 1090$
$1844 \sim 753 \cong 753$	$1890 \sim 969 \cong 969$	$1936 \sim 936 \cong 820$	$1982 \sim 955 \cong 937$
$1845 \sim 807 \cong 771$	$1891 \sim 929 \cong 929$	$1937 \sim 923 \cong 923$	$1983 \sim 1090 \cong 1090$
$1846 \sim 768 \cong 731$	$1892 \sim 920 \cong 920$	$1938 \sim 945 \cong 941$	$1984 \sim 937 \cong 937$
$1847 \sim 741 \cong 741$	$1893 \sim 938 \cong 938$	$1939 \sim 824 \cong 820$	$1985 \sim 775 \cong 775$
$1848 \sim 777 \cong 777$	$1894 \sim 821 \cong 821$	$1940 \sim 743 \cong 739$	$1986 \sim 856 \cong 856$
$1849 \sim 741 \cong 741$	$1895 \sim 741 \cong 741$	$1941 \sim 824 \cong 820$	$1987 \sim 1090 \cong 1090$
$1850 \sim 731 \cong 731$	$1896 \sim 821 \cong 821$	$1942 \sim 963 \cong 963$	$1988 \sim 874 \cong 874$
$1851 \sim 750 \cong 750$	$1897 \sim 956 \cong 956$	$1943 \sim 842 \cong 838$	$1989 \sim 1090 \cong 1090$

$1990 \sim 1091 \cong 731$	$2036 \sim 821 \cong 821$	$2082 \sim 960 \cong 960$	$2128 \sim 933 \cong 849$
$1991 \sim 956 \cong 956$	$2037 \sim 956 \cong 956$	$2083 \sim 924 \cong 870$	$2129 \sim 771 \cong 771$
$1992 \sim 1091 \cong 731$	$2038 \sim 920 \cong 920$	$2084 \sim 744 \cong 744$	$2130 \sim 852 \cong 852$
$1993 \sim 938 \cong 938$	$2039 \sim 741 \cong 741$	$2085 \sim 843 \cong 843$	$2131 \sim 1094 \cong 1090$
$1994 \sim 776 \cong 776$	$2040 \sim 840 \cong 840$	$2086 \sim 942 \cong 942$	$2132 \sim 852 \cong 852$
$1995 \sim 857 \cong 857$	$2041 \sim 938 \cong 938$	$2087 \sim 824 \cong 820$	$2133 \sim 1094 \cong 1090$
$1996 \sim 1091 \cong 731$	$2042 \sim 821 \cong 821$	$2088 \sim 969 \cong 969$	$2134 \sim 1091 \cong 731$
$1997 \sim 875 \cong 875$	$2043 \sim 965 \cong 965$	$2089 \sim 930 \cong 821$	$2135 \sim 930 \cong 821$
$1998 \sim 1091 \cong 731$	$2044 \sim 933 \cong 849$	$2090 \sim 821 \cong 821$	$2136 \sim 1091 \cong 731$
$1999 \sim 1094 \cong 1090$	$2045 \sim 824 \cong 820$	$2091 \sim 957 \cong 957$	$2137 \sim 930 \cong 821$
$2000 \sim 960 \cong 960$	$2046 \sim 960 \cong 960$	$2092 \sim 920 \cong 920$	$2138 \sim 768 \cong 731$
$2001 \sim 1094 \cong 1090$	$2047 \sim 924 \cong 870$	$2093 \sim 740 \cong 740$	$2139 \sim 849 \cong 849$
$2002 \sim 942 \cong 942$	$2048 \sim 744 \cong 744$	$2094 \sim 839 \cong 821$	$2140 \sim 1091 \cong 731$
$2003 \sim 780 \cong 780$	$2049 \sim 843 \cong 843$	$2095 \sim 939 \cong 939$	$2141 \sim 849 \cong 849$
$2004 \sim 861 \cong 861$	$2050 \sim 942 \cong 942$	$2096 \sim 821 \cong 821$	$2142 \sim 1091 \cong 731$
$2005 \sim 1094 \cong 1090$	$2051 \sim 824 \cong 820$	$2097 \sim 966 \cong 966$	$2143 \sim 1090 \cong 1090$
$2006 \sim 879 \cong 879$	$2052 \sim 969 \cong 969$	$2098 \sim 936 \cong 820$	$2144 \sim 928 \cong 820$
$2007 \sim 1094 \cong 1090$	$2053 \sim 929 \cong 929$	$2099 \sim 824 \cong 820$	$2145 \sim 1090 \cong 1090$
$2008 \sim 1091 \cong 731$	$2054 \sim 821 \cong 821$	$2100 \sim 963 \cong 963$	$2146 \sim 928 \cong 820$
$2009 \sim 956 \cong 956$	$2055 \sim 956 \cong 956$	$2101 \sim 923 \cong 923$	$2147 \sim 766 \cong 730$
$2010 \sim 1091 \cong 731$	$2056 \sim 920 \cong 920$	$2102 \sim 743 \cong 739$	$2148 \sim 847 \cong 847$
$2011 \sim 938 \cong 938$	$2057 \sim 741 \cong 741$	$2103 \sim 842 \cong 838$	$2149 \sim 1090 \cong 1090$
$2012 \sim 776 \cong 776$	$2058 \sim 840 \cong 840$	$2104 \sim 945 \cong 941$	$2150 \sim 847 \cong 847$
$2013 \sim 857 \cong 857$	$2059 \sim 938 \cong 938$	$2105 \sim 824 \cong 820$	$2151 \sim 1090 \cong 1090$
$2014 \sim 1091 \cong 731$	$2060 \sim 821 \cong 821$	$2106 \sim 972 \cong 739$	$2152 \sim 1091 \cong 731$
$2015 \sim 875 \cong 875$	$2061 \sim 965 \cong 965$	$2107 \sim 1094 \cong 1090$	$2153 \sim 929 \cong 929$
$2016 \sim 1091 \cong 731$	$2062 \sim 928 \cong 820$	$2108 \sim 936 \cong 820$	$2154 \sim 1091 \cong 731$
$2017 \sim 1094 \cong 1090$	$2063 \sim 820 \cong 820$	$2109 \sim 1094 \cong 1090$	$2155 \sim 929 \cong 929$
$2018 \sim 959 \cong 959$	$2064 \sim 955 \cong 937$	$2110 \sim 936 \cong 820$	$2156 \sim 767 \cong 731$
$2019 \sim 1094 \cong 1090$	$2065 \sim 919 \cong 820$	$2111 \sim 774 \cong 730$	$2157 \sim 848 \cong 750$
$2020 \sim 941 \cong 941$	$2066 \sim 739 \cong 739$	$2112 \sim 855 \cong 847$	$2158 \sim 1091 \cong 731$
$2021 \sim 779 \cong 779$	$2067 \sim 838 \cong 838$	$2113 \sim 1094 \cong 1090$	$2159 \sim 848 \cong 750$
$2022 \sim 860 \cong 860$	$2068 \sim 937 \cong 937$	$2114 \sim 855 \cong 847$	$2160 \sim 1091 \cong 731$
$2023 \sim 1094 \cong 1090$	$2069 \sim 820 \cong 820$	$2115 \sim 1094 \cong 1090$	$2161 \sim 1094 \cong 1090$
$2024 \sim 878 \cong 878$	$2070 \sim 964 \cong 739$	$2116 \sim 1091 \cong 731$	$2162 \sim 933 \cong 849$
$2025 \sim 1094 \cong 1090$	$2071 \sim 930 \cong 821$	$2117 \sim 930 \cong 821$	$2163 \sim 1094 \cong 1090$
$2026 \sim 932 \cong 820$	$2072 \sim 821 \cong 821$	$2118 \sim 1091 \cong 731$	$2164 \sim 933 \cong 849$
$2027 \sim 824 \cong 820$	$2073 \sim 957 \cong 957$	$2119 \sim 930 \cong 821$	$2165 \sim 771 \cong 771$
$2028 \sim 959 \cong 959$	$2074 \sim 920 \cong 920$	$2120 \sim 768 \cong 731$	$2166 \sim 852 \cong 852$
$2029 \sim 923 \cong 923$	$2075 \sim 740 \cong 740$	$2121 \sim 849 \cong 849$	$2167 \sim 1094 \cong 1090$
$2030 \sim 747 \cong 739$	$2076 \sim 839 \cong 821$	$2122 \sim 1091 \cong 731$	$2168 \sim 852 \cong 852$
$2031 \sim 846 \cong 846$	$2077 \sim 939 \cong 939$	$2123 \sim 849 \cong 849$	$2169 \sim 1094 \cong 1090$
$2032 \sim 941 \cong 941$	$2078 \sim 821 \cong 821$	$2124 \sim 1091 \cong 731$	$2170 \sim 1091 \cong 731$
$2033 \sim 824 \cong 820$	$2079 \sim 966 \cong 966$	$2125 \sim 1094 \cong 1090$	$2171 \sim 929 \cong 929$
$2034 \sim 968 \cong 968$	$2080 \sim 933 \cong 849$	$2126 \sim 933 \cong 849$	$2172 \sim 1091 \cong 731$
$2035 \sim 929 \cong 929$	$2081 \sim 824 \cong 820$	$2127 \sim 1094 \cong 1090$	$2173 \sim 929 \cong 929$

$2174 \sim 767 \cong 731$
$2175 \sim 848 \cong 750$
$2176 \sim 1091 \cong 731$
$2177 \sim 848 \cong 750$
$2178 \sim 1091 \cong 731$
$2179 \sim 1094 \cong 1090$
$2180 \sim 932 \cong 820$
$2181 \sim 1094 \cong 1090$
$2182 \sim 932 \cong 820$
$2183 \sim 770 \cong 730$
$2184 \sim 851 \cong 847$
$2185 \sim 1094 \cong 1090$
$2186 \sim 851 \cong 847$
$2187 \sim 1094 \cong 1090$
$2188 \sim 730 \cong 730$
$2189 \sim 730 \cong 730$
$2190 \sim 2190 \cong 750$
$2191 \sim 730 \cong 730$
$2192 \sim 730 \cong 730$
$2193 \sim 2193 \cong 2193$
$2194 \sim 2190 \cong 750$
$2195 \sim 2193 \cong 2193$
$2196 \sim 2196 \cong 802$
$2197 \sim 730 \cong 730$
$2198 \sim 730 \cong 730$
$2199 \sim 2199 \cong 2199$
$2200 \sim 730 \cong 730$
$2201 \sim 730 \cong 730$
$2202 \sim 2202 \cong 2202$
$2203 \sim 2203 \cong 2203$
$2204 \sim 2204 \cong 2204$
$2205 \sim 2205 \cong 775$
$2206 \sim 2206 \cong 748$
$2207 \sim 2207 \cong 2207$
$2208 \sim 731 \cong 731$
$2209 \sim 2209 \cong 2209$
$2210 \sim 2210 \cong 2210$
$2211 \sim 731 \cong 731$
$2212 \sim 2212 \cong 2212$
$2213 \sim 2213 \cong 2213$
$2214 \sim 2214 \cong 748$
$2215 \sim 730 \cong 730$
$2216 \sim 730 \cong 730$
$2217 \sim 2203 \cong 2203$
$2218 \sim 730 \cong 730$
$2219 \sim 730 \cong 730$
$2220 \sim 2204 \cong 2204$
$2221 \sim 2199 \cong 2199$
$2222 \sim 2202 \cong 2202$
$2223 \sim 2205 \cong 775$
$2224 \sim 730 \cong 730$
$2225 \sim 730 \cong 730$
$2226 \sim 2226 \cong 820$
$2227 \sim 730 \cong 730$
$2228 \sim 730 \cong 730$
$2229 \sim 2229 \cong 2229$
$2230 \sim 2226 \cong 820$
$2231 \sim 2229 \cong 2229$
$2232 \sim 2232 \cong 730$
$2233 \sim 2233 \cong 2233$
$2234 \sim 2234 \cong 2234$
$2235 \sim 731 \cong 731$
$2236 \sim 2236 \cong 2236$
$2237 \sim 2237 \cong 2237$
$2238 \sim 731 \cong 731$
$2239 \sim 2239 \cong 2239$
$2240 \sim 2240 \cong 2240$
$2241 \sim 2241 \cong 739$
$2242 \sim 2206 \cong 748$
$2243 \sim 2209 \cong 2209$
$2244 \sim 2212 \cong 2212$
$2245 \sim 2207 \cong 2207$
$2246 \sim 2210 \cong 2210$
$2247 \sim 2213 \cong 2213$
$2248 \sim 731 \cong 731$
$2249 \sim 731 \cong 731$
$2250 \sim 2214 \cong 748$
$2251 \sim 2233 \cong 2233$
$2252 \sim 2236 \cong 2236$
$2253 \sim 2239 \cong 2239$
$2254 \sim 2234 \cong 2234$
$2255 \sim 2237 \cong 2237$
$2256 \sim 2240 \cong 2240$
$2257 \sim 731 \cong 731$
$2258 \sim 731 \cong 731$
$2259 \sim 2241 \cong 739$
$2260 \sim 2260 \cong 802$
$2261 \sim 2261 \cong 2261$
$2262 \sim 2262 \cong 750$
$2263 \sim 2261 \cong 2261$
$2264 \sim 2264 \cong 730$
$2265 \sim 2265 \cong 2265$
$2266 \sim 2262 \cong 750$
$2267 \sim 2265 \cong 2265$
$2268 \sim 734 \cong 730$
$2269 \sim 730 \cong 730$
$2270 \sim 730 \cong 730$
$2271 \sim 2271 \cong 2271$
$2272 \sim 730 \cong 730$
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$2274 \sim 2274 \cong 2274$
$2275 \sim 2271 \cong 2271$
$2276 \sim 2274 \cong 2274$
$2277 \sim 2277 \cong 2277$
$2278 \sim 730 \cong 730$
$2279 \sim 730 \cong 730$
$2280 \sim 2280 \cong 2280$
$2281 \sim 730 \cong 730$
$2282 \sim 730 \cong 730$
$2283 \sim 2283 \cong 2283$
$2284 \sim 2284 \cong 2284$
$2285 \sim 2285 \cong 2285$
$2286 \sim 2286 \cong 2286$
$2287 \sim 2287 \cong 2287$
$2288 \sim 2285 \cong 2285$
$2289 \sim 731 \cong 731$
$2290 \sim 2283 \cong 2283$
$2291 \sim 2274 \cong 2274$
$2292 \sim 731 \cong 731$
$2293 \sim 2293 \cong 2293$
$2294 \sim 2294 \cong 2294$
$2295 \sim 2295 \cong 2295$
$2296 \sim 730 \cong 730$
$2297 \sim 730 \cong 730$
$2298 \sim 2284 \cong 2284$
$2299 \sim 730 \cong 730$
$2300 \sim 730 \cong 730$
$2301 \sim 2285 \cong 2285$
$2302 \sim 2280 \cong 2280$
$2303 \sim 2283 \cong 2283$
$2304 \sim 2286 \cong 2286$
$2305 \sim 730 \cong 730$
$2306 \sim 730 \cong 730$
$2307 \sim 2307 \cong 2307$
$2308 \sim 730 \cong 730$
$2309 \sim 730 \cong 730$
$2310 \sim 2287 \cong 2287$
$2311 \sim 2307 \cong 2307$
$2312 \sim 2287 \cong 2287$
$2313 \sim 2313 \cong 2277$
$2314 \sim 2307 \cong 2307$
$2315 \sim 2284 \cong 2284$
$2316 \sim 731 \cong 731$
$2317 \sim 2280 \cong 2280$
$2318 \sim 2271 \cong 2271$
$2319 \sim 731 \cong 731$
$2320 \sim 2320 \cong 2294$
$2321 \sim 2293 \cong 2293$
$2322 \sim 2322 \cong 2322$
$2323 \sim 2287 \cong 2287$
$2324 \sim 2283 \cong 2283$
$2325 \sim 2293 \cong 2293$
$2326 \sim 2285 \cong 2285$
$2327 \sim 2274 \cong 2274$
$2328 \sim 2294 \cong 2294$
$2329 \sim 731 \cong 731$
$2330 \sim 731 \cong 731$
$2331 \sim 2295 \cong 2295$
$2332 \sim 2307 \cong 2307$
$2333 \sim 2280 \cong 2280$
$2334 \sim 2320 \cong 2294$
$2335 \sim 2284 \cong 2284$
$2336 \sim 2271 \cong 2271$
$2337 \sim 2293 \cong 2293$
$2338 \sim 731 \cong 731$
$2339 \sim 731 \cong 731$
$2340 \sim 2322 \cong 2322$
$2341 \sim 2313 \cong 2277$
$2342 \sim 2286 \cong 2286$
$2343 \sim 2322 \cong 2322$
$2344 \sim 2286 \cong 2286$
$2345 \sim 2277 \cong 2277$
$2346 \sim 2295 \cong 2295$
$2347 \sim 2322 \cong 2322$
$2348 \sim 2295 \cong 2295$
$2349 \sim 734 \cong 730$
$2350 \sim 820 \cong 820$
$2351 \sim 820 \cong 820$
$2352 \sim 2352 \cong 740$
$2353 \sim 820 \cong 820$
$2354 \sim 820 \cong 820$
$2355 \sim 2355 \cong 2355$
$2356 \sim 2352 \cong 740$
$2357 \sim 2355 \cong 2355$

$2358 \sim 2358 \cong 820$	$2404 \sim 2368 \cong 739$	$2450 \sim 2285 \cong 2285$	$2496 \sim 2320 \cong 2294$
$2359 \sim 820 \cong 820$	$2405 \sim 2371 \cong 2371$	$2451 \sim 731 \cong 731$	$2497 \sim 2284 \cong 2284$
$2360 \sim 820 \cong 820$	$2406 \sim 2374 \cong 821$	$2452 \sim 2283 \cong 2283$	$2498 \sim 2271 \cong 2271$
$2361 \sim 2361 \cong 2361$	$2407 \sim 2369 \cong 2369$	$2453 \sim 2274 \cong 2274$	$2499 \sim 2293 \cong 2293$
$2362 \sim 820 \cong 820$	$2408 \sim 2372 \cong 2372$	$2454 \sim 731 \cong 731$	$2500 \sim 731 \cong 731$
$2363 \sim 820 \cong 820$	$2409 \sim 2375 \cong 2375$	$2455 \sim 2293 \cong 2293$	$2501 \sim 731 \cong 731$
$2364 \sim 2364 \cong 2364$	$2410 \sim 821 \cong 821$	$2456 \sim 2294 \cong 2294$	$2502 \sim 2322 \cong 2322$
$2365 \sim 2365 \cong 2365$	$2411 \sim 821 \cong 821$	$2457 \sim 2295 \cong 2295$	$2503 \sim 2313 \cong 2277$
$2366 \sim 2366 \cong 2366$	$2412 \sim 2376 \cong 739$	$2458 \sim 730 \cong 730$	$2504 \sim 2286 \cong 2286$
$2367 \sim 2367 \cong 2367$	$2413 \sim 2395 \cong 2395$	$2459 \sim 730 \cong 730$	$2505 \sim 2322 \cong 2322$
$2368 \sim 2368 \cong 739$	$2414 \sim 2398 \cong 2398$	$2460 \sim 2284 \cong 2284$	$2506 \sim 2286 \cong 2286$
$2369 \sim 2369 \cong 2369$	$2415 \sim 2401 \cong 2401$	$2461 \sim 730 \cong 730$	$2507 \sim 2277 \cong 2277$
$2370 \sim 821 \cong 821$	$2416 \sim 2396 \cong 2396$	$2462 \sim 730 \cong 730$	$2508 \sim 2295 \cong 2295$
$2371 \sim 2371 \cong 2371$	$2417 \sim 2399 \cong 2399$	$2463 \sim 2285 \cong 2285$	$2509 \sim 2322 \cong 2322$
$2372 \sim 2372 \cong 2372$	$2418 \sim 2402 \cong 2402$	$2464 \sim 2280 \cong 2280$	$2510 \sim 2295 \cong 2295$
$2373 \sim 821 \cong 821$	$2419 \sim 821 \cong 821$	$2465 \sim 2283 \cong 2283$	$2511 \sim 734 \cong 730$
$2374 \sim 2374 \cong 821$	$2420 \sim 821 \cong 821$	$2466 \sim 2286 \cong 2286$	$2512 \sim 730 \cong 730$
$2375 \sim 2375 \cong 2375$	$2421 \sim 2403 \cong 2287$	$2467 \sim 730 \cong 730$	$2513 \sim 730 \cong 730$
$2376 \sim 2376 \cong 739$	$2422 \sim 2422 \cong 820$	$2468 \sim 730 \cong 730$	$2514 \sim 2237 \cong 2237$
$2377 \sim 820 \cong 820$	$2423 \sim 2423 \cong 2423$	$2469 \sim 2307 \cong 2307$	$2515 \sim 730 \cong 730$
$2378 \sim 820 \cong 820$	$2424 \sim 2424 \cong 966$	$2470 \sim 730 \cong 730$	$2516 \sim 730 \cong 730$
$2379 \sim 2365 \cong 2365$	$2425 \sim 2423 \cong 2423$	$2471 \sim 730 \cong 730$	$2517 \sim 2210 \cong 2210$
$2380 \sim 820 \cong 820$	$2426 \sim 2426 \cong 2277$	$2472 \sim 2287 \cong 2287$	$2518 \sim 2237 \cong 2237$
$2381 \sim 820 \cong 820$	$2427 \sim 2427 \cong 2427$	$2473 \sim 2307 \cong 2307$	$2519 \sim 2210 \cong 2210$
$2382 \sim 2366 \cong 2366$	$2428 \sim 2424 \cong 966$	$2474 \sim 2287 \cong 2287$	$2520 \sim 2264 \cong 730$
$2383 \sim 2361 \cong 2361$	$2429 \sim 2427 \cong 2427$	$2475 \sim 2313 \cong 2277$	$2521 \sim 730 \cong 730$
$2384 \sim 2364 \cong 2364$	$2430 \sim 824 \cong 820$	$2476 \sim 2307 \cong 2307$	$2522 \sim 730 \cong 730$
$2385 \sim 2367 \cong 2367$	$2431 \sim 730 \cong 730$	$2477 \sim 2284 \cong 2284$	$2523 \sim 2236 \cong 2236$
$2386 \sim 820 \cong 820$	$2432 \sim 730 \cong 730$	$2478 \sim 731 \cong 731$	$2524 \sim 730 \cong 730$
$2387 \sim 820 \cong 820$	$2433 \sim 2271 \cong 2271$	$2479 \sim 2280 \cong 2280$	$2525 \sim 730 \cong 730$
$2388 \sim 2388 \cong 821$	$2434 \sim 730 \cong 730$	$2480 \sim 2271 \cong 2271$	$2526 \sim 2209 \cong 2209$
$2389 \sim 820 \cong 820$	$2435 \sim 730 \cong 730$	$2481 \sim 731 \cong 731$	$2527 \sim 2234 \cong 2234$
$2390 \sim 820 \cong 820$	$2436 \sim 2274 \cong 2274$	$2482 \sim 2320 \cong 2294$	$2528 \sim 2207 \cong 2207$
$2391 \sim 2391 \cong 2391$	$2437 \sim 2271 \cong 2271$	$2483 \sim 2293 \cong 2293$	$2529 \sim 2261 \cong 2261$
$2392 \sim 2388 \cong 821$	$2438 \sim 2274 \cong 2274$	$2484 \sim 2322 \cong 2322$	$2530 \sim 2229 \cong 2229$
$2393 \sim 2391 \cong 2391$	$2439 \sim 2277 \cong 2277$	$2485 \sim 2287 \cong 2287$	$2531 \sim 2204 \cong 2204$
$2394 \sim 2394 \cong 820$	$2440 \sim 730 \cong 730$	$2486 \sim 2283 \cong 2283$	$2532 \sim 731 \cong 731$
$2395 \sim 2395 \cong 2395$	$2441 \sim 730 \cong 730$	$2487 \sim 2293 \cong 2293$	$2533 \sim 2202 \cong 2202$
$2396 \sim 2396 \cong 2396$	$2442 \sim 2280 \cong 2280$	$2488 \sim 2285 \cong 2285$	$2534 \sim 2193 \cong 2193$
$2397 \sim 821 \cong 821$	$2443 \sim 730 \cong 730$	$2489 \sim 2274 \cong 2274$	$2535 \sim 731 \cong 731$
$2398 \sim 2398 \cong 2398$	$2444 \sim 730 \cong 730$	$2490 \sim 2294 \cong 2294$	$2536 \sim 2240 \cong 2240$
$2399 \sim 2399 \cong 2399$	$2445 \sim 2283 \cong 2283$	$2491 \sim 731 \cong 731$	$2537 \sim 2213 \cong 2213$
$2400 \sim 821 \cong 821$	$2446 \sim 2284 \cong 2284$	$2492 \sim 731 \cong 731$	$2538 \sim 2265 \cong 2265$
$2401 \sim 2401 \cong 2401$	$2447 \sim 2285 \cong 2285$	$2493 \sim 2295 \cong 2295$	$2539 \sim 730 \cong 730$
$2402 \sim 2402 \cong 2402$	$2448 \sim 2286 \cong 2286$	$2494 \sim 2307 \cong 2307$	$2540 \sim 730 \cong 730$
$2403 \sim 2403 \cong 2287$	$2449 \sim 2287 \cong 2287$	$2495 \sim 2280 \cong 2280$	$2541 \sim 2234 \cong 2234$

$2542 \sim 730 \cong 730$	$2588 \sim 2196 \cong 802$	$2634 \sim 2368 \cong 739$	$2680 \sim 2352 \cong 740$
$2543 \sim 730 \cong 730$	$2589 \sim 2214 \cong 748$	$2635 \sim 2395 \cong 2395$	$2681 \sim 2355 \cong 2355$
$2544 \sim 2207 \cong 2207$	$2590 \sim 2241 \cong 739$	$2636 \sim 2368 \cong 739$	$2682 \sim 2358 \cong 820$
$2545 \sim 2236 \cong 2236$	$2591 \sim 2214 \cong 748$	$2637 \sim 2422 \cong 820$	$2683 \sim 820 \cong 820$
$2546 \sim 2209 \cong 2209$	$2592 \sim 734 \cong 730$	$2638 \sim 2388 \cong 821$	$2684 \sim 820 \cong 820$
$2547 \sim 2261 \cong 2261$	$2593 \sim 820 \cong 820$	$2639 \sim 2365 \cong 2365$	$2685 \sim 2361 \cong 2361$
$2548 \sim 730 \cong 730$	$2594 \sim 820 \cong 820$	$2640 \sim 821 \cong 821$	$2686 \sim 820 \cong 820$
$2549 \sim 730 \cong 730$	$2595 \sim 2399 \cong 2399$	$2641 \sim 2361 \cong 2361$	$2687 \sim 820 \cong 820$
$2550 \sim 2233 \cong 2233$	$2596 \sim 820 \cong 820$	$2642 \sim 2352 \cong 740$	$2688 \sim 2364 \cong 2364$
$2551 \sim 730 \cong 730$	$2597 \sim 820 \cong 820$	$2643 \sim 821 \cong 821$	$2689 \sim 2365 \cong 2365$
$2552 \sim 730 \cong 730$	$2598 \sim 2372 \cong 2372$	$2644 \sim 2401 \cong 2401$	$2690 \sim 2366 \cong 2366$
$2553 \sim 2206 \cong 748$	$2599 \sim 2399 \cong 2399$	$2645 \sim 2374 \cong 821$	$2691 \sim 2367 \cong 2367$
$2554 \sim 2233 \cong 2233$	$2600 \sim 2372 \cong 2372$	$2646 \sim 2424 \cong 966$	$2692 \sim 2368 \cong 739$
$2555 \sim 2206 \cong 748$	$2601 \sim 2426 \cong 2277$	$2647 \sim 2391 \cong 2391$	$2693 \sim 2369 \cong 2369$
$2556 \sim 2260 \cong 802$	$2602 \sim 820 \cong 820$	$2648 \sim 2364 \cong 2364$	$2694 \sim 821 \cong 821$
$2557 \sim 2226 \cong 820$	$2603 \sim 820 \cong 820$	$2649 \sim 2402 \cong 2402$	$2695 \sim 2371 \cong 2371$
$2558 \sim 2203 \cong 2203$	$2604 \sim 2398 \cong 2398$	$2650 \sim 2366 \cong 2366$	$2696 \sim 2372 \cong 2372$
$2559 \sim 731 \cong 731$	$2605 \sim 820 \cong 820$	$2651 \sim 2355 \cong 2355$	$2697 \sim 821 \cong 821$
$2560 \sim 2199 \cong 2199$	$2606 \sim 820 \cong 820$	$2652 \sim 2375 \cong 2375$	$2698 \sim 2374 \cong 821$
$2561 \sim 2190 \cong 750$	$2607 \sim 2371 \cong 2371$	$2653 \sim 821 \cong 821$	$2699 \sim 2375 \cong 2375$
$2562 \sim 731 \cong 731$	$2608 \sim 2396 \cong 2396$	$2654 \sim 821 \cong 821$	$2700 \sim 2376 \cong 739$
$2563 \sim 2239 \cong 2239$	$2609 \sim 2369 \cong 2369$	$2655 \sim 2427 \cong 2427$	$2701 \sim 820 \cong 820$
$2564 \sim 2212 \cong 2212$	$2610 \sim 2423 \cong 2423$	$2656 \sim 2388 \cong 821$	$2702 \sim 820 \cong 820$
$2565 \sim 2262 \cong 750$	$2611 \sim 2391 \cong 2391$	$2657 \sim 2361 \cong 2361$	$2703 \sim 2365 \cong 2365$
$2566 \sim 2229 \cong 2229$	$2612 \sim 2366 \cong 2366$	$2658 \sim 2401 \cong 2401$	$2704 \sim 820 \cong 820$
$2567 \sim 2202 \cong 2202$	$2613 \sim 821 \cong 821$	$2659 \sim 2365 \cong 2365$	$2705 \sim 820 \cong 820$
$2568 \sim 2240 \cong 2240$	$2614 \sim 2364 \cong 2364$	$2660 \sim 2352 \cong 740$	$2706 \sim 2366 \cong 2366$
$2569 \sim 2204 \cong 2204$	$2615 \sim 2355 \cong 2355$	$2661 \sim 2374 \cong 821$	$2707 \sim 2361 \cong 2361$
$2570 \sim 2193 \cong 2193$	$2616 \sim 821 \cong 821$	$2662 \sim 821 \cong 821$	$2708 \sim 2364 \cong 2364$
$2571 \sim 2213 \cong 2213$	$2617 \sim 2402 \cong 2402$	$2663 \sim 821 \cong 821$	$2709 \sim 2367 \cong 2367$
$2572 \sim 731 \cong 731$	$2618 \sim 2375 \cong 2375$	$2664 \sim 2424 \cong 966$	$2710 \sim 820 \cong 820$
$2573 \sim 731 \cong 731$	$2619 \sim 2427 \cong 2427$	$2665 \sim 2394 \cong 820$	$2711 \sim 820 \cong 820$
$2574 \sim 2265 \cong 2265$	$2620 \sim 820 \cong 820$	$2666 \sim 2367 \cong 2367$	$2712 \sim 2388 \cong 821$
$2575 \sim 2226 \cong 820$	$2621 \sim 820 \cong 820$	$2667 \sim 2403 \cong 2287$	$2713 \sim 820 \cong 820$
$2576 \sim 2199 \cong 2199$	$2622 \sim 2396 \cong 2396$	$2668 \sim 2367 \cong 2367$	$2714 \sim 820 \cong 820$
$2577 \sim 2239 \cong 2239$	$2623 \sim 820 \cong 820$	$2669 \sim 2358 \cong 820$	$2715 \sim 2391 \cong 2391$
$2578 \sim 2203 \cong 2203$	$2624 \sim 820 \cong 820$	$2670 \sim 2376 \cong 739$	$2716 \sim 2388 \cong 821$
$2579 \sim 2190 \cong 750$	$2625 \sim 2369 \cong 2369$	$2671 \sim 2403 \cong 2287$	$2717 \sim 2391 \cong 2391$
$2580 \sim 2212 \cong 2212$	$2626 \sim 2398 \cong 2398$	$2672 \sim 2376 \cong 739$	$2718 \sim 2394 \cong 820$
$2581 \sim 731 \cong 731$	$2627 \sim 2371 \cong 2371$	$2673 \sim 824 \cong 820$	$2719 \sim 2395 \cong 2395$
$2582 \sim 731 \cong 731$	$2628 \sim 2423 \cong 2423$	$2674 \sim 820 \cong 820$	$2720 \sim 2396 \cong 2396$
$2583 \sim 2262 \cong 750$	$2629 \sim 820 \cong 820$	$2675 \sim 820 \cong 820$	$2721 \sim 821 \cong 821$
$2584 \sim 2232 \cong 730$	$2630 \sim 820 \cong 820$	$2676 \sim 2352 \cong 740$	$2722 \sim 2398 \cong 2398$
$2585 \sim 2205 \cong 775$	$2631 \sim 2395 \cong 2395$	$2677 \sim 820 \cong 820$	$2723 \sim 2399 \cong 2399$
$2586 \sim 2241 \cong 739$	$2632 \sim 820 \cong 820$	$2678 \sim 820 \cong 820$	$2724 \sim 821 \cong 821$
$2587 \sim 2205 \cong 775$	$2633 \sim 820 \cong 820$	$2679 \sim 2355 \cong 2355$	$2725 \sim 2401 \cong 2401$

2726	$\sim$	2402	$\cong$	2402
2727	$\sim$	2403	$\cong$	2287
2728	$\sim$	2368	$\cong$	739
2729	$\sim$	2371	$\cong$	2371
2730	$\sim$	2374	$\cong$	821
2731	$\sim$	2369	$\cong$	2369
2732	$\sim$	2372	$\cong$	2372
2733	$\sim$	2375	$\cong$	2375
2734	$\sim$	821	$\cong$	821
2735	$\sim$	821	$\cong$	821
2736	$\sim$	2376	$\cong$	739
2737	$\sim$	2395	$\cong$	2395
2738	$\sim$	2398	$\cong$	2398
2739	$\sim$	2401	$\cong$	2401
2740	$\sim$	2396	$\cong$	2396
2741	$\sim$	2399	$\cong$	2399
2742	$\sim$	2402	$\cong$	2402
2743	$\sim$	821	$\cong$	821
2744	$\sim$	821	$\cong$	821
2745	$\sim$	2403	$\cong$	2287
2746	$\sim$	2422	$\cong$	820
2747	$\sim$	2423	$\cong$	2423
2748	$\sim$	2424	$\cong$	966
2749	$\sim$	2423	$\cong$	2423
2750	$\sim$	2426	$\cong$	2277
2751	$\sim$	2427	$\cong$	2427
2752	$\sim$	2424	$\cong$	966
2753	$\sim$	2427	$\cong$	2427
2754	$\sim$	824	$\cong$	820
2755	$\sim$	820	$\cong$	820
2756	$\sim$	820	$\cong$	820
2757	$\sim$	2399	$\cong$	2399
2758	$\sim$	820	$\cong$	820
2759	$\sim$	820	$\cong$	820
2760	$\sim$	2372	$\cong$	2372
2761	$\sim$	2399	$\cong$	2399
2762	$\sim$	2372	$\cong$	2372
2763	$\sim$	2426	$\cong$	2277
2764	$\sim$	820	$\cong$	820
2765	$\sim$	820	$\cong$	820
2766	$\sim$	2398	$\cong$	2398
2767	$\sim$	820	$\cong$	820
2768	$\sim$	820	$\cong$	820
2769	$\sim$	2371	$\cong$	2371
2770	$\sim$	2396	$\cong$	2396
2771	$\sim$	2369	$\cong$	2369
2772	$\sim$	2423	$\cong$	2423
2773	$\sim$	2391	$\cong$	2391
2774	$\sim$	2366	$\cong$	2366
2775	$\sim$	821	$\cong$	821
2776	$\sim$	2364	$\cong$	2364
2777	$\sim$	2355	$\cong$	2355
2778	$\sim$	821	$\cong$	821
2779	$\sim$	2402	$\cong$	2402
2780	$\sim$	2375	$\cong$	2375
2781	$\sim$	2427	$\cong$	2427
2782	$\sim$	820	$\cong$	820
2783	$\sim$	820	$\cong$	820
2784	$\sim$	2396	$\cong$	2396
2785	$\sim$	820	$\cong$	820
2786	$\sim$	820	$\cong$	820
2787	$\sim$	2369	$\cong$	2369
2788	$\sim$	2398	$\cong$	2398
2789	$\sim$	2371	$\cong$	2371
2790	$\sim$	2423	$\cong$	2423
2791	$\sim$	820	$\cong$	820
2792	$\sim$	820	$\cong$	820
2793	$\sim$	2395	$\cong$	2395
2794	$\sim$	820	$\cong$	820
2795	$\sim$	820	$\cong$	820
2796	$\sim$	2368	$\cong$	739
2797	$\sim$	2395	$\cong$	2395
2798	$\sim$	2368	$\cong$	739
2799	$\sim$	2422	$\cong$	820
2800	$\sim$	2388	$\cong$	821
2801	$\sim$	2365	$\cong$	2365
2802	$\sim$	821	$\cong$	821
2803	$\sim$	2361	$\cong$	2361
2804	$\sim$	2352	$\cong$	740
2805	$\sim$	821	$\cong$	821
2806	$\sim$	2401	$\cong$	2401
2807	$\sim$	2374	$\cong$	821
2808	$\sim$	2424	$\cong$	966
2809	$\sim$	2391	$\cong$	2391
2810	$\sim$	2364	$\cong$	2364
2811	$\sim$	2402	$\cong$	2402
2812	$\sim$	2366	$\cong$	2366
2813	$\sim$	2355	$\cong$	2355
2814	$\sim$	2375	$\cong$	2375
2815	$\sim$	821	$\cong$	821
2816	$\sim$	821	$\cong$	821
2817	$\sim$	2427	$\cong$	2427
2818	$\sim$	2388	$\cong$	821
2819	$\sim$	2361	$\cong$	2361
2820	$\sim$	2401	$\cong$	2401
2821	$\sim$	2365	$\cong$	2365
2822	$\sim$	2352	$\cong$	740
2823	$\sim$	2374	$\cong$	821
2824	$\sim$	821	$\cong$	821
2825	$\sim$	821	$\cong$	821
2826	$\sim$	2424	$\cong$	966
2827	$\sim$	2394	$\cong$	820
2828	$\sim$	2367	$\cong$	2367
2829	$\sim$	2403	$\cong$	2287
2830	$\sim$	2367	$\cong$	2367
2831	$\sim$	2358	$\cong$	820
2832	$\sim$	2376	$\cong$	739
2833	$\sim$	2403	$\cong$	2287
2834	$\sim$	2376	$\cong$	739
2835	$\sim$	824	$\cong$	820
2836	$\sim$	1090	$\cong$	1090
2837	$\sim$	1090	$\cong$	1090
2838	$\sim$	2838	$\cong$	750
2839	$\sim$	1090	$\cong$	1090
2840	$\sim$	1090	$\cong$	1090
2841	$\sim$	2841	$\cong$	2841
2842	$\sim$	2838	$\cong$	750
2843	$\sim$	2841	$\cong$	2841
2844	$\sim$	2844	$\cong$	730
2845	$\sim$	1090	$\cong$	1090
2846	$\sim$	1090	$\cong$	1090
2847	$\sim$	2847	$\cong$	929
2848	$\sim$	1090	$\cong$	1090
2849	$\sim$	1090	$\cong$	1090
2850	$\sim$	2850	$\cong$	2850
2851	$\sim$	2851	$\cong$	929
2852	$\sim$	2852	$\cong$	849
2853	$\sim$	2853	$\cong$	2853
2854	$\sim$	2854	$\cong$	847
2855	$\sim$	2852	$\cong$	849
2856	$\sim$	1091	$\cong$	731
2857	$\sim$	2850	$\cong$	2850
2858	$\sim$	2841	$\cong$	2841
2859	$\sim$	1091	$\cong$	731
2860	$\sim$	2860	$\cong$	2212
2861	$\sim$	2861	$\cong$	731
2862	$\sim$	2862	$\cong$	847
2863	$\sim$	1090	$\cong$	1090
2864	$\sim$	1090	$\cong$	1090
2865	$\sim$	2851	$\cong$	929
2866	$\sim$	1090	$\cong$	1090
2867	$\sim$	1090	$\cong$	1090
2868	$\sim$	2852	$\cong$	849
2869	$\sim$	2847	$\cong$	929
2870	$\sim$	2850	$\cong$	2850
2871	$\sim$	2853	$\cong$	2853
2872	$\sim$	1090	$\cong$	1090
2873	$\sim$	1090	$\cong$	1090
2874	$\sim$	2874	$\cong$	820
2875	$\sim$	1090	$\cong$	1090
2876	$\sim$	1090	$\cong$	1090
2877	$\sim$	2854	$\cong$	847
2878	$\sim$	2874	$\cong$	820
2879	$\sim$	2854	$\cong$	847
2880	$\sim$	2880	$\cong$	730
2881	$\sim$	2874	$\cong$	820
2882	$\sim$	2851	$\cong$	929
2883	$\sim$	1091	$\cong$	731
2884	$\sim$	2847	$\cong$	929
2885	$\sim$	2838	$\cong$	750
2886	$\sim$	1091	$\cong$	731
2887	$\sim$	2887	$\cong$	731
2888	$\sim$	2860	$\cong$	2212
2889	$\sim$	2889	$\cong$	750
2890	$\sim$	2854	$\cong$	847
2891	$\sim$	2850	$\cong$	2850
2892	$\sim$	2860	$\cong$	2212
2893	$\sim$	2852	$\cong$	849
2894	$\sim$	2841	$\cong$	2841
2895	$\sim$	2861	$\cong$	731
2896	$\sim$	1091	$\cong$	731
2897	$\sim$	1091	$\cong$	731
2898	$\sim$	2862	$\cong$	847
2899	$\sim$	2874	$\cong$	820
2900	$\sim$	2847	$\cong$	929
2901	$\sim$	2887	$\cong$	731
2902	$\sim$	2851	$\cong$	929
2903	$\sim$	2838	$\cong$	750
2904	$\sim$	2860	$\cong$	2212
2905	$\sim$	1091	$\cong$	731
2906	$\sim$	1091	$\cong$	731
2907	$\sim$	2889	$\cong$	750
2908	$\sim$	2880	$\cong$	730
2909	$\sim$	2853	$\cong$	2853

$2910 \sim 2889 \cong 750$	$2956 \sim 1094 \cong 1090$	$3002 \sim 1094 \cong 1090$	$3048 \sim 824 \cong 820$
$2911 \sim 2853 \cong 2853$	$2957 \sim 1094 \cong 1090$	$3003 \sim 888 \cong 888$	$3049 \sim 924 \cong 870$
$2912 \sim 2844 \cong 730$	$2958 \sim 855 \cong 847$	$3004 \sim 969 \cong 969$	$3050 \sim 843 \cong 843$
$2913 \sim 2862 \cong 847$	$2959 \sim 936 \cong 820$	$3005 \sim 888 \cong 888$	$3051 \sim 744 \cong 744$
$2914 \sim 2889 \cong 750$	$2960 \sim 855 \cong 847$	$3006 \sim 807 \cong 771$	$3052 \sim 852 \cong 852$
$2915 \sim 2862 \cong 847$	$2961 \sim 774 \cong 730$	$3007 \sim 1094 \cong 1090$	$3053 \sim 861 \cong 861$
$2916 \sim 1094 \cong 1090$	$2962 \sim 932 \cong 820$	$3008 \sim 1094 \cong 1090$	$3054 \sim 843 \cong 843$
$2917 \sim 1094 \cong 1090$	$2963 \sim 959 \cong 959$	$3009 \sim 942 \cong 942$	$3055 \sim 879 \cong 879$
$2918 \sim 1094 \cong 1090$	$2964 \sim 824 \cong 820$	$3010 \sim 1094 \cong 1090$	$3056 \sim 888 \cong 888$
$2919 \sim 972 \cong 739$	$2965 \sim 941 \cong 941$	$3011 \sim 1094 \cong 1090$	$3057 \sim 870 \cong 870$
$2920 \sim 1094 \cong 1090$	$2966 \sim 968 \cong 968$	$3012 \sim 861 \cong 861$	$3058 \sim 824 \cong 820$
$2921 \sim 1094 \cong 1090$	$2967 \sim 824 \cong 820$	$3013 \sim 960 \cong 960$	$3059 \sim 824 \cong 820$
$2922 \sim 891 \cong 891$	$2968 \sim 923 \cong 923$	$3014 \sim 879 \cong 879$	$3060 \sim 753 \cong 753$
$2923 \sim 972 \cong 739$	$2969 \sim 846 \cong 846$	$3015 \sim 780 \cong 780$	$3061 \sim 933 \cong 849$
$2924 \sim 891 \cong 891$	$2970 \sim 747 \cong 739$	$3016 \sim 852 \cong 852$	$3062 \sim 942 \cong 942$
$2925 \sim 810 \cong 802$	$2971 \sim 851 \cong 847$	$3017 \sim 879 \cong 879$	$3063 \sim 924 \cong 870$
$2926 \sim 1094 \cong 1090$	$2972 \sim 860 \cong 860$	$3018 \sim 824 \cong 820$	$3064 \sim 960 \cong 960$
$2927 \sim 1094 \cong 1090$	$2973 \sim 842 \cong 838$	$3019 \sim 861 \cong 861$	$3065 \sim 969 \cong 969$
$2928 \sim 945 \cong 941$	$2974 \sim 878 \cong 878$	$3020 \sim 888 \cong 888$	$3066 \sim 843 \cong 843$
$2929 \sim 1094 \cong 1090$	$2975 \sim 887 \cong 887$	$3021 \sim 824 \cong 820$	$3067 \sim 824 \cong 820$
$2930 \sim 1094 \cong 1090$	$2976 \sim 869 \cong 869$	$3022 \sim 843 \cong 843$	$3068 \sim 824 \cong 820$
$2931 \sim 864 \cong 864$	$2977 \sim 824 \cong 820$	$3023 \sim 870 \cong 870$	$3069 \sim 744 \cong 744$
$2932 \sim 963 \cong 963$	$2978 \sim 824 \cong 820$	$3024 \sim 753 \cong 753$	$3070 \sim 771 \cong 771$
$2933 \sim 882 \cong 882$	$2979 \sim 756 \cong 748$	$3025 \sim 1094 \cong 1090$	$3071 \sim 780 \cong 780$
$2934 \sim 783 \cong 775$	$2980 \sim 932 \cong 820$	$3026 \sim 1094 \cong 1090$	$3072 \sim 744 \cong 744$
$2935 \sim 851 \cong 847$	$2981 \sim 941 \cong 941$	$3027 \sim 960 \cong 960$	$3073 \sim 780 \cong 780$
$2936 \sim 878 \cong 878$	$2982 \sim 923 \cong 923$	$3028 \sim 1094 \cong 1090$	$3074 \sim 807 \cong 771$
$2937 \sim 824 \cong 820$	$2983 \sim 959 \cong 959$	$3029 \sim 1094 \cong 1090$	$3075 \sim 753 \cong 753$
$2938 \sim 860 \cong 860$	$2984 \sim 968 \cong 968$	$3030 \sim 879 \cong 879$	$3076 \sim 744 \cong 744$
$2939 \sim 887 \cong 887$	$2985 \sim 846 \cong 846$	$3031 \sim 942 \cong 942$	$3077 \sim 753 \cong 753$
$2940 \sim 824 \cong 820$	$2986 \sim 824 \cong 820$	$3032 \sim 861 \cong 861$	$3078 \sim 734 \cong 730$
$2941 \sim 842 \cong 838$	$2987 \sim 824 \cong 820$	$3033 \sim 780 \cong 780$	$3079 \sim 1091 \cong 731$
$2942 \sim 869 \cong 869$	$2988 \sim 747 \cong 739$	$3034 \sim 1094 \cong 1090$	$3080 \sim 1091 \cong 731$
$2943 \sim 756 \cong 748$	$2989 \sim 770 \cong 730$	$3035 \sim 1094 \cong 1090$	$3081 \sim 966 \cong 966$
$2944 \sim 1094 \cong 1090$	$2990 \sim 779 \cong 779$	$3036 \sim 933 \cong 849$	$3082 \sim 1091 \cong 731$
$2945 \sim 1094 \cong 1090$	$2991 \sim 743 \cong 739$	$3037 \sim 1094 \cong 1090$	$3083 \sim 1091 \cong 731$
$2946 \sim 963 \cong 963$	$2992 \sim 779 \cong 779$	$3038 \sim 1094 \cong 1090$	$3084 \sim 885 \cong 885$
$2947 \sim 1094 \cong 1090$	$2993 \sim 806 \cong 802$	$3039 \sim 852 \cong 852$	$3085 \sim 966 \cong 966$
$2948 \sim 1094 \cong 1090$	$2994 \sim 752 \cong 752$	$3040 \sim 933 \cong 849$	$3086 \sim 885 \cong 885$
$2949 \sim 882 \cong 882$	$2995 \sim 743 \cong 739$	$3041 \sim 852 \cong 852$	$3087 \sim 804 \cong 731$
$2950 \sim 945 \cong 941$	$2996 \sim 752 \cong 752$	$3042 \sim 771 \cong 771$	$3088 \sim 1091 \cong 731$
$2951 \sim 864 \cong 864$	$2997 \sim 734 \cong 730$	$3043 \sim 933 \cong 849$	$3089 \sim 1091 \cong 731$
$2952 \sim 783 \cong 775$	$2998 \sim 1094 \cong 1090$	$3044 \sim 960 \cong 960$	$3090 \sim 939 \cong 939$
$2953 \sim 1094 \cong 1090$	$2999 \sim 1094 \cong 1090$	$3045 \sim 824 \cong 820$	$3091 \sim 1091 \cong 731$
$2954 \sim 1094 \cong 1090$	$3000 \sim 969 \cong 969$	$3046 \sim 942 \cong 942$	$3092 \sim 1091 \cong 731$
$2955 \sim 936 \cong 820$	$3001 \sim 1094 \cong 1090$	$3047 \sim 969 \cong 969$	$3093 \sim 858 \cong 858$

3094	∼	957	≅	957	3140	∼	821	≅	821	3186	∼	753	≅	753	3232	∼	771	≅	771
3095	∼	876	≅	876	3141	∼	750	≅	750	3187	∼	1094	≅	1090	3233	∼	780	≅	780
3096	∼	777	≅	777	3142	∼	929	≅	929	3188	∼	1094	≅	1090	3234	∼	744	≅	744
3097	∼	848	≅	750	3143	∼	938	≅	938	3189	∼	960	≅	960	3235	∼	780	≅	780
3098	∼	875	≅	875	3144	∼	920	≅	920	3190	∼	1094	≅	1090	3236	∼	807	≅	771
3099	∼	821	≅	821	3145	∼	956	≅	956	3191	∼	1094	≅	1090	3237	∼	753	≅	753
3100	∼	857	≅	857	3146	∼	965	≅	965	3192	∼	879	≅	879	3238	∼	744	≅	744
3101	∼	884	≅	884	3147	∼	840	≅	840	3193	∼	942	≅	942	3239	∼	753	≅	753
3102	∼	821	≅	821	3148	∼	821	≅	821	3194	∼	861	≅	861	3240	∼	734	≅	730
3103	∼	839	≅	821	3149	∼	821	≅	821	3195	∼	780	≅	780	3241	∼	1094	≅	1090
3104	∼	866	≅	866	3150	∼	741	≅	741	3196	∼	1094	≅	1090	3242	∼	1094	≅	1090
3105	∼	750	≅	750	3151	∼	767	≅	731	3197	∼	1094	≅	1090	3243	∼	968	≅	968
3106	∼	1091	≅	731	3152	∼	776	≅	776	3198	∼	933	≅	849	3244	∼	1094	≅	1090
3107	∼	1091	≅	731	3153	∼	740	≅	740	3199	∼	1094	≅	1090	3245	∼	1094	≅	1090
3108	∼	957	≅	957	3154	∼	776	≅	776	3200	∼	1094	≅	1090	3246	∼	887	≅	887
3109	∼	1091	≅	731	3155	∼	803	≅	771	3201	∼	852	≅	852	3247	∼	968	≅	968
3110	∼	1091	≅	731	3156	∼	749	≅	749	3202	∼	933	≅	849	3248	∼	887	≅	887
3111	∼	876	≅	876	3157	∼	740	≅	740	3203	∼	852	≅	852	3249	∼	806	≅	802
3112	∼	939	≅	939	3158	∼	749	≅	749	3204	∼	771	≅	771	3250	∼	1094	≅	1090
3113	∼	858	≅	858	3159	∼	731	≅	731	3205	∼	933	≅	849	3251	∼	1094	≅	1090
3114	∼	777	≅	777	3160	∼	1094	≅	1090	3206	∼	960	≅	960	3252	∼	941	≅	941
3115	∼	1091	≅	731	3161	∼	1094	≅	1090	3207	∼	824	≅	820	3253	∼	1094	≅	1090
3116	∼	1091	≅	731	3162	∼	969	≅	969	3208	∼	942	≅	942	3254	∼	1094	≅	1090
3117	∼	930	≅	821	3163	∼	1094	≅	1090	3209	∼	969	≅	969	3255	∼	860	≅	860
3118	∼	1091	≅	731	3164	∼	1094	≅	1090	3210	∼	824	≅	820	3256	∼	959	≅	959
3119	∼	1091	≅	731	3165	∼	888	≅	888	3211	∼	924	≅	870	3257	∼	878	≅	878
3120	∼	849	≅	849	3166	∼	969	≅	969	3212	∼	843	≅	843	3258	∼	779	≅	779
3121	∼	930	≅	821	3167	∼	888	≅	888	3213	∼	744	≅	744	3259	∼	855	≅	847
3122	∼	849	≅	849	3168	∼	807	≅	771	3214	∼	852	≅	852	3260	∼	882	≅	882
3123	∼	768	≅	731	3169	∼	1094	≅	1090	3215	∼	861	≅	861	3261	∼	824	≅	820
3124	∼	929	≅	929	3170	∼	1094	≅	1090	3216	∼	843	≅	843	3262	∼	864	≅	864
3125	∼	956	≅	956	3171	∼	942	≅	942	3217	∼	879	≅	879	3263	∼	891	≅	891
3126	∼	821	≅	821	3172	∼	1094	≅	1090	3218	∼	888	≅	888	3264	∼	824	≅	820
3127	∼	938	≅	938	3173	∼	1094	≅	1090	3219	∼	870	≅	870	3265	∼	846	≅	846
3128	∼	965	≅	965	3174	∼	861	≅	861	3220	∼	824	≅	820	3266	∼	869	≅	869
3129	∼	821	≅	821	3175	∼	960	≅	960	3221	∼	824	≅	820	3267	∼	752	≅	752
3130	∼	920	≅	920	3176	∼	879	≅	879	3222	∼	753	≅	753	3268	∼	1094	≅	1090
3131	∼	840	≅	840	3177	∼	780	≅	780	3223	∼	933	≅	849	3269	∼	1094	≅	1090
3132	∼	741	≅	741	3178	∼	852	≅	852	3224	∼	942	≅	942	3270	∼	959	≅	959
3133	∼	848	≅	750	3179	∼	879	≅	879	3225	∼	924	≅	870	3271	∼	1094	≅	1090
3134	∼	857	≅	857	3180	∼	824	≅	820	3226	∼	960	≅	960	3272	∼	1094	≅	1090
3135	∼	839	≅	821	3181	∼	861	≅	861	3227	∼	969	≅	969	3273	∼	878	≅	878
3136	∼	875	≅	875	3182	∼	888	≅	888	3228	∼	843	≅	843	3274	∼	941	≅	941
3137	∼	884	≅	884	3183	∼	824	≅	820	3229	∼	824	≅	820	3275	∼	860	≅	860
3138	∼	866	≅	866	3184	∼	843	≅	843	3230	∼	824	≅	820	3276	∼	779	≅	779
3139	∼	821	≅	821	3185	∼	870	≅	870	3231	∼	744	≅	744	3277	∼	1094	≅	1090

3278	∼	1094	≅	1090	3324	∼	965	≅	965	3370	∼	939	≅	939	3416	∼	1091	≅	731
3279	∼	932	≅	820	3325	∼	1091	≅	731	3371	∼	966	≅	966	3417	∼	858	≅	858
3280	∼	1094	≅	1090	3326	∼	1091	≅	731	3372	∼	821	≅	821	3418	∼	957	≅	957
3281	∼	1094	≅	1090	3327	∼	884	≅	884	3373	∼	920	≅	920	3419	∼	876	≅	876
3282	∼	851	≅	847	3328	∼	965	≅	965	3374	∼	839	≅	821	3420	∼	777	≅	777
3283	∼	932	≅	820	3329	∼	884	≅	884	3375	∼	740	≅	740	3421	∼	848	≅	750
3284	∼	851	≅	847	3330	∼	803	≅	771	3376	∼	849	≅	849	3422	∼	875	≅	875
3285	∼	770	≅	730	3331	∼	1091	≅	731	3377	∼	858	≅	858	3423	∼	821	≅	821
3286	∼	936	≅	820	3332	∼	1091	≅	731	3378	∼	840	≅	840	3424	∼	857	≅	857
3287	∼	963	≅	963	3333	∼	938	≅	938	3379	∼	876	≅	876	3425	∼	884	≅	884
3288	∼	824	≅	820	3334	∼	1091	≅	731	3380	∼	885	≅	885	3426	∼	821	≅	821
3289	∼	945	≅	941	3335	∼	1091	≅	731	3381	∼	866	≅	866	3427	∼	839	≅	821
3290	∼	972	≅	739	3336	∼	857	≅	857	3382	∼	821	≅	821	3428	∼	866	≅	866
3291	∼	824	≅	820	3337	∼	956	≅	956	3383	∼	821	≅	821	3429	∼	750	≅	750
3292	∼	923	≅	923	3338	∼	875	≅	875	3384	∼	749	≅	749	3430	∼	1091	≅	731
3293	∼	842	≅	838	3339	∼	776	≅	776	3385	∼	930	≅	821	3431	∼	1091	≅	731
3294	∼	743	≅	739	3340	∼	849	≅	849	3386	∼	939	≅	939	3432	∼	957	≅	957
3295	∼	855	≅	847	3341	∼	876	≅	876	3387	∼	920	≅	920	3433	∼	1091	≅	731
3296	∼	864	≅	864	3342	∼	821	≅	821	3388	∼	957	≅	957	3434	∼	1091	≅	731
3297	∼	846	≅	846	3343	∼	858	≅	858	3389	∼	966	≅	966	3435	∼	876	≅	876
3298	∼	882	≅	882	3344	∼	885	≅	885	3390	∼	839	≅	821	3436	∼	939	≅	939
3299	∼	891	≅	891	3345	∼	821	≅	821	3391	∼	821	≅	821	3437	∼	858	≅	858
3300	∼	869	≅	869	3346	∼	840	≅	840	3392	∼	821	≅	821	3438	∼	777	≅	777
3301	∼	824	≅	820	3347	∼	866	≅	866	3393	∼	740	≅	740	3439	∼	1091	≅	731
3302	∼	824	≅	820	3348	∼	749	≅	749	3394	∼	768	≅	731	3440	∼	1091	≅	731
3303	∼	752	≅	752	3349	∼	1091	≅	731	3395	∼	777	≅	777	3441	∼	930	≅	821
3304	∼	936	≅	820	3350	∼	1091	≅	731	3396	∼	741	≅	741	3442	∼	1091	≅	731
3305	∼	945	≅	941	3351	∼	956	≅	956	3397	∼	777	≅	777	3443	∼	1091	≅	731
3306	∼	923	≅	923	3352	∼	1091	≅	731	3398	∼	804	≅	731	3444	∼	849	≅	849
3307	∼	963	≅	963	3353	∼	1091	≅	731	3399	∼	750	≅	750	3445	∼	930	≅	821
3308	∼	972	≅	739	3354	∼	875	≅	875	3400	∼	741	≅	741	3446	∼	849	≅	849
3309	∼	842	≅	838	3355	∼	938	≅	938	3401	∼	750	≅	750	3447	∼	768	≅	731
3310	∼	824	≅	820	3356	∼	857	≅	857	3402	∼	731	≅	731	3448	∼	929	≅	929
3311	∼	824	≅	820	3357	∼	776	≅	776	3403	∼	1091	≅	731	3449	∼	956	≅	956
3312	∼	743	≅	739	3358	∼	1091	≅	731	3404	∼	1091	≅	731	3450	∼	821	≅	821
3313	∼	774	≅	730	3359	∼	1091	≅	731	3405	∼	966	≅	966	3451	∼	938	≅	938
3314	∼	783	≅	775	3360	∼	929	≅	929	3406	∼	1091	≅	731	3452	∼	965	≅	965
3315	∼	747	≅	739	3361	∼	1091	≅	731	3407	∼	1091	≅	731	3453	∼	821	≅	821
3316	∼	783	≅	775	3362	∼	1091	≅	731	3408	∼	885	≅	885	3454	∼	920	≅	920
3317	∼	810	≅	802	3363	∼	848	≅	750	3409	∼	966	≅	966	3455	∼	840	≅	840
3318	∼	756	≅	748	3364	∼	929	≅	929	3410	∼	885	≅	885	3456	∼	741	≅	741
3319	∼	747	≅	739	3365	∼	848	≅	750	3411	∼	804	≅	731	3457	∼	848	≅	750
3320	∼	756	≅	748	3366	∼	767	≅	731	3412	∼	1091	≅	731	3458	∼	857	≅	857
3321	∼	734	≅	730	3367	∼	930	≅	821	3413	∼	1091	≅	731	3459	∼	839	≅	821
3322	∼	1091	≅	731	3368	∼	957	≅	957	3414	∼	939	≅	939	3460	∼	875	≅	875
3323	∼	1091	≅	731	3369	∼	821	≅	821	3415	∼	1091	≅	731	3461	∼	884	≅	884

3462	$\sim$	866	$\cong$	866
3463	$\sim$	821	$\cong$	821
3464	$\sim$	821	$\cong$	821
3465	$\sim$	750	$\cong$	750
3466	$\sim$	929	$\cong$	929
3467	$\sim$	938	$\cong$	938
3468	$\sim$	920	$\cong$	920
3469	$\sim$	956	$\cong$	956
3470	$\sim$	965	$\cong$	965
3471	$\sim$	840	$\cong$	840
3472	$\sim$	821	$\cong$	821
3473	$\sim$	821	$\cong$	821
3474	$\sim$	741	$\cong$	741
3475	$\sim$	767	$\cong$	731
3476	$\sim$	776	$\cong$	776
3477	$\sim$	740	$\cong$	740
3478	$\sim$	776	$\cong$	776
3479	$\sim$	803	$\cong$	771
3480	$\sim$	749	$\cong$	749
3481	$\sim$	740	$\cong$	740
3482	$\sim$	749	$\cong$	749
3483	$\sim$	731	$\cong$	731
3484	$\sim$	1091	$\cong$	731
3485	$\sim$	1091	$\cong$	731
3486	$\sim$	965	$\cong$	965
3487	$\sim$	1091	$\cong$	731
3488	$\sim$	1091	$\cong$	731
3489	$\sim$	884	$\cong$	884
3490	$\sim$	965	$\cong$	965
3491	$\sim$	884	$\cong$	884
3492	$\sim$	803	$\cong$	771
3493	$\sim$	1091	$\cong$	731
3494	$\sim$	1091	$\cong$	731
3495	$\sim$	938	$\cong$	938
3496	$\sim$	1091	$\cong$	731
3497	$\sim$	1091	$\cong$	731
3498	$\sim$	857	$\cong$	857
3499	$\sim$	956	$\cong$	956
3500	$\sim$	875	$\cong$	875
3501	$\sim$	776	$\cong$	776
3502	$\sim$	849	$\cong$	849
3503	$\sim$	876	$\cong$	876
3504	$\sim$	821	$\cong$	821
3505	$\sim$	858	$\cong$	858
3506	$\sim$	885	$\cong$	885
3507	$\sim$	821	$\cong$	821

3508	$\sim$	840	$\cong$	840
3509	$\sim$	866	$\cong$	866
3510	$\sim$	749	$\cong$	749
3511	$\sim$	1091	$\cong$	731
3512	$\sim$	1091	$\cong$	731
3513	$\sim$	956	$\cong$	956
3514	$\sim$	1091	$\cong$	731
3515	$\sim$	1091	$\cong$	731
3516	$\sim$	875	$\cong$	875
3517	$\sim$	938	$\cong$	938
3518	$\sim$	857	$\cong$	857
3519	$\sim$	776	$\cong$	776
3520	$\sim$	1091	$\cong$	731
3521	$\sim$	1091	$\cong$	731
3522	$\sim$	929	$\cong$	929
3523	$\sim$	1091	$\cong$	731
3524	$\sim$	1091	$\cong$	731
3525	$\sim$	848	$\cong$	750
3526	$\sim$	929	$\cong$	929
3527	$\sim$	848	$\cong$	750
3528	$\sim$	767	$\cong$	731
3529	$\sim$	930	$\cong$	821
3530	$\sim$	957	$\cong$	957
3531	$\sim$	821	$\cong$	821
3532	$\sim$	939	$\cong$	939
3533	$\sim$	966	$\cong$	966
3534	$\sim$	821	$\cong$	821
3535	$\sim$	920	$\cong$	920
3536	$\sim$	839	$\cong$	821
3537	$\sim$	740	$\cong$	740
3538	$\sim$	849	$\cong$	849
3539	$\sim$	858	$\cong$	858
3540	$\sim$	840	$\cong$	840
3541	$\sim$	876	$\cong$	876
3542	$\sim$	885	$\cong$	885
3543	$\sim$	866	$\cong$	866
3544	$\sim$	821	$\cong$	821
3545	$\sim$	821	$\cong$	821
3546	$\sim$	749	$\cong$	749
3547	$\sim$	930	$\cong$	821
3548	$\sim$	939	$\cong$	939
3549	$\sim$	920	$\cong$	920
3550	$\sim$	957	$\cong$	957
3551	$\sim$	966	$\cong$	966
3552	$\sim$	839	$\cong$	821
3553	$\sim$	821	$\cong$	821

3554	$\sim$	821	$\cong$	821
3555	$\sim$	740	$\cong$	740
3556	$\sim$	768	$\cong$	731
3557	$\sim$	777	$\cong$	777
3558	$\sim$	741	$\cong$	741
3559	$\sim$	777	$\cong$	777
3560	$\sim$	804	$\cong$	731
3561	$\sim$	750	$\cong$	750
3562	$\sim$	741	$\cong$	741
3563	$\sim$	750	$\cong$	750
3564	$\sim$	731	$\cong$	731
3565	$\sim$	1090	$\cong$	1090
3566	$\sim$	1090	$\cong$	1090
3567	$\sim$	964	$\cong$	739
3568	$\sim$	1090	$\cong$	1090
3569	$\sim$	1090	$\cong$	1090
3570	$\sim$	883	$\cong$	883
3571	$\sim$	964	$\cong$	739
3572	$\sim$	883	$\cong$	883
3573	$\sim$	802	$\cong$	802
3574	$\sim$	1090	$\cong$	1090
3575	$\sim$	1090	$\cong$	1090
3576	$\sim$	937	$\cong$	937
3577	$\sim$	1090	$\cong$	1090
3578	$\sim$	1090	$\cong$	1090
3579	$\sim$	856	$\cong$	856
3580	$\sim$	955	$\cong$	937
3581	$\sim$	874	$\cong$	874
3582	$\sim$	775	$\cong$	775
3583	$\sim$	847	$\cong$	847
3584	$\sim$	874	$\cong$	874
3585	$\sim$	820	$\cong$	820
3586	$\sim$	856	$\cong$	856
3587	$\sim$	883	$\cong$	883
3588	$\sim$	820	$\cong$	820
3589	$\sim$	838	$\cong$	838
3590	$\sim$	865	$\cong$	820
3591	$\sim$	748	$\cong$	748
3592	$\sim$	1090	$\cong$	1090
3593	$\sim$	1090	$\cong$	1090
3594	$\sim$	955	$\cong$	937
3595	$\sim$	1090	$\cong$	1090
3596	$\sim$	1090	$\cong$	1090
3597	$\sim$	874	$\cong$	874
3598	$\sim$	937	$\cong$	937
3599	$\sim$	856	$\cong$	856

3600	$\sim$	775	$\cong$	775
3601	$\sim$	1090	$\cong$	1090
3602	$\sim$	1090	$\cong$	1090
3603	$\sim$	928	$\cong$	820
3604	$\sim$	1090	$\cong$	1090
3605	$\sim$	1090	$\cong$	1090
3606	$\sim$	847	$\cong$	847
3607	$\sim$	928	$\cong$	820
3608	$\sim$	847	$\cong$	847
3609	$\sim$	766	$\cong$	730
3610	$\sim$	928	$\cong$	820
3611	$\sim$	955	$\cong$	937
3612	$\sim$	820	$\cong$	820
3613	$\sim$	937	$\cong$	937
3614	$\sim$	964	$\cong$	739
3615	$\sim$	820	$\cong$	820
3616	$\sim$	919	$\cong$	820
3617	$\sim$	838	$\cong$	838
3618	$\sim$	739	$\cong$	739
3619	$\sim$	847	$\cong$	847
3620	$\sim$	856	$\cong$	856
3621	$\sim$	838	$\cong$	838
3622	$\sim$	874	$\cong$	874
3623	$\sim$	883	$\cong$	883
3624	$\sim$	865	$\cong$	820
3625	$\sim$	820	$\cong$	820
3626	$\sim$	820	$\cong$	820
3627	$\sim$	748	$\cong$	748
3628	$\sim$	928	$\cong$	820
3629	$\sim$	937	$\cong$	937
3630	$\sim$	919	$\cong$	820
3631	$\sim$	955	$\cong$	937
3632	$\sim$	964	$\cong$	739
3633	$\sim$	838	$\cong$	838
3634	$\sim$	820	$\cong$	820
3635	$\sim$	820	$\cong$	820
3636	$\sim$	739	$\cong$	739
3637	$\sim$	766	$\cong$	730
3638	$\sim$	775	$\cong$	775
3639	$\sim$	739	$\cong$	739
3640	$\sim$	775	$\cong$	775
3641	$\sim$	802	$\cong$	802
3642	$\sim$	748	$\cong$	748
3643	$\sim$	739	$\cong$	739
3644	$\sim$	748	$\cong$	748
3645	$\sim$	730	$\cong$	730

$3646 \sim 730 \cong 730$	$3692 \sim 2399 \cong 2399$	$3738 \sim 2369 \cong 2369$	$3784 \sim 2293 \cong 2293$
$3647 \sim 2190 \cong 750$	$3693 \sim 820 \cong 820$	$3739 \sim 2374 \cong 821$	$3785 \sim 2295 \cong 2295$
$3648 \sim 730 \cong 730$	$3694 \sim 2399 \cong 2399$	$3740 \sim 2376 \cong 739$	$3786 \sim 2294 \cong 2294$
$3649 \sim 2190 \cong 750$	$3695 \sim 2426 \cong 2277$	$3741 \sim 2375 \cong 2375$	$3787 \sim 2283 \cong 2283$
$3650 \sim 2196 \cong 802$	$3696 \sim 2372 \cong 2372$	$3742 \sim 2371 \cong 2371$	$3788 \sim 731 \cong 731$
$3651 \sim 2193 \cong 2193$	$3697 \sim 820 \cong 820$	$3743 \sim 821 \cong 821$	$3789 \sim 2274 \cong 2274$
$3652 \sim 730 \cong 730$	$3698 \sim 2372 \cong 2372$	$3744 \sim 2372 \cong 2372$	$3790 \sim 2391 \cong 2391$
$3653 \sim 2193 \cong 2193$	$3699 \sim 820 \cong 820$	$3745 \sim 2287 \cong 2287$	$3791 \sim 821 \cong 821$
$3654 \sim 730 \cong 730$	$3700 \sim 730 \cong 730$	$3746 \sim 731 \cong 731$	$3792 \sim 2366 \cong 2366$
$3655 \sim 820 \cong 820$	$3701 \sim 2271 \cong 2271$	$3747 \sim 2285 \cong 2285$	$3793 \sim 2402 \cong 2402$
$3656 \sim 2352 \cong 740$	$3702 \sim 730 \cong 730$	$3748 \sim 2293 \cong 2293$	$3794 \sim 2427 \cong 2427$
$3657 \sim 820 \cong 820$	$3703 \sim 2271 \cong 2271$	$3749 \sim 2295 \cong 2295$	$3795 \sim 2375 \cong 2375$
$3658 \sim 2352 \cong 740$	$3704 \sim 2277 \cong 2277$	$3750 \sim 2294 \cong 2294$	$3796 \sim 2364 \cong 2364$
$3659 \sim 2358 \cong 820$	$3705 \sim 2274 \cong 2274$	$3751 \sim 2283 \cong 2283$	$3797 \sim 821 \cong 821$
$3660 \sim 2355 \cong 2355$	$3706 \sim 730 \cong 730$	$3752 \sim 731 \cong 731$	$3798 \sim 2355 \cong 2355$
$3661 \sim 820 \cong 820$	$3707 \sim 2274 \cong 2274$	$3753 \sim 2274 \cong 2274$	$3799 \sim 2229 \cong 2229$
$3662 \sim 2355 \cong 2355$	$3708 \sim 730 \cong 730$	$3754 \sim 2368 \cong 739$	$3800 \sim 731 \cong 731$
$3663 \sim 820 \cong 820$	$3709 \sim 820 \cong 820$	$3755 \sim 821 \cong 821$	$3801 \sim 2204 \cong 2204$
$3664 \sim 730 \cong 730$	$3710 \sim 2399 \cong 2399$	$3756 \sim 2369 \cong 2369$	$3802 \sim 2240 \cong 2240$
$3665 \sim 2271 \cong 2271$	$3711 \sim 820 \cong 820$	$3757 \sim 2374 \cong 821$	$3803 \sim 2265 \cong 2265$
$3666 \sim 730 \cong 730$	$3712 \sim 2399 \cong 2399$	$3758 \sim 2376 \cong 739$	$3804 \sim 2213 \cong 2213$
$3667 \sim 2271 \cong 2271$	$3713 \sim 2426 \cong 2277$	$3759 \sim 2375 \cong 2375$	$3805 \sim 2202 \cong 2202$
$3668 \sim 2277 \cong 2277$	$3714 \sim 2372 \cong 2372$	$3760 \sim 2371 \cong 2371$	$3806 \sim 731 \cong 731$
$3669 \sim 2274 \cong 2274$	$3715 \sim 820 \cong 820$	$3761 \sim 821 \cong 821$	$3807 \sim 2193 \cong 2193$
$3670 \sim 730 \cong 730$	$3716 \sim 2372 \cong 2372$	$3762 \sim 2372 \cong 2372$	$3808 \sim 730 \cong 730$
$3671 \sim 2274 \cong 2274$	$3717 \sim 820 \cong 820$	$3763 \sim 2854 \cong 847$	$3809 \sim 2199 \cong 2199$
$3672 \sim 730 \cong 730$	$3718 \sim 730 \cong 730$	$3764 \sim 1091 \cong 731$	$3810 \sim 730 \cong 730$
$3673 \sim 820 \cong 820$	$3719 \sim 2237 \cong 2237$	$3765 \sim 2852 \cong 849$	$3811 \sim 2203 \cong 2203$
$3674 \sim 2352 \cong 740$	$3720 \sim 730 \cong 730$	$3766 \sim 2860 \cong 2212$	$3812 \sim 2205 \cong 775$
$3675 \sim 820 \cong 820$	$3721 \sim 2237 \cong 2237$	$3767 \sim 2862 \cong 847$	$3813 \sim 2204 \cong 2204$
$3676 \sim 2352 \cong 740$	$3722 \sim 2264 \cong 730$	$3768 \sim 2861 \cong 731$	$3814 \sim 730 \cong 730$
$3677 \sim 2358 \cong 820$	$3723 \sim 2210 \cong 2210$	$3769 \sim 2850 \cong 2850$	$3815 \sim 2202 \cong 2202$
$3678 \sim 2355 \cong 2355$	$3724 \sim 730 \cong 730$	$3770 \sim 1091 \cong 731$	$3816 \sim 730 \cong 730$
$3679 \sim 820 \cong 820$	$3725 \sim 2210 \cong 2210$	$3771 \sim 2841 \cong 2841$	$3817 \sim 820 \cong 820$
$3680 \sim 2355 \cong 2355$	$3726 \sim 730 \cong 730$	$3772 \sim 2391 \cong 2391$	$3818 \sim 2361 \cong 2361$
$3681 \sim 820 \cong 820$	$3727 \sim 2206 \cong 748$	$3773 \sim 821 \cong 821$	$3819 \sim 820 \cong 820$
$3682 \sim 1090 \cong 1090$	$3728 \sim 731 \cong 731$	$3774 \sim 2366 \cong 2366$	$3820 \sim 2365 \cong 2365$
$3683 \sim 2838 \cong 750$	$3729 \sim 2207 \cong 2207$	$3775 \sim 2402 \cong 2402$	$3821 \sim 2367 \cong 2367$
$3684 \sim 1090 \cong 1090$	$3730 \sim 2212 \cong 2212$	$3776 \sim 2427 \cong 2427$	$3822 \sim 2366 \cong 2366$
$3685 \sim 2838 \cong 750$	$3731 \sim 2214 \cong 748$	$3777 \sim 2375 \cong 2375$	$3823 \sim 820 \cong 820$
$3686 \sim 2844 \cong 730$	$3732 \sim 2213 \cong 2213$	$3778 \sim 2364 \cong 2364$	$3824 \sim 2364 \cong 2364$
$3687 \sim 2841 \cong 2841$	$3733 \sim 2209 \cong 2209$	$3779 \sim 821 \cong 821$	$3825 \sim 820 \cong 820$
$3688 \sim 1090 \cong 1090$	$3734 \sim 731 \cong 731$	$3780 \sim 2355 \cong 2355$	$3826 \sim 730 \cong 730$
$3689 \sim 2841 \cong 2841$	$3735 \sim 2210 \cong 2210$	$3781 \sim 2287 \cong 2287$	$3827 \sim 2280 \cong 2280$
$3690 \sim 1090 \cong 1090$	$3736 \sim 2368 \cong 739$	$3782 \sim 731 \cong 731$	$3828 \sim 730 \cong 730$
$3691 \sim 820 \cong 820$	$3737 \sim 821 \cong 821$	$3783 \sim 2285 \cong 2285$	$3829 \sim 2284 \cong 2284$

3830 ∼ 2286 ≅ 2286	3876 ∼ 2369 ≅ 2369	3922 ∼ 2369 ≅ 2369	3968 ∼ 2213 ≅ 2213
3831 ∼ 2285 ≅ 2285	3877 ∼ 820 ≅ 820	3923 ∼ 2375 ≅ 2375	3969 ∼ 2193 ≅ 2193
3832 ∼ 730 ≅ 730	3878 ∼ 2371 ≅ 2371	3924 ∼ 2372 ≅ 2372	3970 ∼ 2260 ≅ 802
3833 ∼ 2283 ≅ 2283	3879 ∼ 820 ≅ 820	3925 ∼ 2854 ≅ 847	3971 ∼ 2262 ≅ 750
3834 ∼ 730 ≅ 730	3880 ∼ 730 ≅ 730	3926 ∼ 2860 ≅ 2212	3972 ∼ 2261 ≅ 2261
3835 ∼ 820 ≅ 820	3881 ∼ 2236 ≅ 2236	3927 ∼ 2850 ≅ 2850	3973 ∼ 2262 ≅ 750
3836 ∼ 2361 ≅ 2361	3882 ∼ 730 ≅ 730	3928 ∼ 1091 ≅ 731	3974 ∼ 734 ≅ 730
3837 ∼ 820 ≅ 820	3883 ∼ 2234 ≅ 2234	3929 ∼ 2862 ≅ 847	3975 ∼ 2265 ≅ 2265
3838 ∼ 2365 ≅ 2365	3884 ∼ 2261 ≅ 2261	3930 ∼ 1091 ≅ 731	3976 ∼ 2261 ≅ 2261
3839 ∼ 2367 ≅ 2367	3885 ∼ 2207 ≅ 2207	3931 ∼ 2852 ≅ 849	3977 ∼ 2265 ≅ 2265
3840 ∼ 2366 ≅ 2366	3886 ∼ 730 ≅ 730	3932 ∼ 2861 ≅ 731	3978 ∼ 2264 ≅ 730
3841 ∼ 820 ≅ 820	3887 ∼ 2209 ≅ 2209	3933 ∼ 2841 ≅ 2841	3979 ∼ 2422 ≅ 820
3842 ∼ 2364 ≅ 2364	3888 ∼ 730 ≅ 730	3934 ∼ 2391 ≅ 2391	3980 ∼ 2424 ≅ 966
3843 ∼ 820 ≅ 820	3889 ∼ 2206 ≅ 748	3935 ∼ 2402 ≅ 2402	3981 ∼ 2423 ≅ 2423
3844 ∼ 1090 ≅ 1090	3890 ∼ 2212 ≅ 2212	3936 ∼ 2364 ≅ 2364	3982 ∼ 2424 ≅ 966
3845 ∼ 2847 ≅ 929	3891 ∼ 2209 ≅ 2209	3937 ∼ 821 ≅ 821	3983 ∼ 824 ≅ 820
3846 ∼ 1090 ≅ 1090	3892 ∼ 731 ≅ 731	3938 ∼ 2427 ≅ 2427	3984 ∼ 2427 ≅ 2427
3847 ∼ 2851 ≅ 929	3893 ∼ 2214 ≅ 748	3939 ∼ 821 ≅ 821	3985 ∼ 2423 ≅ 2423
3848 ∼ 2853 ≅ 2853	3894 ∼ 731 ≅ 731	3940 ∼ 2366 ≅ 2366	3986 ∼ 2427 ≅ 2427
3849 ∼ 2852 ≅ 849	3895 ∼ 2207 ≅ 2207	3941 ∼ 2375 ≅ 2375	3987 ∼ 2426 ≅ 2277
3850 ∼ 1090 ≅ 1090	3896 ∼ 2213 ≅ 2213	3942 ∼ 2355 ≅ 2355	3988 ∼ 2313 ≅ 2277
3851 ∼ 2850 ≅ 2850	3897 ∼ 2210 ≅ 2210	3943 ∼ 2287 ≅ 2287	3989 ∼ 2322 ≅ 2322
3852 ∼ 1090 ≅ 1090	3898 ∼ 2368 ≅ 739	3944 ∼ 2293 ≅ 2293	3990 ∼ 2286 ≅ 2286
3853 ∼ 820 ≅ 820	3899 ∼ 2374 ≅ 821	3945 ∼ 2283 ≅ 2283	3991 ∼ 2322 ≅ 2322
3854 ∼ 2398 ≅ 2398	3900 ∼ 2371 ≅ 2371	3946 ∼ 731 ≅ 731	3992 ∼ 734 ≅ 730
3855 ∼ 820 ≅ 820	3901 ∼ 821 ≅ 821	3947 ∼ 2295 ≅ 2295	3993 ∼ 2295 ≅ 2295
3856 ∼ 2396 ≅ 2396	3902 ∼ 2376 ≅ 739	3948 ∼ 731 ≅ 731	3994 ∼ 2286 ≅ 2286
3857 ∼ 2423 ≅ 2423	3903 ∼ 821 ≅ 821	3949 ∼ 2285 ≅ 2285	3995 ∼ 2295 ≅ 2295
3858 ∼ 2369 ≅ 2369	3904 ∼ 2369 ≅ 2369	3950 ∼ 2294 ≅ 2294	3996 ∼ 2277 ≅ 2277
3859 ∼ 820 ≅ 820	3905 ∼ 2375 ≅ 2375	3951 ∼ 2274 ≅ 2274	3997 ∼ 2422 ≅ 820
3860 ∼ 2371 ≅ 2371	3906 ∼ 2372 ≅ 2372	3952 ∼ 2391 ≅ 2391	3998 ∼ 2424 ≅ 966
3861 ∼ 820 ≅ 820	3907 ∼ 2287 ≅ 2287	3953 ∼ 2402 ≅ 2402	3999 ∼ 2423 ≅ 2423
3862 ∼ 730 ≅ 730	3908 ∼ 2293 ≅ 2293	3954 ∼ 2364 ≅ 2364	4000 ∼ 2424 ≅ 966
3863 ∼ 2280 ≅ 2280	3909 ∼ 2283 ≅ 2283	3955 ∼ 821 ≅ 821	4001 ∼ 824 ≅ 820
3864 ∼ 730 ≅ 730	3910 ∼ 731 ≅ 731	3956 ∼ 2427 ≅ 2427	4002 ∼ 2427 ≅ 2427
3865 ∼ 2284 ≅ 2284	3911 ∼ 2295 ≅ 2295	3957 ∼ 821 ≅ 821	4003 ∼ 2423 ≅ 2423
3866 ∼ 2286 ≅ 2286	3912 ∼ 731 ≅ 731	3958 ∼ 2366 ≅ 2366	4004 ∼ 2427 ≅ 2427
3867 ∼ 2285 ≅ 2285	3913 ∼ 2285 ≅ 2285	3959 ∼ 2375 ≅ 2375	4005 ∼ 2426 ≅ 2277
3868 ∼ 730 ≅ 730	3914 ∼ 2294 ≅ 2294	3960 ∼ 2355 ≅ 2355	4006 ∼ 2880 ≅ 730
3869 ∼ 2283 ≅ 2283	3915 ∼ 2274 ≅ 2274	3961 ∼ 2229 ≅ 2229	4007 ∼ 2889 ≅ 750
3870 ∼ 730 ≅ 730	3916 ∼ 2368 ≅ 739	3962 ∼ 2240 ≅ 2240	4008 ∼ 2853 ≅ 2853
3871 ∼ 820 ≅ 820	3917 ∼ 2374 ≅ 821	3963 ∼ 2202 ≅ 2202	4009 ∼ 2889 ≅ 750
3872 ∼ 2398 ≅ 2398	3918 ∼ 2371 ≅ 2371	3964 ∼ 731 ≅ 731	4010 ∼ 1094 ≅ 1090
3873 ∼ 820 ≅ 820	3919 ∼ 821 ≅ 821	3965 ∼ 2265 ≅ 2265	4011 ∼ 2862 ≅ 847
3874 ∼ 2396 ≅ 2396	3920 ∼ 2376 ≅ 739	3966 ∼ 731 ≅ 731	4012 ∼ 2853 ≅ 2853
3875 ∼ 2423 ≅ 2423	3921 ∼ 821 ≅ 821	3967 ∼ 2204 ≅ 2204	4013 ∼ 2862 ≅ 847

4014 $\sim$ 2844 $\cong$ 730	4060 $\sim$ 2395 $\cong$ 2395	4106 $\sim$ 2320 $\cong$ 2294	4152 $\sim$ 730 $\cong$ 730
4015 $\sim$ 2394 $\cong$ 820	4061 $\sim$ 2401 $\cong$ 2401	4107 $\sim$ 2280 $\cong$ 2280	4153 $\sim$ 2280 $\cong$ 2280
4016 $\sim$ 2403 $\cong$ 2287	4062 $\sim$ 2398 $\cong$ 2398	4108 $\sim$ 731 $\cong$ 731	4154 $\sim$ 2286 $\cong$ 2286
4017 $\sim$ 2367 $\cong$ 2367	4063 $\sim$ 821 $\cong$ 821	4109 $\sim$ 2322 $\cong$ 2322	4155 $\sim$ 2283 $\cong$ 2283
4018 $\sim$ 2403 $\cong$ 2287	4064 $\sim$ 2403 $\cong$ 2287	4110 $\sim$ 731 $\cong$ 731	4156 $\sim$ 730 $\cong$ 730
4019 $\sim$ 824 $\cong$ 820	4065 $\sim$ 821 $\cong$ 821	4111 $\sim$ 2284 $\cong$ 2284	4157 $\sim$ 2285 $\cong$ 2285
4020 $\sim$ 2376 $\cong$ 739	4066 $\sim$ 2396 $\cong$ 2396	4112 $\sim$ 2293 $\cong$ 2293	4158 $\sim$ 730 $\cong$ 730
4021 $\sim$ 2367 $\cong$ 2367	4067 $\sim$ 2402 $\cong$ 2402	4113 $\sim$ 2271 $\cong$ 2271	4159 $\sim$ 820 $\cong$ 820
4022 $\sim$ 2376 $\cong$ 739	4068 $\sim$ 2399 $\cong$ 2399	4114 $\sim$ 2388 $\cong$ 821	4160 $\sim$ 2365 $\cong$ 2365
4023 $\sim$ 2358 $\cong$ 820	4069 $\sim$ 2307 $\cong$ 2307	4115 $\sim$ 2401 $\cong$ 2401	4161 $\sim$ 820 $\cong$ 820
4024 $\sim$ 2313 $\cong$ 2277	4070 $\sim$ 2320 $\cong$ 2294	4116 $\sim$ 2361 $\cong$ 2361	4162 $\sim$ 2361 $\cong$ 2361
4025 $\sim$ 2322 $\cong$ 2322	4071 $\sim$ 2280 $\cong$ 2280	4117 $\sim$ 821 $\cong$ 821	4163 $\sim$ 2367 $\cong$ 2367
4026 $\sim$ 2286 $\cong$ 2286	4072 $\sim$ 731 $\cong$ 731	4118 $\sim$ 2424 $\cong$ 966	4164 $\sim$ 2364 $\cong$ 2364
4027 $\sim$ 2322 $\cong$ 2322	4073 $\sim$ 2322 $\cong$ 2322	4119 $\sim$ 821 $\cong$ 821	4165 $\sim$ 820 $\cong$ 820
4028 $\sim$ 734 $\cong$ 730	4074 $\sim$ 731 $\cong$ 731	4120 $\sim$ 2365 $\cong$ 2365	4166 $\sim$ 2366 $\cong$ 2366
4029 $\sim$ 2295 $\cong$ 2295	4075 $\sim$ 2284 $\cong$ 2284	4121 $\sim$ 2374 $\cong$ 821	4167 $\sim$ 820 $\cong$ 820
4030 $\sim$ 2286 $\cong$ 2286	4076 $\sim$ 2293 $\cong$ 2293	4122 $\sim$ 2352 $\cong$ 740	4168 $\sim$ 1090 $\cong$ 1090
4031 $\sim$ 2295 $\cong$ 2295	4077 $\sim$ 2271 $\cong$ 2271	4123 $\sim$ 2226 $\cong$ 820	4169 $\sim$ 2851 $\cong$ 929
4032 $\sim$ 2277 $\cong$ 2277	4078 $\sim$ 2395 $\cong$ 2395	4124 $\sim$ 2239 $\cong$ 2239	4170 $\sim$ 1090 $\cong$ 1090
4033 $\sim$ 2394 $\cong$ 820	4079 $\sim$ 2401 $\cong$ 2401	4125 $\sim$ 2199 $\cong$ 2199	4171 $\sim$ 2847 $\cong$ 929
4034 $\sim$ 2403 $\cong$ 2287	4080 $\sim$ 2398 $\cong$ 2398	4126 $\sim$ 731 $\cong$ 731	4172 $\sim$ 2853 $\cong$ 2853
4035 $\sim$ 2367 $\cong$ 2367	4081 $\sim$ 821 $\cong$ 821	4127 $\sim$ 2262 $\cong$ 750	4173 $\sim$ 2850 $\cong$ 2850
4036 $\sim$ 2403 $\cong$ 2287	4082 $\sim$ 2403 $\cong$ 2287	4128 $\sim$ 731 $\cong$ 731	4174 $\sim$ 1090 $\cong$ 1090
4037 $\sim$ 824 $\cong$ 820	4083 $\sim$ 821 $\cong$ 821	4129 $\sim$ 2203 $\cong$ 2203	4175 $\sim$ 2852 $\cong$ 849
4038 $\sim$ 2376 $\cong$ 739	4084 $\sim$ 2396 $\cong$ 2396	4130 $\sim$ 2212 $\cong$ 2212	4176 $\sim$ 1090 $\cong$ 1090
4039 $\sim$ 2367 $\cong$ 2367	4085 $\sim$ 2402 $\cong$ 2402	4131 $\sim$ 2190 $\cong$ 750	4177 $\sim$ 820 $\cong$ 820
4040 $\sim$ 2376 $\cong$ 739	4086 $\sim$ 2399 $\cong$ 2399	4132 $\sim$ 730 $\cong$ 730	4178 $\sim$ 2396 $\cong$ 2396
4041 $\sim$ 2358 $\cong$ 820	4087 $\sim$ 2874 $\cong$ 820	4133 $\sim$ 2203 $\cong$ 2203	4179 $\sim$ 820 $\cong$ 820
4042 $\sim$ 2232 $\cong$ 730	4088 $\sim$ 2887 $\cong$ 731	4134 $\sim$ 730 $\cong$ 730	4180 $\sim$ 2398 $\cong$ 2398
4043 $\sim$ 2241 $\cong$ 739	4089 $\sim$ 2847 $\cong$ 929	4135 $\sim$ 2199 $\cong$ 2199	4181 $\sim$ 2423 $\cong$ 2423
4044 $\sim$ 2205 $\cong$ 775	4090 $\sim$ 1091 $\cong$ 731	4136 $\sim$ 2205 $\cong$ 775	4182 $\sim$ 2371 $\cong$ 2371
4045 $\sim$ 2241 $\cong$ 739	4091 $\sim$ 2889 $\cong$ 750	4137 $\sim$ 2202 $\cong$ 2202	4183 $\sim$ 820 $\cong$ 820
4046 $\sim$ 734 $\cong$ 730	4092 $\sim$ 1091 $\cong$ 731	4138 $\sim$ 730 $\cong$ 730	4184 $\sim$ 2369 $\cong$ 2369
4047 $\sim$ 2214 $\cong$ 748	4093 $\sim$ 2851 $\cong$ 929	4139 $\sim$ 2204 $\cong$ 2204	4185 $\sim$ 820 $\cong$ 820
4048 $\sim$ 2205 $\cong$ 775	4094 $\sim$ 2860 $\cong$ 2212	4140 $\sim$ 730 $\cong$ 730	4186 $\sim$ 730 $\cong$ 730
4049 $\sim$ 2214 $\cong$ 748	4095 $\sim$ 2838 $\cong$ 750	4141 $\sim$ 820 $\cong$ 820	4187 $\sim$ 2284 $\cong$ 2284
4050 $\sim$ 2196 $\cong$ 802	4096 $\sim$ 2388 $\cong$ 821	4142 $\sim$ 2365 $\cong$ 2365	4188 $\sim$ 730 $\cong$ 730
4051 $\sim$ 2233 $\cong$ 2233	4097 $\sim$ 2401 $\cong$ 2401	4143 $\sim$ 820 $\cong$ 820	4189 $\sim$ 2280 $\cong$ 2280
4052 $\sim$ 2239 $\cong$ 2239	4098 $\sim$ 2361 $\cong$ 2361	4144 $\sim$ 2361 $\cong$ 2361	4190 $\sim$ 2286 $\cong$ 2286
4053 $\sim$ 2236 $\cong$ 2236	4099 $\sim$ 821 $\cong$ 821	4145 $\sim$ 2367 $\cong$ 2367	4191 $\sim$ 2283 $\cong$ 2283
4054 $\sim$ 731 $\cong$ 731	4100 $\sim$ 2424 $\cong$ 966	4146 $\sim$ 2364 $\cong$ 2364	4192 $\sim$ 730 $\cong$ 730
4055 $\sim$ 2241 $\cong$ 739	4101 $\sim$ 821 $\cong$ 821	4147 $\sim$ 820 $\cong$ 820	4193 $\sim$ 2285 $\cong$ 2285
4056 $\sim$ 731 $\cong$ 731	4102 $\sim$ 2365 $\cong$ 2365	4148 $\sim$ 2366 $\cong$ 2366	4194 $\sim$ 730 $\cong$ 730
4057 $\sim$ 2234 $\cong$ 2234	4103 $\sim$ 2374 $\cong$ 821	4149 $\sim$ 820 $\cong$ 820	4195 $\sim$ 820 $\cong$ 820
4058 $\sim$ 2240 $\cong$ 2240	4104 $\sim$ 2352 $\cong$ 740	4150 $\sim$ 730 $\cong$ 730	4196 $\sim$ 2396 $\cong$ 2396
4059 $\sim$ 2237 $\cong$ 2237	4105 $\sim$ 2307 $\cong$ 2307	4151 $\sim$ 2284 $\cong$ 2284	4197 $\sim$ 820 $\cong$ 820

$4198 \sim 2398 \cong 2398$	$4244 \sim 2403 \cong 2287$	$4290 \sim 2212 \cong 2212$	$4336 \sim 1090 \cong 1090$
$4199 \sim 2423 \cong 2423$	$4245 \sim 2402 \cong 2402$	$4291 \sim 2199 \cong 2199$	$4337 \sim 2854 \cong 847$
$4200 \sim 2371 \cong 2371$	$4246 \sim 2398 \cong 2398$	$4292 \sim 731 \cong 731$	$4338 \sim 1090 \cong 1090$
$4201 \sim 820 \cong 820$	$4247 \sim 821 \cong 821$	$4293 \sim 2190 \cong 750$	$4339 \sim 820 \cong 820$
$4202 \sim 2369 \cong 2369$	$4248 \sim 2399 \cong 2399$	$4294 \sim 730 \cong 730$	$4340 \sim 2395 \cong 2395$
$4203 \sim 820 \cong 820$	$4249 \sim 2874 \cong 820$	$4295 \sim 2226 \cong 820$	$4341 \sim 820 \cong 820$
$4204 \sim 730 \cong 730$	$4250 \sim 1091 \cong 731$	$4296 \sim 730 \cong 730$	$4342 \sim 2395 \cong 2395$
$4205 \sim 2234 \cong 2234$	$4251 \sim 2851 \cong 929$	$4297 \sim 2226 \cong 820$	$4343 \sim 2422 \cong 820$
$4206 \sim 730 \cong 730$	$4252 \sim 2887 \cong 731$	$4298 \sim 2232 \cong 730$	$4344 \sim 2368 \cong 739$
$4207 \sim 2236 \cong 2236$	$4253 \sim 2889 \cong 750$	$4299 \sim 2229 \cong 2229$	$4345 \sim 820 \cong 820$
$4208 \sim 2261 \cong 2261$	$4254 \sim 2860 \cong 2212$	$4300 \sim 730 \cong 730$	$4346 \sim 2368 \cong 739$
$4209 \sim 2209 \cong 2209$	$4255 \sim 2847 \cong 929$	$4301 \sim 2229 \cong 2229$	$4347 \sim 820 \cong 820$
$4210 \sim 730 \cong 730$	$4256 \sim 1091 \cong 731$	$4302 \sim 730 \cong 730$	$4348 \sim 730 \cong 730$
$4211 \sim 2207 \cong 2207$	$4257 \sim 2838 \cong 750$	$4303 \sim 820 \cong 820$	$4349 \sim 2307 \cong 2307$
$4212 \sim 730 \cong 730$	$4258 \sim 2388 \cong 821$	$4304 \sim 2388 \cong 821$	$4350 \sim 730 \cong 730$
$4213 \sim 2233 \cong 2233$	$4259 \sim 821 \cong 821$	$4305 \sim 820 \cong 820$	$4351 \sim 2307 \cong 2307$
$4214 \sim 731 \cong 731$	$4260 \sim 2365 \cong 2365$	$4306 \sim 2388 \cong 821$	$4352 \sim 2313 \cong 2277$
$4215 \sim 2234 \cong 2234$	$4261 \sim 2401 \cong 2401$	$4307 \sim 2394 \cong 820$	$4353 \sim 2287 \cong 2287$
$4216 \sim 2239 \cong 2239$	$4262 \sim 2424 \cong 966$	$4308 \sim 2391 \cong 2391$	$4354 \sim 730 \cong 730$
$4217 \sim 2241 \cong 739$	$4263 \sim 2374 \cong 821$	$4309 \sim 820 \cong 820$	$4355 \sim 2287 \cong 2287$
$4218 \sim 2240 \cong 2240$	$4264 \sim 2361 \cong 2361$	$4310 \sim 2391 \cong 2391$	$4356 \sim 730 \cong 730$
$4219 \sim 2236 \cong 2236$	$4265 \sim 821 \cong 821$	$4311 \sim 820 \cong 820$	$4357 \sim 820 \cong 820$
$4220 \sim 731 \cong 731$	$4266 \sim 2352 \cong 740$	$4312 \sim 730 \cong 730$	$4358 \sim 2395 \cong 2395$
$4221 \sim 2237 \cong 2237$	$4267 \sim 2307 \cong 2307$	$4313 \sim 2307 \cong 2307$	$4359 \sim 820 \cong 820$
$4222 \sim 2395 \cong 2395$	$4268 \sim 731 \cong 731$	$4314 \sim 730 \cong 730$	$4360 \sim 2395 \cong 2395$
$4223 \sim 821 \cong 821$	$4269 \sim 2284 \cong 2284$	$4315 \sim 2307 \cong 2307$	$4361 \sim 2422 \cong 820$
$4224 \sim 2396 \cong 2396$	$4270 \sim 2320 \cong 2294$	$4316 \sim 2313 \cong 2277$	$4362 \sim 2368 \cong 739$
$4225 \sim 2401 \cong 2401$	$4271 \sim 2322 \cong 2322$	$4317 \sim 2287 \cong 2287$	$4363 \sim 820 \cong 820$
$4226 \sim 2403 \cong 2287$	$4272 \sim 2293 \cong 2293$	$4318 \sim 730 \cong 730$	$4364 \sim 2368 \cong 739$
$4227 \sim 2402 \cong 2402$	$4273 \sim 2280 \cong 2280$	$4319 \sim 2287 \cong 2287$	$4365 \sim 820 \cong 820$
$4228 \sim 2398 \cong 2398$	$4274 \sim 731 \cong 731$	$4320 \sim 730 \cong 730$	$4366 \sim 730 \cong 730$
$4229 \sim 821 \cong 821$	$4275 \sim 2271 \cong 2271$	$4321 \sim 820 \cong 820$	$4367 \sim 2233 \cong 2233$
$4230 \sim 2399 \cong 2399$	$4276 \sim 2388 \cong 821$	$4322 \sim 2388 \cong 821$	$4368 \sim 730 \cong 730$
$4231 \sim 2307 \cong 2307$	$4277 \sim 821 \cong 821$	$4323 \sim 820 \cong 820$	$4369 \sim 2233 \cong 2233$
$4232 \sim 731 \cong 731$	$4278 \sim 2365 \cong 2365$	$4324 \sim 2388 \cong 821$	$4370 \sim 2260 \cong 802$
$4233 \sim 2284 \cong 2284$	$4279 \sim 2401 \cong 2401$	$4325 \sim 2394 \cong 820$	$4371 \sim 2206 \cong 748$
$4234 \sim 2320 \cong 2294$	$4280 \sim 2424 \cong 966$	$4326 \sim 2391 \cong 2391$	$4372 \sim 730 \cong 730$
$4235 \sim 2322 \cong 2322$	$4281 \sim 2374 \cong 821$	$4327 \sim 820 \cong 820$	$4373 \sim 2206 \cong 748$
$4236 \sim 2293 \cong 2293$	$4282 \sim 2361 \cong 2361$	$4328 \sim 2391 \cong 2391$	$4374 \sim 730 \cong 730$
$4237 \sim 2280 \cong 2280$	$4283 \sim 821 \cong 821$	$4329 \sim 820 \cong 820$	$4375 \sim 1094 \cong 1090$
$4238 \sim 731 \cong 731$	$4284 \sim 2352 \cong 740$	$4330 \sim 1090 \cong 1090$	$4376 \sim 824 \cong 820$
$4239 \sim 2271 \cong 2271$	$4285 \sim 2226 \cong 820$	$4331 \sim 2874 \cong 820$	$4377 \sim 824 \cong 820$
$4240 \sim 2395 \cong 2395$	$4286 \sim 731 \cong 731$	$4332 \sim 1090 \cong 1090$	$4378 \sim 824 \cong 820$
$4241 \sim 821 \cong 821$	$4287 \sim 2203 \cong 2203$	$4333 \sim 2874 \cong 820$	$4379 \sim 734 \cong 730$
$4242 \sim 2396 \cong 2396$	$4288 \sim 2239 \cong 2239$	$4334 \sim 2880 \cong 730$	$4380 \sim 734 \cong 730$
$4243 \sim 2401 \cong 2401$	$4289 \sim 2262 \cong 750$	$4335 \sim 2854 \cong 847$	$4381 \sim 824 \cong 820$

$4382 \sim 734 \cong 730$	$4428 \sim 2205 \cong 775$	$4474 \sim 1091 \cong 731$	$4520 \sim 2369 \cong 2369$
$4383 \sim 734 \cong 730$	$4429 \sim 2862 \cong 847$	$4475 \sim 821 \cong 821$	$4521 \sim 2366 \cong 2366$
$4384 \sim 2889 \cong 750$	$4430 \sim 2427 \cong 2427$	$4476 \sim 821 \cong 821$	$4522 \sim 2369 \cong 2369$
$4385 \sim 2424 \cong 966$	$4431 \sim 2376 \cong 739$	$4477 \sim 821 \cong 821$	$4523 \sim 2207 \cong 2207$
$4386 \sim 2403 \cong 2287$	$4432 \sim 2427 \cong 2427$	$4478 \sim 731 \cong 731$	$4524 \sim 2285 \cong 2285$
$4387 \sim 2424 \cong 966$	$4433 \sim 2265 \cong 2265$	$4479 \sim 731 \cong 731$	$4525 \sim 2366 \cong 2366$
$4388 \sim 2262 \cong 750$	$4434 \sim 2295 \cong 2295$	$4480 \sim 821 \cong 821$	$4526 \sim 2285 \cong 2285$
$4389 \sim 2322 \cong 2322$	$4435 \sim 2376 \cong 739$	$4481 \sim 731 \cong 731$	$4527 \sim 2204 \cong 2204$
$4390 \sim 2403 \cong 2287$	$4436 \sim 2295 \cong 2295$	$4482 \sim 731 \cong 731$	$4528 \sim 2841 \cong 2841$
$4391 \sim 2322 \cong 2322$	$4437 \sim 2214 \cong 748$	$4483 \sim 2860 \cong 2212$	$4529 \sim 2372 \cong 2372$
$4392 \sim 2241 \cong 739$	$4438 \sim 2853 \cong 2853$	$4484 \sim 2374 \cong 821$	$4530 \sim 2355 \cong 2355$
$4393 \sim 2862 \cong 847$	$4439 \sim 2423 \cong 2423$	$4485 \sim 2402 \cong 2402$	$4531 \sim 2372 \cong 2372$
$4394 \sim 2427 \cong 2427$	$4440 \sim 2367 \cong 2367$	$4486 \sim 2374 \cong 821$	$4532 \sim 2210 \cong 2210$
$4395 \sim 2376 \cong 739$	$4441 \sim 2423 \cong 2423$	$4487 \sim 2212 \cong 2212$	$4533 \sim 2274 \cong 2274$
$4396 \sim 2427 \cong 2427$	$4442 \sim 2261 \cong 2261$	$4488 \sim 2293 \cong 2293$	$4534 \sim 2355 \cong 2355$
$4397 \sim 2265 \cong 2265$	$4443 \sim 2286 \cong 2286$	$4489 \sim 2402 \cong 2402$	$4535 \sim 2274 \cong 2274$
$4398 \sim 2295 \cong 2295$	$4444 \sim 2367 \cong 2367$	$4490 \sim 2293 \cong 2293$	$4536 \sim 2193 \cong 2193$
$4399 \sim 2376 \cong 739$	$4445 \sim 2286 \cong 2286$	$4491 \sim 2240 \cong 2240$	$4537 \sim 2889 \cong 750$
$4400 \sim 2295 \cong 2295$	$4446 \sim 2205 \cong 775$	$4492 \sim 2854 \cong 847$	$4538 \sim 2403 \cong 2287$
$4401 \sim 2214 \cong 748$	$4447 \sim 2844 \cong 730$	$4493 \sim 2368 \cong 739$	$4539 \sim 2424 \cong 966$
$4402 \sim 2889 \cong 750$	$4448 \sim 2426 \cong 2277$	$4494 \sim 2391 \cong 2391$	$4540 \sim 2403 \cong 2287$
$4403 \sim 2424 \cong 966$	$4449 \sim 2358 \cong 820$	$4495 \sim 2368 \cong 739$	$4541 \sim 2241 \cong 739$
$4404 \sim 2403 \cong 2287$	$4450 \sim 2426 \cong 2277$	$4496 \sim 2206 \cong 748$	$4542 \sim 2322 \cong 2322$
$4405 \sim 2424 \cong 966$	$4451 \sim 2264 \cong 730$	$4497 \sim 2287 \cong 2287$	$4543 \sim 2424 \cong 966$
$4406 \sim 2262 \cong 750$	$4452 \sim 2277 \cong 2277$	$4498 \sim 2391 \cong 2391$	$4544 \sim 2322 \cong 2322$
$4407 \sim 2322 \cong 2322$	$4453 \sim 2358 \cong 820$	$4499 \sim 2287 \cong 2287$	$4545 \sim 2262 \cong 750$
$4408 \sim 2403 \cong 2287$	$4454 \sim 2277 \cong 2277$	$4500 \sim 2229 \cong 2229$	$4546 \sim 1091 \cong 731$
$4409 \sim 2322 \cong 2322$	$4455 \sim 2196 \cong 802$	$4501 \sim 2850 \cong 2850$	$4547 \sim 821 \cong 821$
$4410 \sim 2241 \cong 739$	$4456 \sim 2862 \cong 847$	$4502 \sim 2371 \cong 2371$	$4548 \sim 821 \cong 821$
$4411 \sim 2880 \cong 730$	$4457 \sim 2376 \cong 739$	$4503 \sim 2364 \cong 2364$	$4549 \sim 821 \cong 821$
$4412 \sim 2422 \cong 820$	$4458 \sim 2427 \cong 2427$	$4504 \sim 2371 \cong 2371$	$4550 \sim 731 \cong 731$
$4413 \sim 2394 \cong 820$	$4459 \sim 2376 \cong 739$	$4505 \sim 2209 \cong 2209$	$4551 \sim 731 \cong 731$
$4414 \sim 2422 \cong 820$	$4460 \sim 2214 \cong 748$	$4506 \sim 2283 \cong 2283$	$4552 \sim 821 \cong 821$
$4415 \sim 2260 \cong 802$	$4461 \sim 2295 \cong 2295$	$4507 \sim 2364 \cong 2364$	$4553 \sim 731 \cong 731$
$4416 \sim 2313 \cong 2277$	$4462 \sim 2427 \cong 2427$	$4508 \sim 2283 \cong 2283$	$4554 \sim 731 \cong 731$
$4417 \sim 2394 \cong 820$	$4463 \sim 2295 \cong 2295$	$4509 \sim 2202 \cong 2202$	$4555 \sim 1091 \cong 731$
$4418 \sim 2313 \cong 2277$	$4464 \sim 2265 \cong 2265$	$4510 \sim 2861 \cong 731$	$4556 \sim 821 \cong 821$
$4419 \sim 2232 \cong 730$	$4465 \sim 1091 \cong 731$	$4511 \sim 2375 \cong 2375$	$4557 \sim 821 \cong 821$
$4420 \sim 2853 \cong 2853$	$4466 \sim 821 \cong 821$	$4512 \sim 2375 \cong 2375$	$4558 \sim 821 \cong 821$
$4421 \sim 2423 \cong 2423$	$4467 \sim 821 \cong 821$	$4513 \sim 2375 \cong 2375$	$4559 \sim 731 \cong 731$
$4422 \sim 2367 \cong 2367$	$4468 \sim 821 \cong 821$	$4514 \sim 2213 \cong 2213$	$4560 \sim 731 \cong 731$
$4423 \sim 2423 \cong 2423$	$4469 \sim 731 \cong 731$	$4515 \sim 2294 \cong 2294$	$4561 \sim 821 \cong 821$
$4424 \sim 2261 \cong 2261$	$4470 \sim 731 \cong 731$	$4516 \sim 2375 \cong 2375$	$4562 \sim 731 \cong 731$
$4425 \sim 2286 \cong 2286$	$4471 \sim 821 \cong 821$	$4517 \sim 2294 \cong 2294$	$4563 \sim 731 \cong 731$
$4426 \sim 2367 \cong 2367$	$4472 \sim 731 \cong 731$	$4518 \sim 2213 \cong 2213$	$4564 \sim 2887 \cong 731$
$4427 \sim 2286 \cong 2286$	$4473 \sim 731 \cong 731$	$4519 \sim 2852 \cong 849$	$4565 \sim 2401 \cong 2401$

$4566 \sim 2401 \cong 2401$	$4612 \sim 2399 \cong 2399$	$4658 \sim 2206 \cong 748$	$4704 \sim 2277 \cong 2277$
$4567 \sim 2401 \cong 2401$	$4613 \sim 2237 \cong 2237$	$4659 \sim 2287 \cong 2287$	$4705 \sim 2426 \cong 2277$
$4568 \sim 2239 \cong 2239$	$4614 \sim 2271 \cong 2271$	$4660 \sim 2391 \cong 2391$	$4706 \sim 2277 \cong 2277$
$4569 \sim 2320 \cong 2294$	$4615 \sim 2352 \cong 740$	$4661 \sim 2287 \cong 2287$	$4707 \sim 2264 \cong 730$
$4570 \sim 2401 \cong 2401$	$4616 \sim 2271 \cong 2271$	$4662 \sim 2229 \cong 2229$	$4708 \sim 2838 \cong 750$
$4571 \sim 2320 \cong 2294$	$4617 \sim 2190 \cong 750$	$4663 \sim 2852 \cong 849$	$4709 \sim 2352 \cong 740$
$4572 \sim 2239 \cong 2239$	$4618 \sim 2862 \cong 847$	$4664 \sim 2369 \cong 2369$	$4710 \sim 2399 \cong 2399$
$4573 \sim 2874 \cong 820$	$4619 \sim 2376 \cong 739$	$4665 \sim 2366 \cong 2366$	$4711 \sim 2352 \cong 740$
$4574 \sim 2395 \cong 2395$	$4620 \sim 2427 \cong 2427$	$4666 \sim 2369 \cong 2369$	$4712 \sim 2190 \cong 750$
$4575 \sim 2388 \cong 821$	$4621 \sim 2376 \cong 739$	$4667 \sim 2207 \cong 2207$	$4713 \sim 2271 \cong 2271$
$4576 \sim 2395 \cong 2395$	$4622 \sim 2214 \cong 748$	$4668 \sim 2285 \cong 2285$	$4714 \sim 2399 \cong 2399$
$4577 \sim 2233 \cong 2233$	$4623 \sim 2295 \cong 2295$	$4669 \sim 2366 \cong 2366$	$4715 \sim 2271 \cong 2271$
$4578 \sim 2307 \cong 2307$	$4624 \sim 2427 \cong 2427$	$4670 \sim 2285 \cong 2285$	$4716 \sim 2237 \cong 2237$
$4579 \sim 2388 \cong 821$	$4625 \sim 2295 \cong 2295$	$4671 \sim 2204 \cong 2204$	$4717 \sim 2841 \cong 2841$
$4580 \sim 2307 \cong 2307$	$4626 \sim 2265 \cong 2265$	$4672 \sim 1091 \cong 731$	$4718 \sim 2355 \cong 2355$
$4581 \sim 2226 \cong 820$	$4627 \sim 2860 \cong 2212$	$4673 \sim 821 \cong 821$	$4719 \sim 2372 \cong 2372$
$4582 \sim 2847 \cong 929$	$4628 \sim 2374 \cong 821$	$4674 \sim 821 \cong 821$	$4720 \sim 2355 \cong 2355$
$4583 \sim 2398 \cong 2398$	$4629 \sim 2402 \cong 2402$	$4675 \sim 821 \cong 821$	$4721 \sim 2193 \cong 2193$
$4584 \sim 2361 \cong 2361$	$4630 \sim 2374 \cong 821$	$4676 \sim 731 \cong 731$	$4722 \sim 2274 \cong 2274$
$4585 \sim 2398 \cong 2398$	$4631 \sim 2212 \cong 2212$	$4677 \sim 731 \cong 731$	$4723 \sim 2372 \cong 2372$
$4586 \sim 2236 \cong 2236$	$4632 \sim 2293 \cong 2293$	$4678 \sim 821 \cong 821$	$4724 \sim 2274 \cong 2274$
$4587 \sim 2280 \cong 2280$	$4633 \sim 2402 \cong 2402$	$4679 \sim 731 \cong 731$	$4725 \sim 2210 \cong 2210$
$4588 \sim 2361 \cong 2361$	$4634 \sim 2293 \cong 2293$	$4680 \sim 731 \cong 731$	$4726 \sim 2838 \cong 750$
$4589 \sim 2280 \cong 2280$	$4635 \sim 2240 \cong 2240$	$4681 \sim 2850 \cong 2850$	$4727 \sim 2352 \cong 740$
$4590 \sim 2199 \cong 2199$	$4636 \sim 2861 \cong 731$	$4682 \sim 2371 \cong 2371$	$4728 \sim 2399 \cong 2399$
$4591 \sim 2860 \cong 2212$	$4637 \sim 2375 \cong 2375$	$4683 \sim 2364 \cong 2364$	$4729 \sim 2352 \cong 740$
$4592 \sim 2402 \cong 2402$	$4638 \sim 2375 \cong 2375$	$4684 \sim 2371 \cong 2371$	$4730 \sim 2190 \cong 750$
$4593 \sim 2374 \cong 821$	$4639 \sim 2375 \cong 2375$	$4685 \sim 2209 \cong 2209$	$4731 \sim 2271 \cong 2271$
$4594 \sim 2402 \cong 2402$	$4640 \sim 2213 \cong 2213$	$4686 \sim 2283 \cong 2283$	$4732 \sim 2399 \cong 2399$
$4595 \sim 2240 \cong 2240$	$4641 \sim 2294 \cong 2294$	$4687 \sim 2364 \cong 2364$	$4733 \sim 2271 \cong 2271$
$4596 \sim 2293 \cong 2293$	$4642 \sim 2375 \cong 2375$	$4688 \sim 2283 \cong 2283$	$4734 \sim 2237 \cong 2237$
$4597 \sim 2374 \cong 821$	$4643 \sim 2294 \cong 2294$	$4689 \sim 2202 \cong 2202$	$4735 \sim 1090 \cong 1090$
$4598 \sim 2293 \cong 2293$	$4644 \sim 2213 \cong 2213$	$4690 \sim 2841 \cong 2841$	$4736 \sim 820 \cong 820$
$4599 \sim 2212 \cong 2212$	$4645 \sim 1091 \cong 731$	$4691 \sim 2372 \cong 2372$	$4737 \sim 820 \cong 820$
$4600 \sim 2851 \cong 929$	$4646 \sim 821 \cong 821$	$4692 \sim 2355 \cong 2355$	$4738 \sim 820 \cong 820$
$4601 \sim 2396 \cong 2396$	$4647 \sim 821 \cong 821$	$4693 \sim 2372 \cong 2372$	$4739 \sim 730 \cong 730$
$4602 \sim 2365 \cong 2365$	$4648 \sim 821 \cong 821$	$4694 \sim 2210 \cong 2210$	$4740 \sim 730 \cong 730$
$4603 \sim 2396 \cong 2396$	$4649 \sim 731 \cong 731$	$4695 \sim 2274 \cong 2274$	$4741 \sim 820 \cong 820$
$4604 \sim 2234 \cong 2234$	$4650 \sim 731 \cong 731$	$4696 \sim 2355 \cong 2355$	$4742 \sim 730 \cong 730$
$4605 \sim 2284 \cong 2284$	$4651 \sim 821 \cong 821$	$4697 \sim 2274 \cong 2274$	$4743 \sim 730 \cong 730$
$4606 \sim 2365 \cong 2365$	$4652 \sim 731 \cong 731$	$4698 \sim 2193 \cong 2193$	$4744 \sim 1090 \cong 1090$
$4607 \sim 2284 \cong 2284$	$4653 \sim 731 \cong 731$	$4699 \sim 2844 \cong 730$	$4745 \sim 820 \cong 820$
$4608 \sim 2203 \cong 2203$	$4654 \sim 2854 \cong 847$	$4700 \sim 2358 \cong 820$	$4746 \sim 820 \cong 820$
$4609 \sim 2838 \cong 750$	$4655 \sim 2368 \cong 739$	$4701 \sim 2426 \cong 2277$	$4747 \sim 820 \cong 820$
$4610 \sim 2399 \cong 2399$	$4656 \sim 2391 \cong 2391$	$4702 \sim 2358 \cong 820$	$4748 \sim 730 \cong 730$
$4611 \sim 2352 \cong 740$	$4657 \sim 2368 \cong 739$	$4703 \sim 2196 \cong 802$	$4749 \sim 730 \cong 730$

4750	∼	820	≅	820	4796	∼	2284	≅	2284	4842	∼	2209	≅	2209	4888	∼	1091	≅	731
4751	∼	730	≅	730	4797	∼	2234	≅	2234	4843	∼	1090	≅	1090	4889	∼	821	≅	821
4752	∼	730	≅	730	4798	∼	2852	≅	849	4844	∼	820	≅	820	4890	∼	821	≅	821
4753	∼	2841	≅	2841	4799	∼	2366	≅	2366	4845	∼	820	≅	820	4891	∼	821	≅	821
4754	∼	2355	≅	2355	4800	∼	2369	≅	2369	4846	∼	820	≅	820	4892	∼	731	≅	731
4755	∼	2372	≅	2372	4801	∼	2366	≅	2366	4847	∼	730	≅	730	4893	∼	731	≅	731
4756	∼	2355	≅	2355	4802	∼	2204	≅	2204	4848	∼	730	≅	730	4894	∼	821	≅	821
4757	∼	2193	≅	2193	4803	∼	2285	≅	2285	4849	∼	820	≅	820	4895	∼	731	≅	731
4758	∼	2274	≅	2274	4804	∼	2369	≅	2369	4850	∼	730	≅	730	4896	∼	731	≅	731
4759	∼	2372	≅	2372	4805	∼	2285	≅	2285	4851	∼	730	≅	730	4897	∼	2874	≅	820
4760	∼	2274	≅	2274	4806	∼	2207	≅	2207	4852	∼	1090	≅	1090	4898	∼	2395	≅	2395
4761	∼	2210	≅	2210	4807	∼	2847	≅	929	4853	∼	820	≅	820	4899	∼	2388	≅	821
4762	∼	1090	≅	1090	4808	∼	2361	≅	2361	4854	∼	820	≅	820	4900	∼	2395	≅	2395
4763	∼	820	≅	820	4809	∼	2398	≅	2398	4855	∼	820	≅	820	4901	∼	2233	≅	2233
4764	∼	820	≅	820	4810	∼	2361	≅	2361	4856	∼	730	≅	730	4902	∼	2307	≅	2307
4765	∼	820	≅	820	4811	∼	2199	≅	2199	4857	∼	730	≅	730	4903	∼	2388	≅	821
4766	∼	730	≅	730	4812	∼	2280	≅	2280	4858	∼	820	≅	820	4904	∼	2307	≅	2307
4767	∼	730	≅	730	4813	∼	2398	≅	2398	4859	∼	730	≅	730	4905	∼	2226	≅	820
4768	∼	820	≅	820	4814	∼	2280	≅	2280	4860	∼	730	≅	730	4906	∼	2851	≅	929
4769	∼	730	≅	730	4815	∼	2236	≅	2236	4861	∼	2889	≅	750	4907	∼	2396	≅	2396
4770	∼	730	≅	730	4816	∼	1090	≅	1090	4862	∼	2403	≅	2287	4908	∼	2365	≅	2365
4771	∼	1090	≅	1090	4817	∼	820	≅	820	4863	∼	2424	≅	966	4909	∼	2396	≅	2396
4772	∼	820	≅	820	4818	∼	820	≅	820	4864	∼	2403	≅	2287	4910	∼	2234	≅	2234
4773	∼	820	≅	820	4819	∼	820	≅	820	4865	∼	2241	≅	739	4911	∼	2284	≅	2284
4774	∼	820	≅	820	4820	∼	730	≅	730	4866	∼	2322	≅	2322	4912	∼	2365	≅	2365
4775	∼	730	≅	730	4821	∼	730	≅	730	4867	∼	2424	≅	966	4913	∼	2284	≅	2284
4776	∼	730	≅	730	4822	∼	820	≅	820	4868	∼	2322	≅	2322	4914	∼	2203	≅	2203
4777	∼	820	≅	820	4823	∼	730	≅	730	4869	∼	2262	≅	750	4915	∼	1091	≅	731
4778	∼	730	≅	730	4824	∼	730	≅	730	4870	∼	2887	≅	731	4916	∼	821	≅	821
4779	∼	730	≅	730	4825	∼	1090	≅	1090	4871	∼	2401	≅	2401	4917	∼	821	≅	821
4780	∼	2853	≅	2853	4826	∼	820	≅	820	4872	∼	2401	≅	2401	4918	∼	821	≅	821
4781	∼	2367	≅	2367	4827	∼	820	≅	820	4873	∼	2401	≅	2401	4919	∼	731	≅	731
4782	∼	2423	≅	2423	4828	∼	820	≅	820	4874	∼	2239	≅	2239	4920	∼	731	≅	731
4783	∼	2367	≅	2367	4829	∼	730	≅	730	4875	∼	2320	≅	2294	4921	∼	821	≅	821
4784	∼	2205	≅	775	4830	∼	730	≅	730	4876	∼	2401	≅	2401	4922	∼	731	≅	731
4785	∼	2286	≅	2286	4831	∼	820	≅	820	4877	∼	2320	≅	2294	4923	∼	731	≅	731
4786	∼	2423	≅	2423	4832	∼	730	≅	730	4878	∼	2239	≅	2239	4924	∼	2847	≅	929
4787	∼	2286	≅	2286	4833	∼	730	≅	730	4879	∼	2860	≅	2212	4925	∼	2398	≅	2398
4788	∼	2261	≅	2261	4834	∼	2850	≅	2850	4880	∼	2402	≅	2402	4926	∼	2361	≅	2361
4789	∼	2851	≅	929	4835	∼	2364	≅	2364	4881	∼	2374	≅	821	4927	∼	2398	≅	2398
4790	∼	2365	≅	2365	4836	∼	2371	≅	2371	4882	∼	2402	≅	2402	4928	∼	2236	≅	2236
4791	∼	2396	≅	2396	4837	∼	2364	≅	2364	4883	∼	2240	≅	2240	4929	∼	2280	≅	2280
4792	∼	2365	≅	2365	4838	∼	2202	≅	2202	4884	∼	2293	≅	2293	4930	∼	2361	≅	2361
4793	∼	2203	≅	2203	4839	∼	2283	≅	2283	4885	∼	2374	≅	821	4931	∼	2280	≅	2280
4794	∼	2284	≅	2284	4840	∼	2371	≅	2371	4886	∼	2293	≅	2293	4932	∼	2199	≅	2199
4795	∼	2396	≅	2396	4841	∼	2283	≅	2283	4887	∼	2212	≅	2212	4933	∼	2838	≅	750

$4934 \sim 2399 \cong 2399$	$4977 \sim 2234 \cong 2234$	$5020 \sim 820 \cong 820$	$5063 \sim 730 \cong 730$
$4935 \sim 2352 \cong 740$	$4978 \sim 1090 \cong 1090$	$5021 \sim 730 \cong 730$	$5064 \sim 730 \cong 730$
$4936 \sim 2399 \cong 2399$	$4979 \sim 820 \cong 820$	$5022 \sim 730 \cong 730$	$5065 \sim 820 \cong 820$
$4937 \sim 2237 \cong 2237$	$4980 \sim 820 \cong 820$	$5023 \sim 2880 \cong 730$	$5066 \sim 730 \cong 730$
$4938 \sim 2271 \cong 2271$	$4981 \sim 820 \cong 820$	$5024 \sim 2394 \cong 820$	$5067 \sim 730 \cong 730$
$4939 \sim 2352 \cong 740$	$4982 \sim 730 \cong 730$	$5025 \sim 2422 \cong 820$	$5068 \sim 1090 \cong 1090$
$4940 \sim 2271 \cong 2271$	$4983 \sim 730 \cong 730$	$5026 \sim 2394 \cong 820$	$5069 \sim 820 \cong 820$
$4941 \sim 2190 \cong 750$	$4984 \sim 820 \cong 820$	$5027 \sim 2232 \cong 730$	$5070 \sim 820 \cong 820$
$4942 \sim 2853 \cong 2853$	$4985 \sim 730 \cong 730$	$5028 \sim 2313 \cong 2277$	$5071 \sim 820 \cong 820$
$4943 \sim 2367 \cong 2367$	$4986 \sim 730 \cong 730$	$5029 \sim 2422 \cong 820$	$5072 \sim 730 \cong 730$
$4944 \sim 2423 \cong 2423$	$4987 \sim 1090 \cong 1090$	$5030 \sim 2313 \cong 2277$	$5073 \sim 730 \cong 730$
$4945 \sim 2367 \cong 2367$	$4988 \sim 820 \cong 820$	$5031 \sim 2260 \cong 802$	$5074 \sim 820 \cong 820$
$4946 \sim 2205 \cong 775$	$4989 \sim 820 \cong 820$	$5032 \sim 2874 \cong 820$	$5075 \sim 730 \cong 730$
$4947 \sim 2286 \cong 2286$	$4990 \sim 820 \cong 820$	$5033 \sim 2388 \cong 821$	$5076 \sim 730 \cong 730$
$4948 \sim 2423 \cong 2423$	$4991 \sim 730 \cong 730$	$5034 \sim 2395 \cong 2395$	$5077 \sim 2854 \cong 847$
$4949 \sim 2286 \cong 2286$	$4992 \sim 730 \cong 730$	$5035 \sim 2388 \cong 821$	$5078 \sim 2391 \cong 2391$
$4950 \sim 2261 \cong 2261$	$4993 \sim 820 \cong 820$	$5036 \sim 2226 \cong 820$	$5079 \sim 2368 \cong 739$
$4951 \sim 2847 \cong 929$	$4994 \sim 730 \cong 730$	$5037 \sim 2307 \cong 2307$	$5080 \sim 2391 \cong 2391$
$4952 \sim 2361 \cong 2361$	$4995 \sim 730 \cong 730$	$5038 \sim 2395 \cong 2395$	$5081 \sim 2229 \cong 2229$
$4953 \sim 2398 \cong 2398$	$4996 \sim 2852 \cong 849$	$5039 \sim 2307 \cong 2307$	$5082 \sim 2287 \cong 2287$
$4954 \sim 2361 \cong 2361$	$4997 \sim 2366 \cong 2366$	$5040 \sim 2233 \cong 2233$	$5083 \sim 2368 \cong 739$
$4955 \sim 2199 \cong 2199$	$4998 \sim 2369 \cong 2369$	$5041 \sim 2854 \cong 847$	$5084 \sim 2287 \cong 2287$
$4956 \sim 2280 \cong 2280$	$4999 \sim 2366 \cong 2366$	$5042 \sim 2391 \cong 2391$	$5085 \sim 2206 \cong 748$
$4957 \sim 2398 \cong 2398$	$5000 \sim 2204 \cong 2204$	$5043 \sim 2368 \cong 739$	$5086 \sim 1090 \cong 1090$
$4958 \sim 2280 \cong 2280$	$5001 \sim 2285 \cong 2285$	$5044 \sim 2391 \cong 2391$	$5087 \sim 820 \cong 820$
$4959 \sim 2236 \cong 2236$	$5002 \sim 2369 \cong 2369$	$5045 \sim 2229 \cong 2229$	$5088 \sim 820 \cong 820$
$4960 \sim 2850 \cong 2850$	$5003 \sim 2285 \cong 2285$	$5046 \sim 2287 \cong 2287$	$5089 \sim 820 \cong 820$
$4961 \sim 2364 \cong 2364$	$5004 \sim 2207 \cong 2207$	$5047 \sim 2368 \cong 739$	$5090 \sim 730 \cong 730$
$4962 \sim 2371 \cong 2371$	$5005 \sim 1090 \cong 1090$	$5048 \sim 2287 \cong 2287$	$5091 \sim 730 \cong 730$
$4963 \sim 2364 \cong 2364$	$5006 \sim 820 \cong 820$	$5049 \sim 2206 \cong 748$	$5092 \sim 820 \cong 820$
$4964 \sim 2202 \cong 2202$	$5007 \sim 820 \cong 820$	$5050 \sim 2874 \cong 820$	$5093 \sim 730 \cong 730$
$4965 \sim 2283 \cong 2283$	$5008 \sim 820 \cong 820$	$5051 \sim 2388 \cong 821$	$5094 \sim 730 \cong 730$
$4966 \sim 2371 \cong 2371$	$5009 \sim 730 \cong 730$	$5052 \sim 2395 \cong 2395$	$5095 \sim 1090 \cong 1090$
$4967 \sim 2283 \cong 2283$	$5010 \sim 730 \cong 730$	$5053 \sim 2388 \cong 821$	$5096 \sim 820 \cong 820$
$4968 \sim 2209 \cong 2209$	$5011 \sim 820 \cong 820$	$5054 \sim 2226 \cong 820$	$5097 \sim 820 \cong 820$
$4969 \sim 2851 \cong 929$	$5012 \sim 730 \cong 730$	$5055 \sim 2307 \cong 2307$	$5098 \sim 820 \cong 820$
$4970 \sim 2365 \cong 2365$	$5013 \sim 730 \cong 730$	$5056 \sim 2395 \cong 2395$	$5099 \sim 730 \cong 730$
$4971 \sim 2396 \cong 2396$	$5014 \sim 1090 \cong 1090$	$5057 \sim 2307 \cong 2307$	$5100 \sim 730 \cong 730$
$4972 \sim 2365 \cong 2365$	$5015 \sim 820 \cong 820$	$5058 \sim 2233 \cong 2233$	$5101 \sim 820 \cong 820$
$4973 \sim 2203 \cong 2203$	$5016 \sim 820 \cong 820$	$5059 \sim 1090 \cong 1090$	$5102 \sim 730 \cong 730$
$4974 \sim 2284 \cong 2284$	$5017 \sim 820 \cong 820$	$5060 \sim 820 \cong 820$	$5103 \sim 730 \cong 730$
$4975 \sim 2396 \cong 2396$	$5018 \sim 730 \cong 730$	$5061 \sim 820 \cong 820$	
$4976 \sim 2284 \cong 2284$	$5019 \sim 730 \cong 730$	$5062 \sim 820 \cong 820$	

5104 through $5832 \sim 1090 \simeq 1090$.

8 Group information

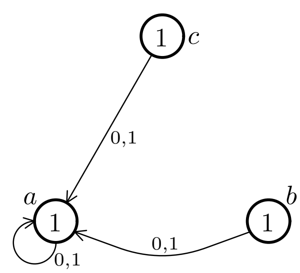
We use the following notation:

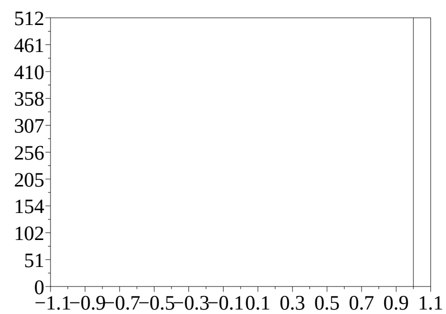
- Rels - a list of some relators in the group. In most cases these are the first few relators in the length-lexicographic order, but in some cases (more precisely, for the automata numbered by 744, 753, 776, 840, 843, 858, 885, 888, 956, 965, 2209, 2210, 2213, 2234, 2261, 2274, 2293, 2355, 2364, 2396, 2402, 2423) there could be some shorter relators. In most cases the given list does not give a presentation of the group (exception are the finite and abelian groups, and the automata numbered by 820, 846, 870, 2212, 2240, 2294).
- SF - these numbers represent the size of the factors $G/\operatorname{Stab}_G(n)$, for $n \geq 0$.
- Gr - these numbers represent the first few values of the growth function $\gamma_G(n)$, for $n \geq 0$, with respect to the generating system a, b, c ($\gamma_G(n)$ counts the number of elements of length at most n in G).

Automaton number 1

$a = (a, a)$ Group: *Trivial Group*
$b = (a, a)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *yes*

Rels: a, b, c
SF: 2^0,2^0,2^0,2^0,2^0,2^0,2^0,2^0,2^0
Gr: 1,1,1,1,1,1,1,1,1,1,1





Automaton number 730

$a = \sigma(a, a)$ Group: *Klein Group*

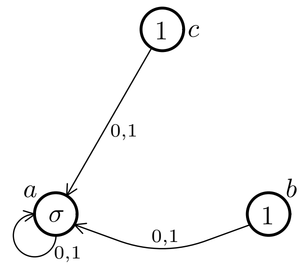
$b = (a, a)$ Contracting: *yes*

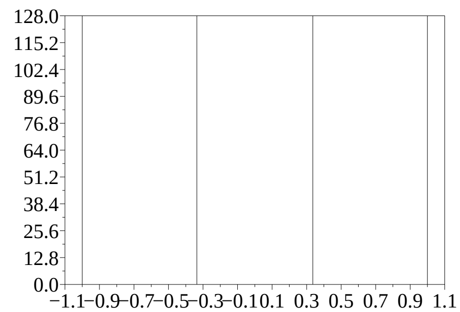
$c = (a, a)$ Self-replicating: *no*

Rels: $b^{-1}c$, a^2, b^2, $abab$

SF: 2^0,2^1,2^2,2^2,2^2,2^2,2^2,2^2,2^2

Gr: 1,3,4,4,4,4,4,4,4,4,4





Automaton number 731

$a = \sigma(b, a)$ Group: $\mathbb{Z}$

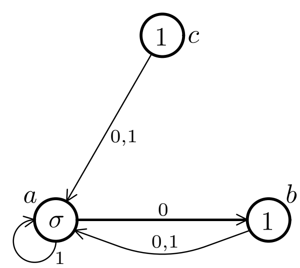
$b = (a, a)$ Contracting: *yes*

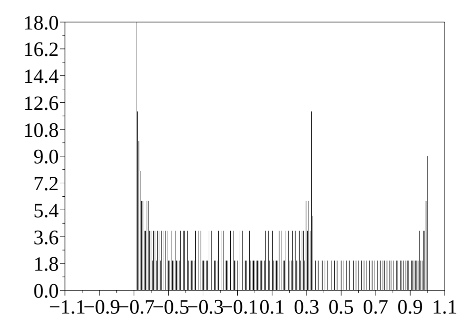
$c = (a, a)$ Self-replicating: *yes*

Rels: $b^{-1}c$, ba^2

SF: 2^0,2^1,2^2,2^3,2^4,2^5,2^6,2^7,2^8

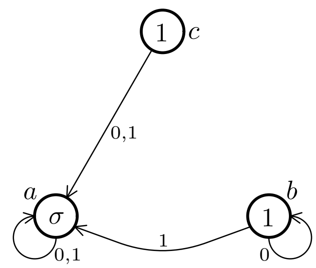
Gr: 1,5,9,13,17,21,25,29,33,37,41

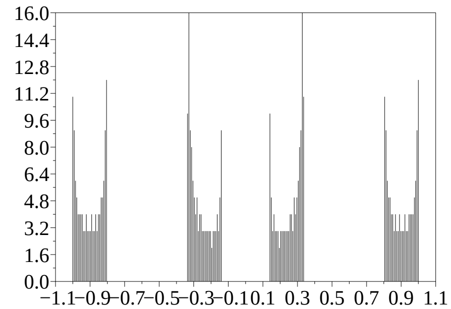




Automaton number 739

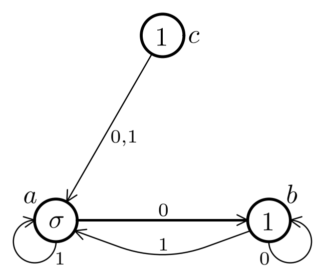
$a = \sigma(a, a)$ Group: $C_2 \ltimes (\mathbb{Z} \wr C_2)$
$b = (b, a)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *no*
Rels: a^2, b^2, c^2, $(ac)^2$, $(acbab)^2$
SF: 2^0,2^1,2^3,2^6,2^8,2^{10},2^{12},2^{14},2^{16}
Gr: 1,4,9,17,30,47,68,93,122,155,192

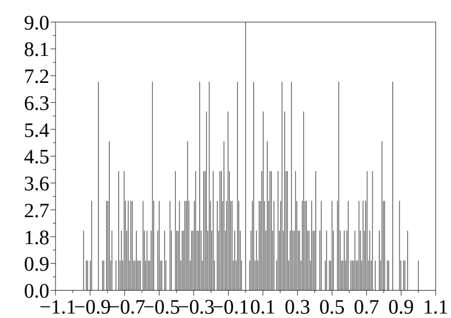




Automaton number 740

$a = \sigma(b, a)$ Group:
$b = (b, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *no*
Rels: $(a^{-1}b)^2$, $(b^{-1}c)^2$, $a^{-1}c^{-1}ac^{-1}b^2$,
$[a, b]^2$
SF: 2^0,2^1,2^3,2^6,2^9,2^{11},2^{14},2^{16},2^{18}
Gr: 1,7,33,135,495,1725





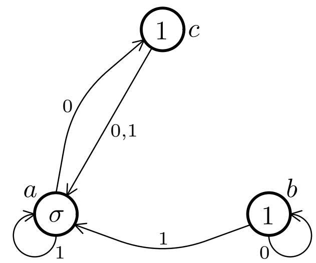
Automaton number 741

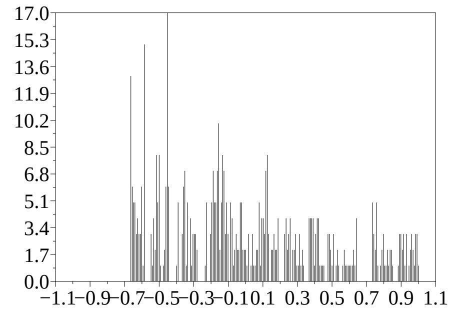
$a = \sigma(c, a)$ Group:
$b = (b, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*

Rels: ca^2, $b^{-1}a^{-3}b^{-1}ababa$,
$b^{-1}a^{-6}b^{-1}a^{-2}ba^{-2}ba^{-2}$

SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}

Gr: 1,7,29,115,441,1643





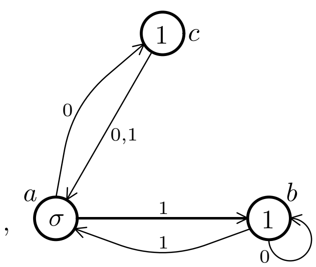
Automaton number 744

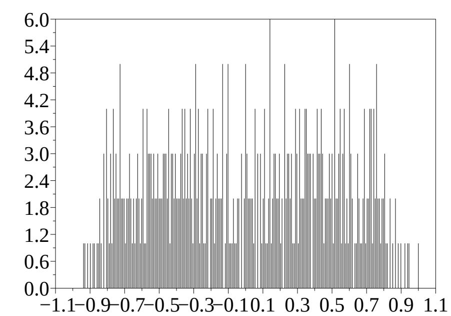
$a = \sigma(c, b)$ Group:
$b = (b, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*

Rels:
$[a^2ca^{-1}bc^{-1}b^{-1}a^{-1}, aca^{-1}bc^{-1}b^{-1}]$,
$abcb^{-1}ac^{-1}a^{-2}bcb^{-1}ab^{-1}aca^{-1}bc^{-1}a^{-1}bc^{-1}b^{-1}$,
$abcb^{-1}ab^{-1}a^{-2}bcb^{-1}ac^{-1}aba^{-1}bc^{-1}b^{-1}ca^{-1}bc^{-1}b^{-1}$,
$abcb^{-1}ab^{-1}a^{-2}bcb^{-1}ab^{-1}a\cdot$
$ba^{-1}bc^{-1}a^{-1}bc^{-1}b^{-1}$

SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}

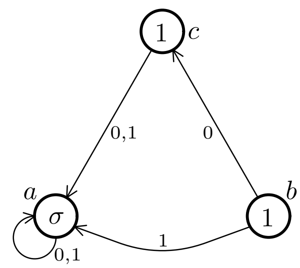
Gr: 1,7,37,187,937,4687

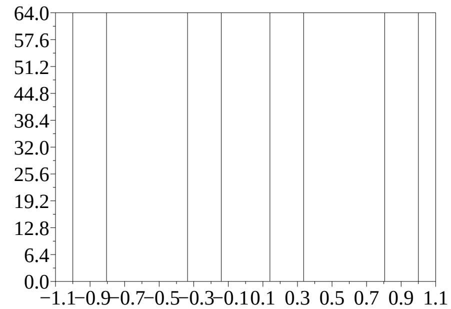




Automaton number 748

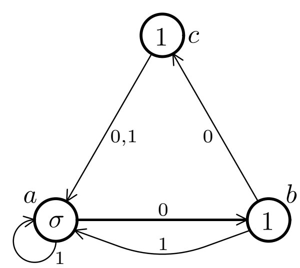
$a = \sigma(a, a)$ Group: $D_4 \times C_2$
$b = (c, a)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *no*
Rels: a^2, b^2, c^2, $acac$, $bcbc$, $abababab$
SF: 2^0,2^1,2^3,2^4,2^4,2^4,2^4,2^4,2^4
Gr: 1,4,8,12,15,16,16,16,16,16,16

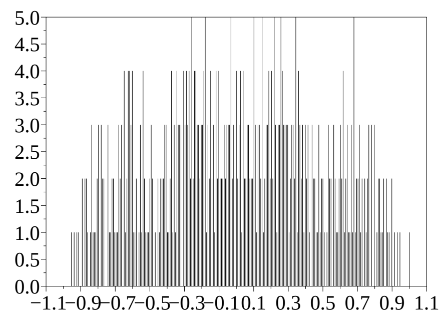




Automaton number 749

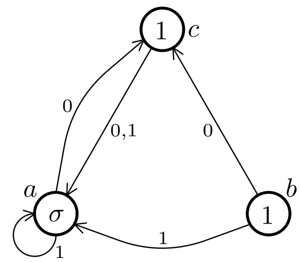
$a = \sigma(b, a)$ Group:
$b = (c, a)$ Contracting: *n/a*
$c = (a, a)$ Self-replicating: *yes*
Rels: $a^{-1}c^{-1}bab^{-1}a^{-1}cb^{-1}ab$,
$a^{-1}c^{-1}bac^{-1}a^{-1}cb^{-1}ac$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,37,187,937,4667

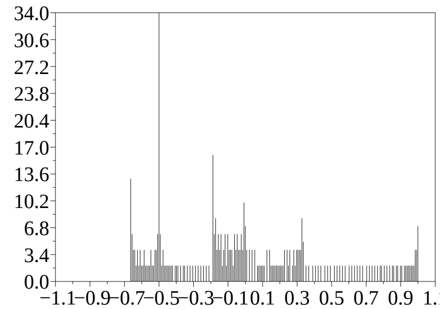




Automaton number 750

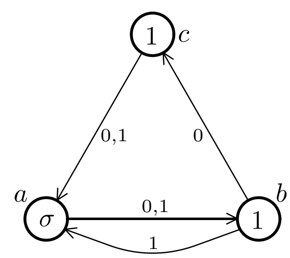
$a = \sigma(c, a)$ Group: $C_2 \wr \mathbb{Z}$
$b = (c, a)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *no*
Rels: ca^2, $(a^{-1}b)^2$, $[b, c]$
SF: 2^0,2^1,2^3,2^5,2^7,2^9,2^{11},2^{13},2^{15}
Gr: 1,7,23,49,87,137,199,273,359

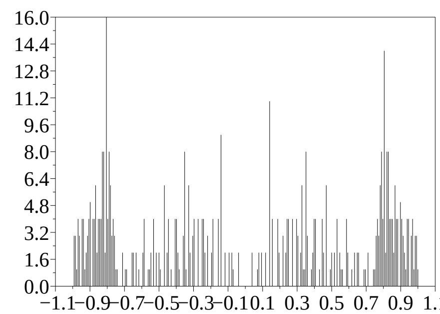




Automaton number 752

$a = \sigma(b, b)$ Group: virtually $\mathbb{Z}^3$
$b = (c, a)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *no*
Rels: a^2, b^2, c^2, $(acbab)^2$, $(acacb)^2$,
$(abc)^2(acb)^2$, $acbcbabacbcbab$, $abcbacbabcbacb$,
$acbcacbacbcacb$, $acacbcbacacbcb$, $abc(bca)^2cbcbacb$,
$a(cb)^3aba(cb)^3ab$, $abcbcbacbabcbcbacb$,
$acbcbcacbacbcbcacb$
SF: 2^0,2^1,2^3,2^5,2^7,2^8,2^{10},2^{11},2^{13}
Gr: 1,4,10,22,46,84,140,217,319,448





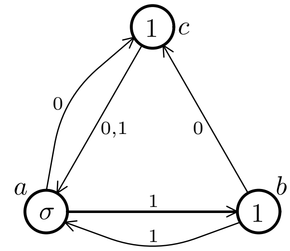
Automaton number 753

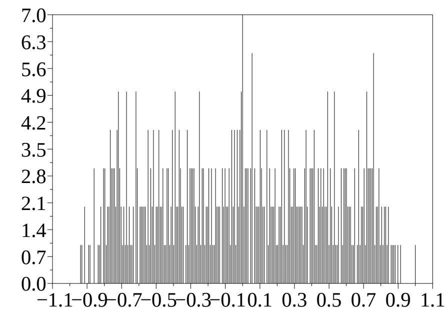
$a = \sigma(c, b)$ Group:
$b = (c, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*

Rels: $aba^{-1}b^{-1}ab^{-1}ca^{-1}ba^{-1}b^{-1}ab^{-1}cac^{-1}b\cdot$
$a^{-1}bab^{-1}a^{-1}c^{-1}ba^{-1}bab^{-1}$,
$aba^{-1}b^{-1}ab^{-1}ca^{-1}c^{-1}ba^{-1}c^{-1}bab^{-1}ca\cdot$
$c^{-1}ba^{-1}bab^{-1}a^{-1}c^{-1}ba^{-1}b^{-1}cab^{-1}c$

SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}

Gr: 1,7,37,187,937,4687





Automaton number 771

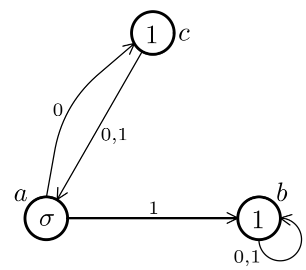
$a = \sigma(c, b)$ Group: $\mathbb{Z}^2$
$b = (b, b)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *yes*

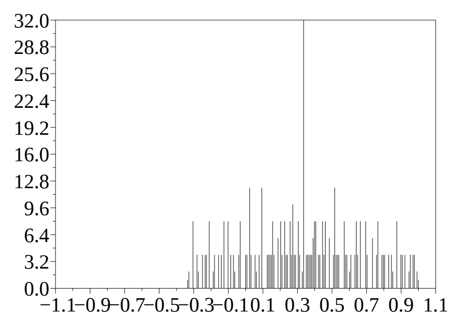
Rels: b, $a^{-1}c^{-1}ac$

SF: 2^0,2^1,2^2,2^3,2^4,2^5,2^6,2^7,2^8

Gr: 1,5,13,25,41,61,85,113,145,181,221

Limit space: 2-dimensional torus T_2

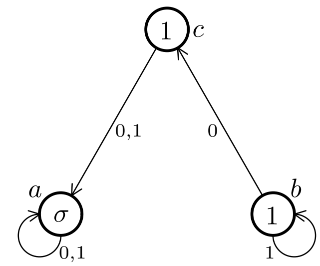


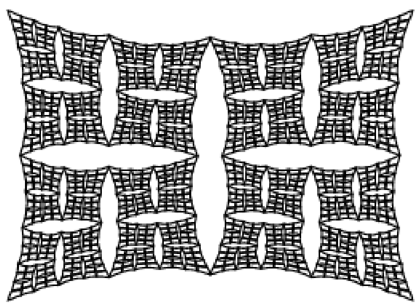


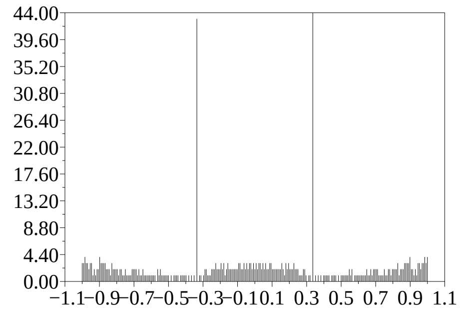
Automaton number 775

$a = \sigma(a,a)$ Group: $C_2 \ltimes IMG\left(\left(\frac{z-1}{z+1}\right)^2\right)$
$b = (c,b)$ Contracting: *yes*
$c = (a,a)$ Self-replicating: *yes*

Rels: a^2, b^2, c^2, $acac$, $acbcbabcbcabcbabcb$
SF: 2^0,2^1,2^2,2^4,2^6,2^9,2^{15},2^{26},2^{48}
Gr: 1,4,9,17,30,51,85,140,229,367,579
Limit space:



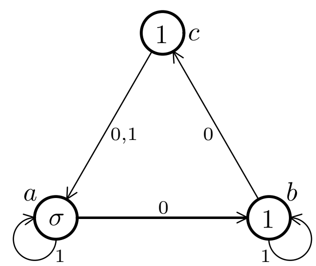


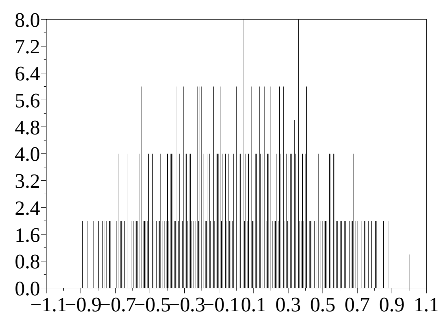


Automaton number 776

$a = \sigma(b,a)$ Group:
$b = (c,b)$ Contracting: *no*
$c = (a,a)$ Self-replicating: *yes*

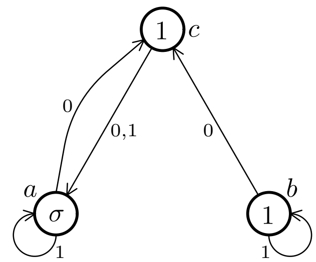
Rels: $aba^{-1}b^{-1}a^2c^{-1}ab^{-1}a^{-1}bcb^{-1}ac^{-1}a^{-1}ba^{-1}\cdot$
$b^{-1}a^2c^{-1}ab^{-1}a^{-1}bcb^{-1}ac^{-1}aca^{-1}bc^{-1}b^{-1}ab\cdot$
$a^{-1}ca^{-2}bab^{-1}a^{-1}ca^{-1}bc^{-1}b^{-1}aba^{-1}ca^{-2}bab^{-1}$,
$aba^{-1}b^{-1}a^2c^{-1}ab^{-1}a^{-1}bcb^{-1}ac^{-1}a^{-1}cba^{-1}\cdot$
$b^{-1}a^2c^{-1}ab^{-1}a^{-1}bc^{-1}b^{-1}aba^{-1}ca^{-2}\cdot$
$bab^{-1}aca^{-1}bc^{-1}b^{-1}aba^{-1}ca^{-2}bab^{-1}\cdot$
$a^{-1}ba^{-1}b^{-1}a^2c^{-1}ab^{-1}a^{-1}bcb^{-1}\cdot$
$aba^{-1}ca^{-2}bab^{-1}c^{-1}$
SF: 2^0,2^1,2^2,2^4,2^7,2^{13},2^{24},2^{46},2^{89}
Gr: 1,7,37,187,937,4687

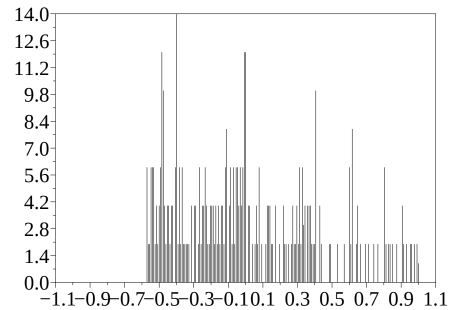




Automaton number 777

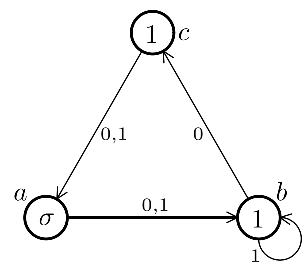
$a = \sigma(c, a)$ Group:
$b = (c, b)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*
Rels: ca^2, $b^{-1}a^5b^{-1}a^{-1}ba^{-3}ba^{-1}$
SF: 2^0,2^1,2^2,2^4,2^7,2^{13},2^{24},2^{46},2^{89}
Gr: 1,7,29,115,441,1695

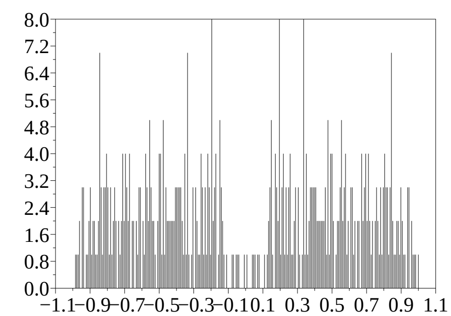




Automaton number 779

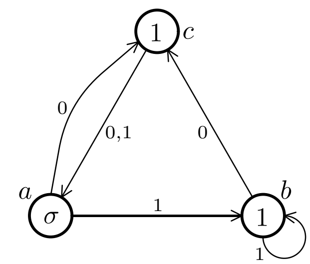
$a = \sigma(b, b)$ Group:
$b = (c, b)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *yes*
Rels: a^2, b^2, c^2, $acabcabcbabacabcabcbab$,
$acbcbacacabcbcabcbabcb$
SF: 2^0,2^1,2^2,2^4,2^6,2^9,2^{15},2^{26},2^{48}
Gr: 1,4,10,22,46,94,190,382,766,1534,3070,6120

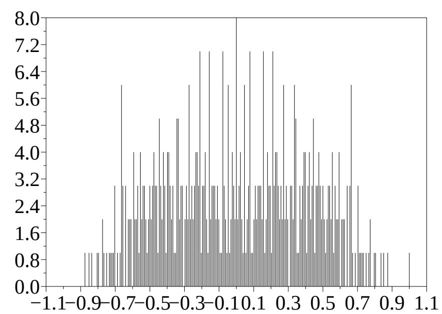




Automaton number 780

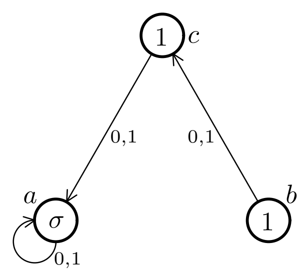
$a = \sigma(c, b)$ Group:
$b = (c, b)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*
Rels: $(a^{-1}b)^2$, $[ba^{-1}, c]$
SF: 2^0,2^1,2^2,2^4,2^6,2^9,2^{15},2^{27},2^{49}
Gr: 1,7,35,159,705,3107

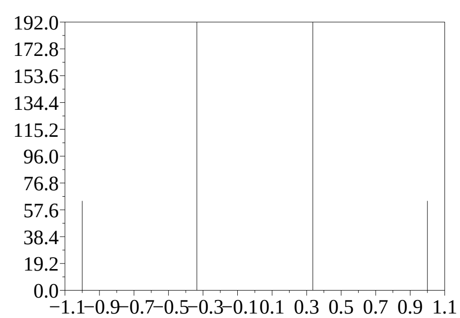




Automaton number 802

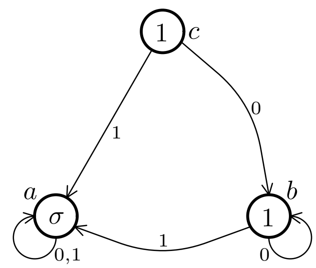
$a = \sigma(a, a)$ Group: $C_2 \times C_2 \times C_2$
$b = (c, c)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *no*
Rels: a^2, b^2, c^2, $[a, b]$, $[a, c]$, $[b, c]$
SF: 2^0,2^1,2^2,2^3,2^3,2^3,2^3,2^3,2^3
Gr: 1,4,7,8,8,8,8,8,8,8,8

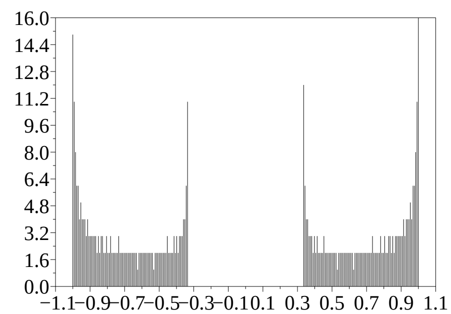




Automaton number 820

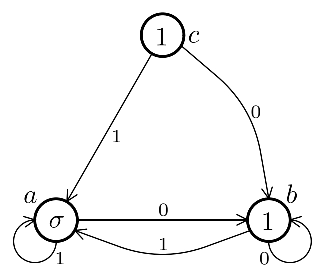
$a = \sigma(a, a)$ Group: D_∞
$b = (b, a)$ Contracting: *yes*
$c = (b, a)$ Self-replicating: *yes*
Rels: $b^{-1}c$, a^2, b^2
SF: 2^0,2^1,2^3,2^4,2^5,2^6,2^7,2^8,2^9
Gr: 1,3,5,7,9,11,13,15,17,19,21

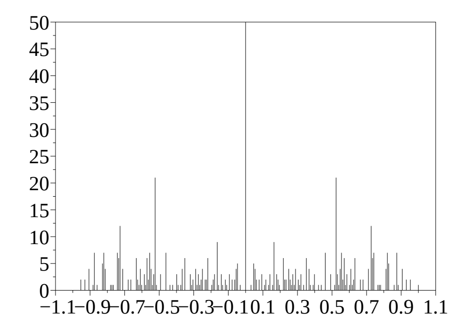




Automaton number 821

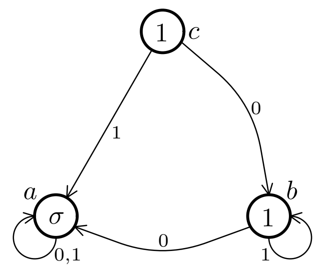
$a = \sigma(b, a)$ Group: *Lamplighter group* $\mathbb{Z} \wr C_2$
$b = (b, a)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $b^{-1}c$, $(a^{-1}b)^2$, $[a, b]^2$,
$a^{-3}baba^{-2}b^{-1}a^2b$
SF: 2^0,2^1,2^3,2^5,2^6,2^8,2^9,2^{10},2^{11}
Gr: 1,5,15,39,92,208,452,964,2016

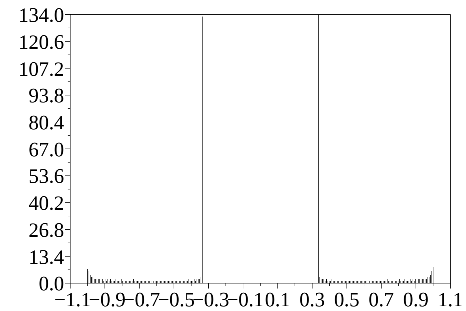




Automaton number 838

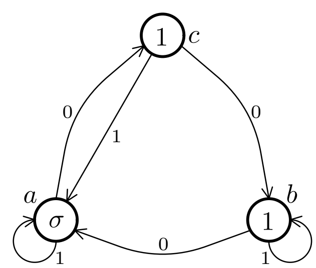
$a = \sigma(a,a)$ Group: $C_2 \ltimes \langle s,t \mid s^2 = t^2 \rangle$
$b = (a,b)$ Contracting: *yes*
$c = (b,a)$ Self-replicating: *no*
Rels: a^2, b^2, c^2, $abcacb$
SF: 2^0,2^1,2^3,2^5,2^7,2^9,2^{11},2^{13},2^{15}
Gr: 1,4,10,19,31,46,64,85,109,136

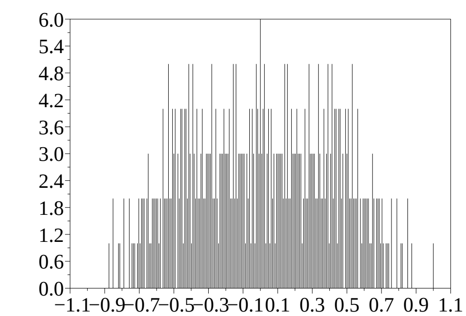




Automaton number 840

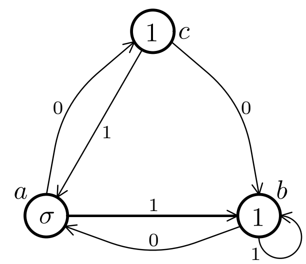
$a = \sigma(c,a)$ Group:
$b = (a,b)$ Contracting: *no*
$c = (b,a)$ Self-replicating: *yes*
Rels: $abac^{-1}a^{-2}bac^{-1}aca^{-1}b^{-1}ca^{-1}b^{-1}$,
$abac^{-1}a^{-2}cac^{-1}b^{-1}caca^{-1}b^{-1}c^{-1}bca^{-1}c^{-1}$,
$acac^{-1}b^{-1}ca^{-2}bac^{-1}ac^{-1}bca^{-2}b^{-1}$,
$acac^{-1}b^{-1}ca^{-2}cac^{-1}b^{-1}cac^{-1}bca^{-1}c^{-2}bca^{-1}c^{-1}$
SF: 2^0,2^1,2^3,2^5,2^7,2^{10},2^{15},2^{25},2^{41}
Gr: 1,7,37,187,937,4687



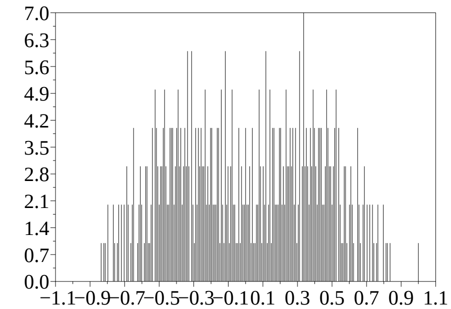


Automaton number 843

$a = \sigma(c, b)$ Group:
$b = (a, b)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*

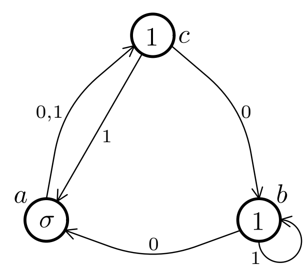


Rels: $acab^{-1}a^{-2}cab^{-1}aba^{-1}c^{-1}ba^{-1}c^{-1}$,
$acab^{-1}a^{-2}cb^{-1}ab^{-1}caba^{-1}c^{-2}ba^{-1}bc^{-1}$,
$acb^{-1}ab^{-1}ca^{-2}cab^{-1}ac^{-1}ba^{-1}bc^{-1}ba^{-1}c^{-1}$,
$acb^{-1}ab^{-1}ca^{-2}cb^{-1}ab^{-1}cac^{-1}ba^{-1}bc^{-2}ba^{-1}bc^{-1}$
SF: 2^0,2^1,2^3,2^5,2^8,2^{14},2^{24},2^{43},2^{81}
Gr: 1,7,37,187,937,4687

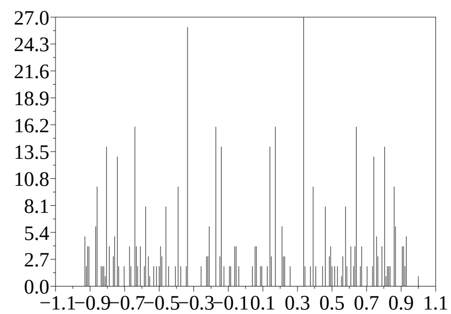


Automaton number 846

$a = \sigma(c, c)$ Group: $C_2 * C_2 * C_2$
$b = (a, b)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *no*

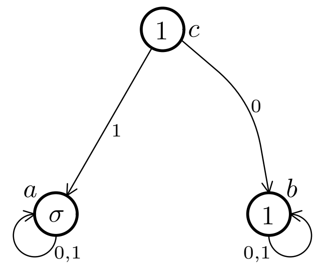


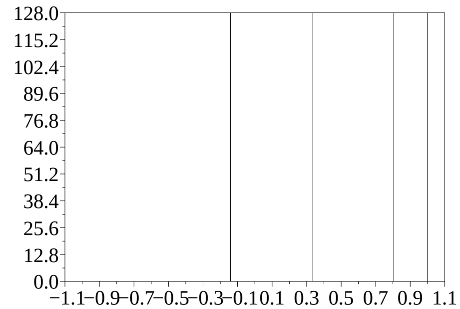
Rels: a^2, b^2, c^2
SF: 2^0,2^1,2^3,2^5,2^7,2^{10},2^{13},2^{16},2^{19}
Gr: 1,4,10,22,46,94,190,382,766,1534



Automaton number 847

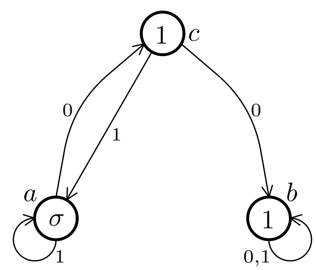
$a = \sigma(a, a)$ Group: D_4
$b = (b, b)$ Contracting: *yes*
$c = (b, a)$ Self-replicating: *no*
Rels: b, a^2, c^2, $acacacac$
SF: 2^0,2^1,2^3,2^3,2^3,2^3,2^3,2^3,2^3
Gr: 1,3,5,7,8,8,8,8,8,8,8

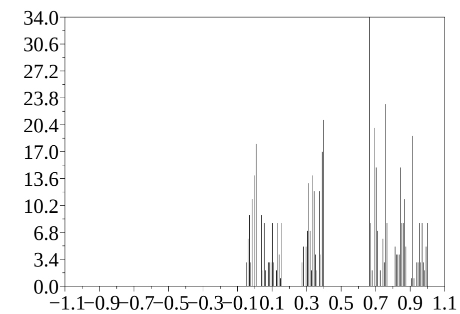




Automaton number 849

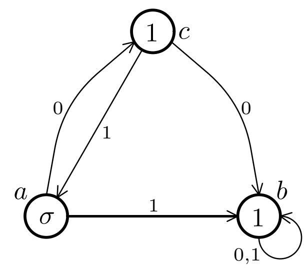
$a = \sigma(c, a)$ Group:
$b = (b, b)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: b, $[ac^{-1}a^{-1}, c]$,
$[a^2, c^{-1}] \cdot [c, a^{-2}]$, $[a^{-1}, c^{-2}] \cdot [a^{-1}, c^2]$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,5,17,53,153,421,1125,2945,7589

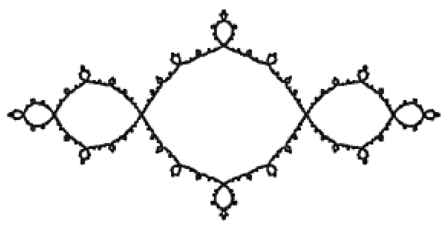


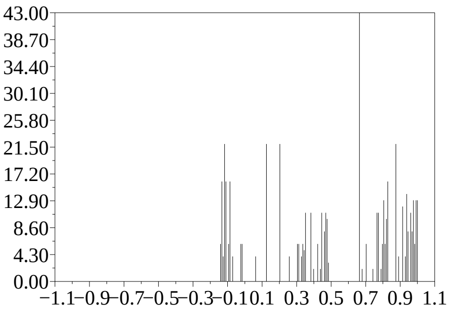


Automaton number 852

$a = \sigma(c, b)$ Group: $IMG(z^2 - 1)$
$b = (b, b)$ Contracting: *yes*
$c = (b, a)$ Self-replicating: *yes*
Rels: b, $[ac^{-1}a^{-1}, c]$,
$[c, a^2] \cdot [c, a^{-2}]$, $[a^{-1}, c^{-2}] \cdot [a^{-1}, c^2]$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,5,17,53,153,421,1125,2945,7545
Limit space:

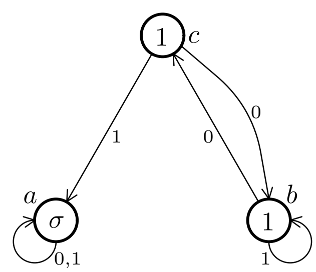


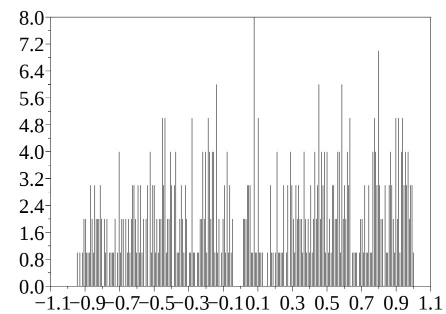




Automaton number 856

$a = \sigma(a, a)$ Group: $C_2 \ltimes G_{2850}$
$b = (c, b)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: a^2, b^2, c^2, $acbcacbcabcacacacb$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,4,10,22,46,94,190,382,766,
1525,3025,5998,11890,23532





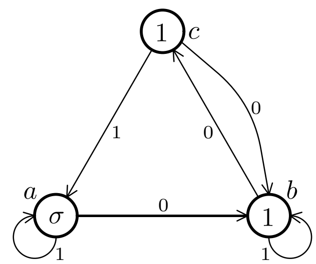
Automaton number 857

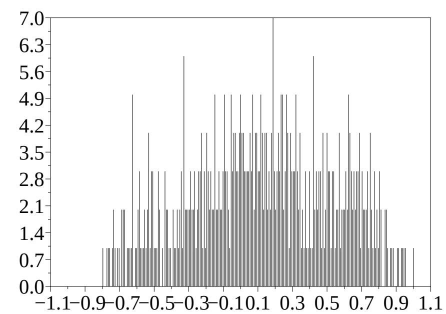
$a = \sigma(b, a)$ Group:
$b = (c, b)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*

Rels: $(a^{-1}c)^2$, $(a^{-1}b)^4$, $(a^{-1}b^{-1}ac)^2$, $(b^{-1}c)^4$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}

Gr: 1,7,35,165,758,3460





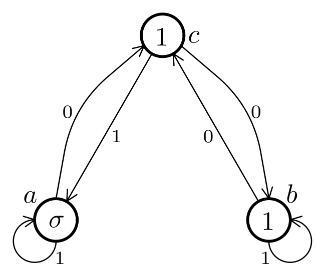
Automaton number 858

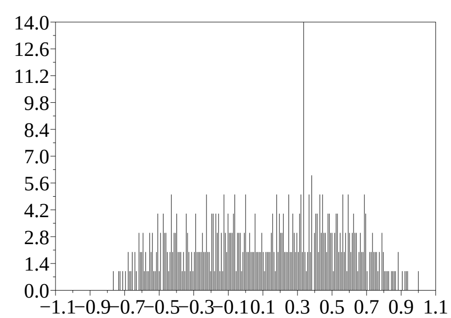
$a = \sigma(c, a)$ Group:
$b = (c, b)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*

Rels: $abca^{-1}c^{-1}ab^{-1}a^2c^{-1}b^{-1}a^{-1}bca^{-1}c^{-1}a\cdot b^{-1}a^2c^{-1}b^{-1}abca^{-2}ba^{-1}cac^{-1}b^{-1}a^{-1}\cdot bca^{-2}ba^{-1}cac^{-1}b^{-1}$,
$abca^{-1}c^{-1}ab^{-1}a^2c^{-1}b^{-1}a^{-1}cba^{-1}b^{-1}ab^{-1}a\cdot bca^{-2}ba^{-1}cac^{-1}b^{-1}a^{-1}ba^{-1}bab^{-1}c^{-1}$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{90},2^{176}

Gr: 1,7,37,187,937,4687





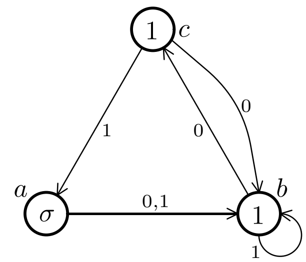
Automaton number 860

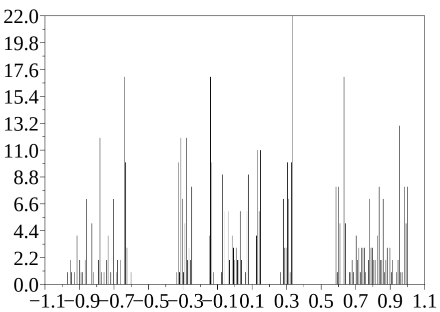
$a = \sigma(b, b)$ Group:
$b = (c, b)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*

Rels: a^2, b^2, c^2, $acbacacabcabab$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}

Gr: 1,4,10,22,46,94,190,375,731,1422,2762,5350





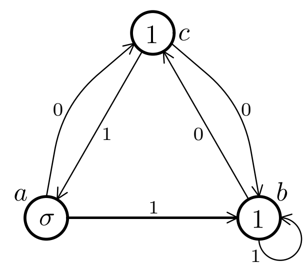
Automaton number 861

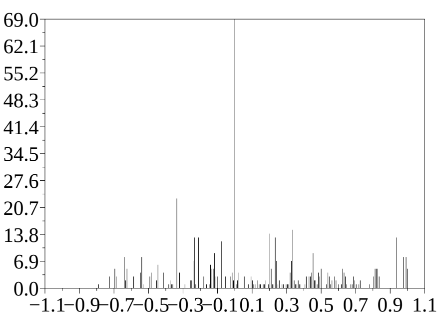
$a = \sigma(c, b)$ Group:
$b = (c, b)$ Contracting: *n/a*
$c = (b, a)$ Self-replicating: *yes*

Rels: $(a^{-1}b)^2$, $(b^{-1}c)^2$, $[a, b]^2$, $[b, c]^2$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}

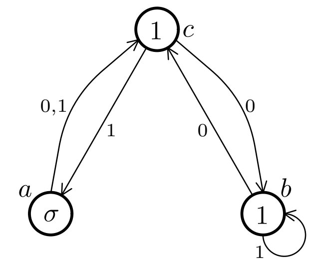
Gr: 1,7,33,143,599,2485

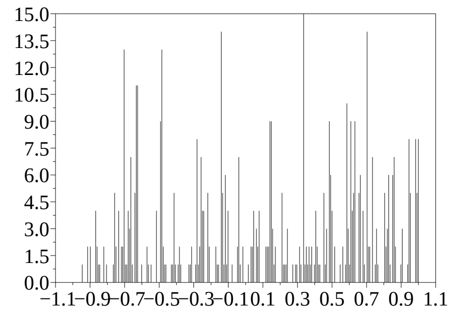




Automaton number 864

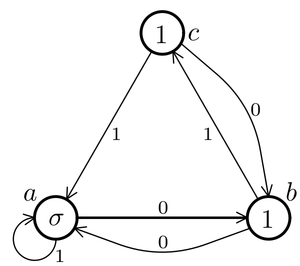
$a = \sigma(c, c)$ Group:
$b = (c, b)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: a^2, b^2, c^2, $abcabcbabcbacbabab$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,4,10,22,46,94,190,382,766,1525,
3025,5998,11890

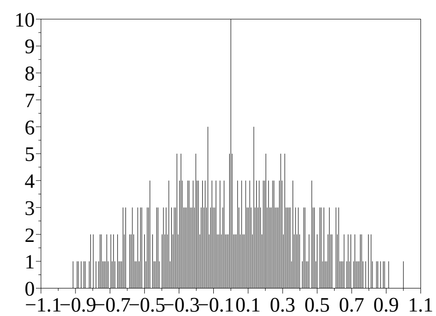




Automaton number 866

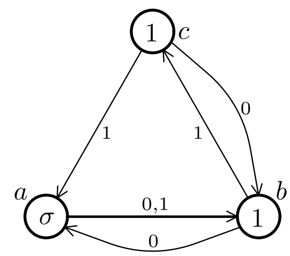
$a = \sigma(b, a)$ Group:
$b = (a, c)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $(ca^{-1})^2$, $ba^{-2}cab^{-1}ab^{-1}c^{-1}aba^{-1}$,
$cab^{-1}cb^{-1}a^{-1}cbc^{-1}ba^{-2}$
SF: 2^0,2^1,2^3,2^5,2^9,2^{15},2^{26},2^{48},2^{92}
Gr: 1,7,35,165,769,3575

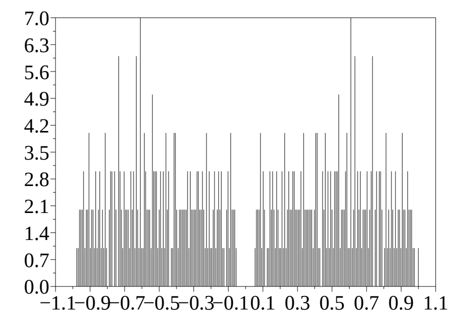




Automaton number 869

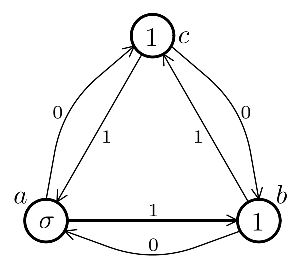
$a = \sigma(b, b)$ Group:
$b = (a, c)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: a^2, b^2, c^2, $acbcacbcabcacacacb$
SF: 2^0,2^1,2^3,2^4,2^6,2^9,2^{15},2^{26},2^{48}
Gr: 1,4,10,22,46,94,190,382,766,1525

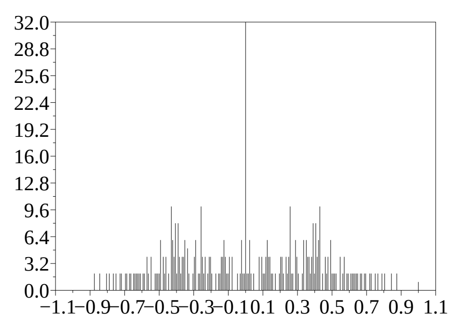




Automaton number 870

$a = \sigma(c, b)$ Group: $BS(1, 3)$
$b = (a, c)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $a^{-1}ca^{-1}b$, $(b^{-1}a)^{b^{-1}}(b^{-1}a)^{-3}$
SF: 2^0,2^1,2^3,2^4,2^6,2^8,2^{10},2^{12},2^{14}
Gr: 1,7,33,127,433,1415





Automaton number 874

$a = \sigma(a, a)$ Group: $C_2 \ltimes G_{2852}$

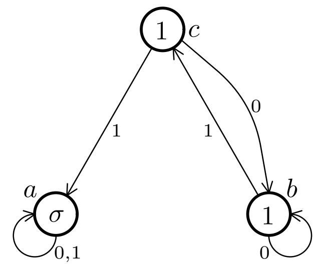
$b = (b, c)$ Contracting: *no*

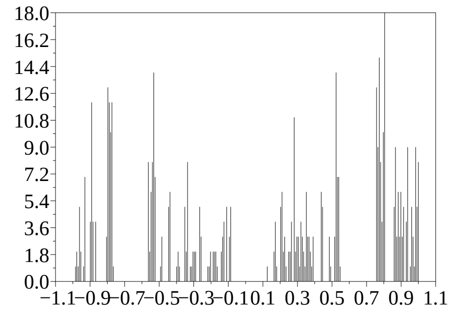
$c = (b, a)$ Self-replicating: *yes*

Rels: a^2, b^2, c^2, $abcabcacbacb$, $abcbcabcacbcbacb$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}

Gr: 1,4,10,22,46,94,184,352,664,1244,2320,4288





Automaton number 875

$a = \sigma(b, a)$ Group:

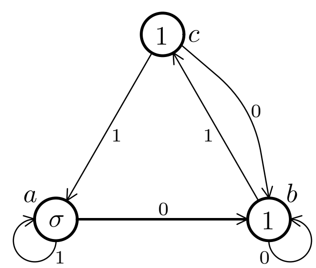
$b = (b, c)$ Contracting: *no*

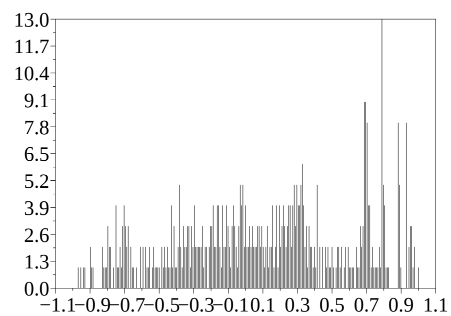
$c = (b, a)$ Self-replicating: *yes*

Rels: $(a^{-1}c)^2$, $(b^{-1}c)^2$, $(a^{-1}b)^4$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}

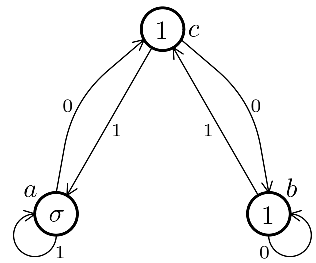
Gr: 1,7,33,143,607,2563

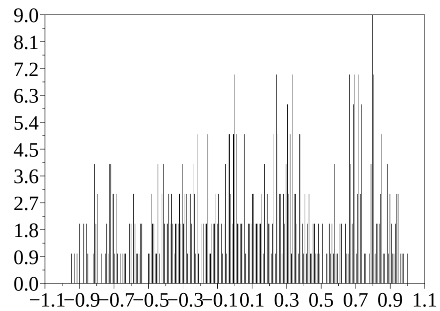




Automaton number 876

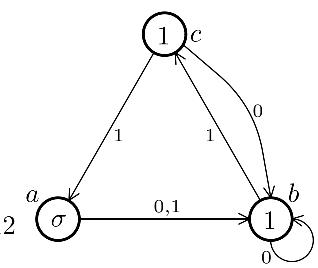
$a = \sigma(c, a)$ Group:
$b = (b, c)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $a^{-2}bcb^{-2}a^2c^{-1}b$, $a^{-2}cb^{-1}a^2c^{-2}bc$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,7,37,187,937,4667

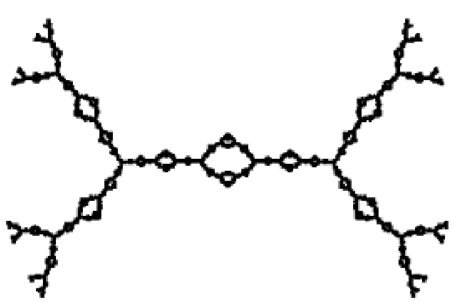


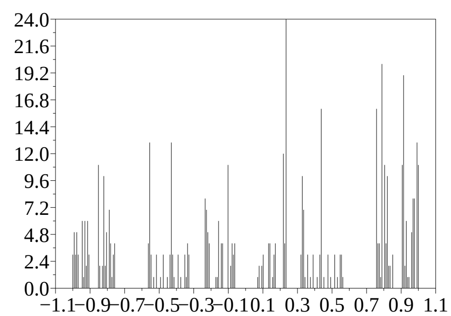


Automaton number 878

$a = \sigma(b, b)$ Group: $C_2 \ltimes IMG(1 - \frac{1}{z^2})$
$b = (b, c)$ Contracting: *yes*
$c = (b, a)$ Self-replicating: *yes*
Rels: a^2, b^2, c^2, $abcabcacbacb$,
$abcbcabcacbcbacb$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,4,10,22,46,94,184,352,664,1244,2296,4198,7612
Limit space:

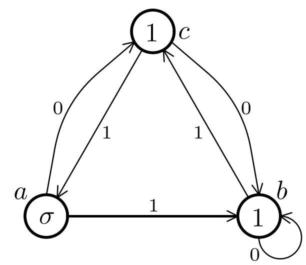


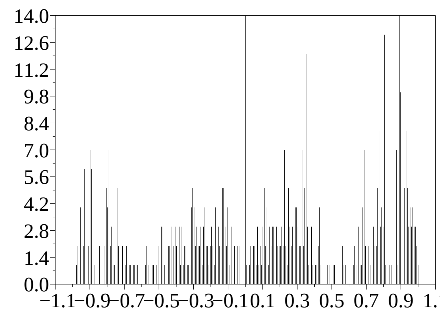




Automaton number 879

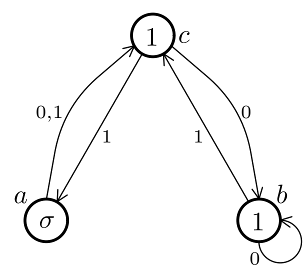
$a = \sigma(c, b)$ Group:
$b = (b, c)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $(a^{-1}b)^2$, $a^{-1}ca^{-1}cb^{-1}ac^{-1}ac^{-1}b$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,35,165,769,3567

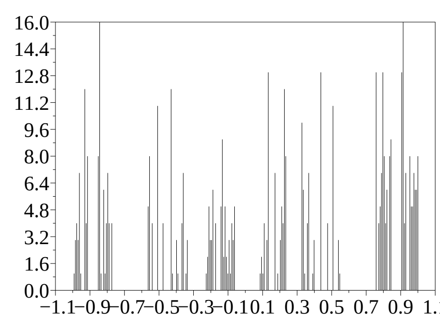




Automaton number 882

$a = \sigma(c, c)$ Group:
$b = (b, c)$ Contracting: *n/a*
$c = (b, a)$ Self-replicating: *yes*
Rels: a^2, b^2, c^2, $abcabcacbacb$,
$abcbcabcacbcbacb$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,4,10,22,46,94,184,352,664,1244





Automaton number 883

$a = \sigma(a, a)$ Group: $C_2 \ltimes G_{2841}$

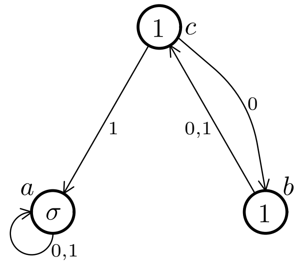
$b = (c, c)$ Contracting: *no*

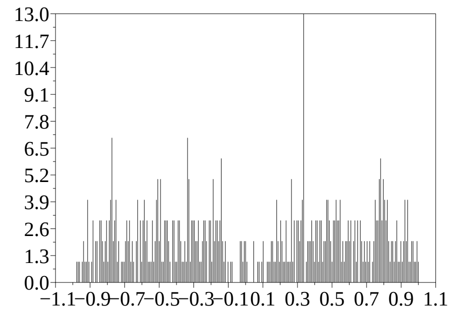
$c = (b, a)$ Self-replicating: *yes*

Rels: a^2, b^2, c^2, $acbcbacbcacbcabcbcabab$, $acbacbcacabacbacbcacab$

SF: 2^0,2^1,2^3,2^6,2^9,2^{14},2^{24},2^{43},2^{80}

Gr: 1,4,10,22,46,94,190,382,766,1534,3070,6120





Automaton number 884

$a = \sigma(b, a)$ Group:

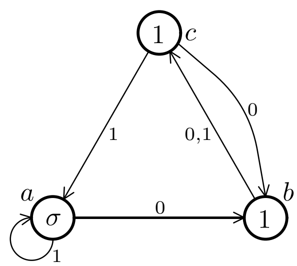
$b = (c, c)$ Contracting: *no*

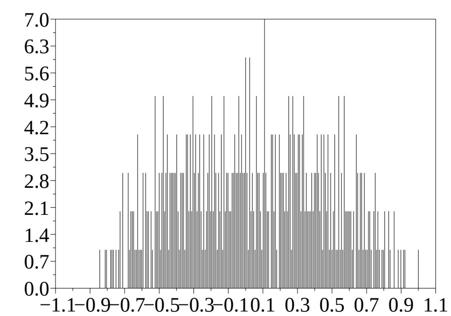
$c = (b, a)$ Self-replicating: *yes*

Rels: $(a^{-1}c)^2$, $(b^{-1}c)^2$, $[b, ac^{-1}]$

SF: 2^0,2^1,2^3,2^6,2^9,2^{15},2^{27},2^{49},2^{93}

Gr: 1,7,33,135,529,2051





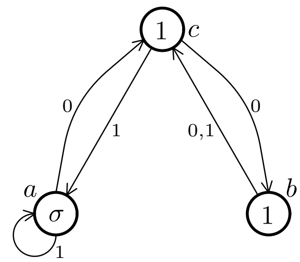
Automaton number 885

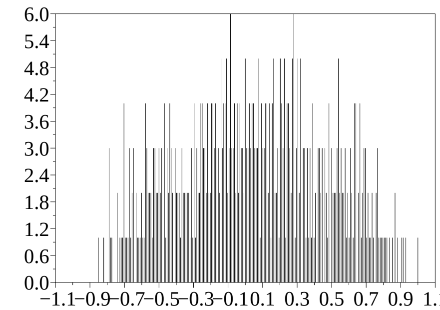
$a = \sigma(c, a)$ Group:
$b = (c, c)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*

Rels: $acba^{-1}b^{-1}ac^{-1}a^{-1}cba^{-1}b^{-1}ac^{-1}aca^{-1}\cdot bab^{-1}c^{-1}a^{-1}ca^{-1}bab^{-1}c^{-1}$,
$acba^{-1}b^{-1}ac^{-1}a^{-1}ca^{-1}c^{-1}b^{-1}a^{3}c^{-1}aca^{-1}b\cdot ab^{-1}c^{-1}a^{-1}ca^{-3}bcac^{-1}$

SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}

Gr: 1,7,37,187,937,4687





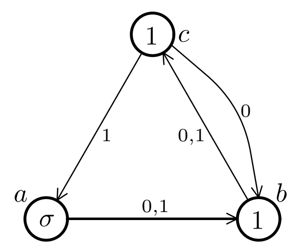
Automaton number 887

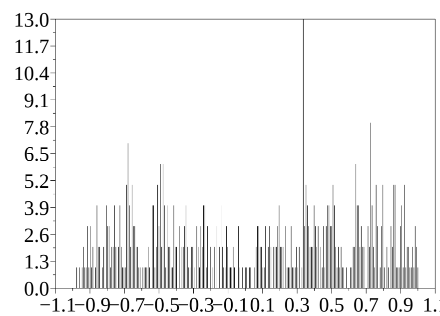
$a = \sigma(b, b)$ Group:
$b = (c, c)$ Contracting: *n/a*
$c = (b, a)$ Self-replicating: *yes*

Rels: a^2, b^2, c^2, *babacbcbacbcacbcabcbca*,
bacacbcabcabacacbcabca

SF: 2^0,2^1,2^3,2^6,2^9,2^{14},2^{24},2^{43},2^{80}

Gr: 1,4,10,22,46,94,190,382,766,1534,3070,6120





Automaton number 888

$a = \sigma(c, b)$ Group:

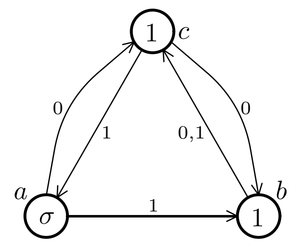
$b = (c, c)$ Contracting: *no*

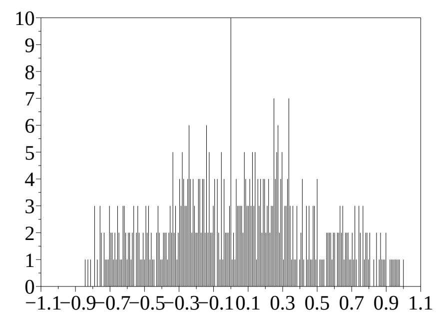
$c = (b, a)$ Self-replicating: *yes*

Rels: $aca^{-1}ba^{-2}ca^{-1}bab^{-1}ac^{-1}b^{-1}ac^{-1}$,
$aca^{-1}ba^{-3}bab^{-1}a^2b^{-1}ac^{-1}a^{-1}ba^{-1}b^{-1}a$,
$bab^{-1}a^{-1}ca^{-1}b^2a^{-1}b^{-1}ab^{-1}ac^{-1}$,
$bab^{-1}a^{-2}bab^{-1}aba^{-2}b^{-1}a$

SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}

Gr: 1,7,37,187,937,4687





Automaton number 891

$a = \sigma(c, c)$ Group: $C_2 \ltimes$ *Lampighter*

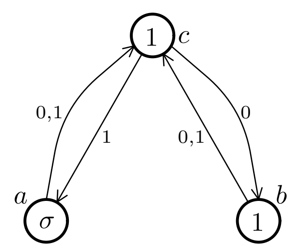
$b = (c, c)$ Contracting: *no*

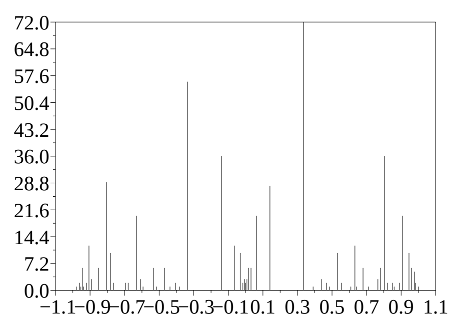
$c = (b, a)$ Self-replicating: *yes*

Rels: a^2, b^2, c^2, $abab$, $(acb)^4$,
$[acaca, bcacb]$, $[acaca, bcbcb]$

SF: 2^0,2^1,2^3,2^6,2^7,2^9,2^{10},2^{11},2^{12}

Gr: 1,4,9,17,30,51,82,128,198,304





Automaton number 920

$a = \sigma(b, a)$ Group:

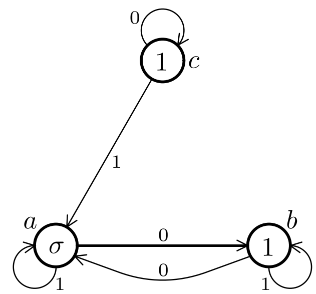
$b = (a, b)$ Contracting: *n/a*

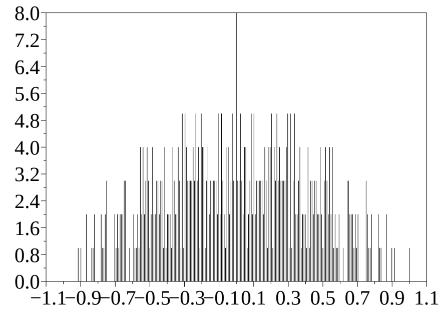
$c = (c, a)$ Self-replicating: *yes*

Rels: $(a^{-1}b)^2$, $[a, b]^2$, $(a^{-1}c^{-1}ab)^2$

SF: 2^0,2^1,2^3,2^5,2^9,2^{15},2^{26},2^{48},2^{92}

Gr: 1,7,35,165,757,3447





Automaton number 923

$a = \sigma(b, b)$ Group:

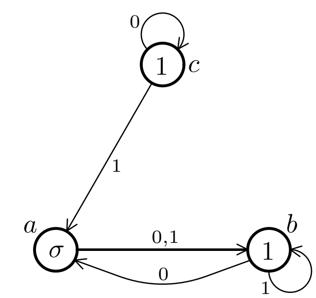
$b = (a, b)$ Contracting: *yes*

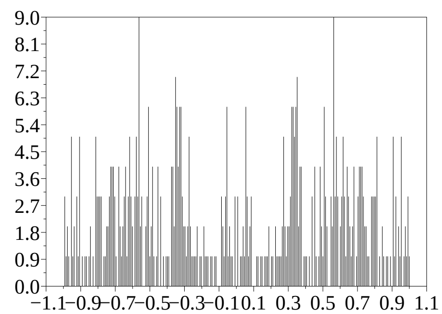
$c = (c, a)$ Self-replicating: *yes*

Rels: a^2, b^2, c^2, $abcabcbabcbacbabab$

SF: 2^0,2^1,2^3,2^4,2^6,2^9,2^{15},2^{26},2^{48}

Gr: 1,4,10,22,46,94,190,382,766,
1525,3025,5998,11890

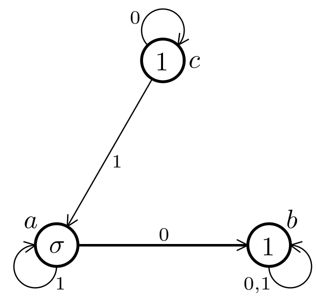


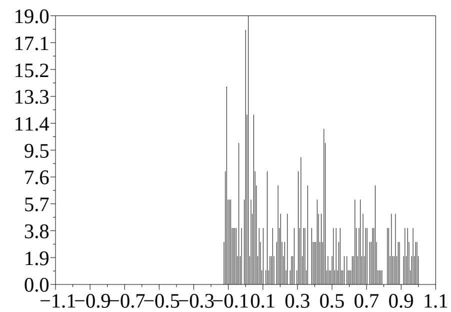


Automaton number 929

$a = \sigma(b, a)$ Group:
$b = (b, b)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*

Rels: b, $a^{-3}cac^{-1}ac^{-1}ac$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,5,17,53,161,475,1387

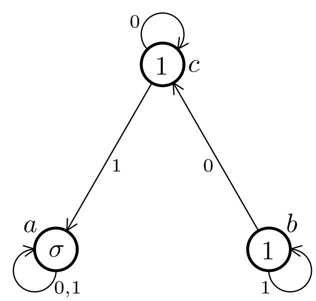


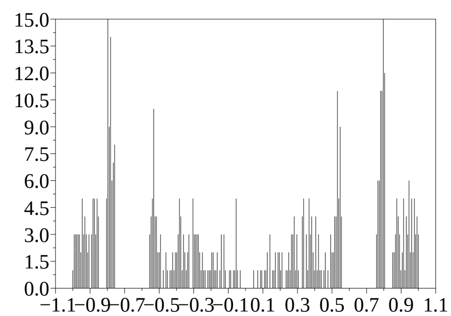


Automaton number 937

$a = \sigma(a, a)$ Group: $C_2 \ltimes G_{929}$
$b = (c, b)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*

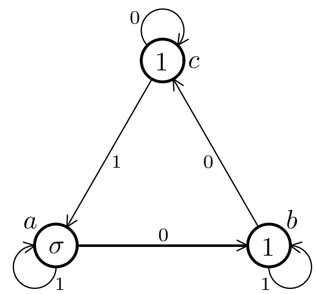
Rels: a^2, b^2, c^2, *abcabcacbacb*,
abcbcabcacbcbacb
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,4,10,22,46,94,184,352,664,1244

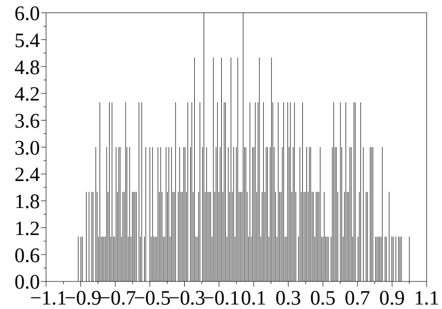




Automaton number 938

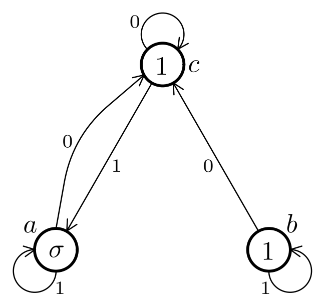
$a = \sigma(b, a)$ Group:
$b = (c, b)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $a^{-2}bcb^{-2}a^2c^{-1}b$, $a^{-2}cb^{-1}a^2c^{-2}bc$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,7,37,187,937,4667

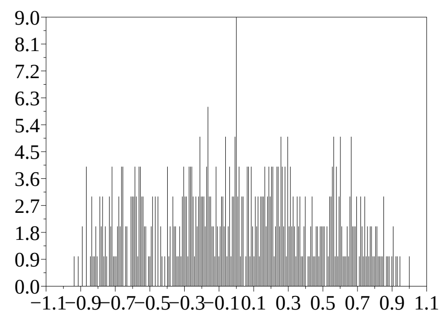




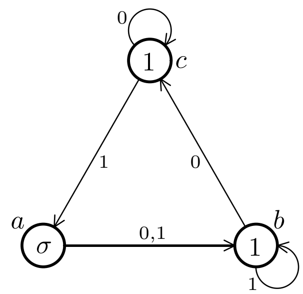
Automaton number 939

$a = \sigma(c, a)$ Group:
$b = (c, b)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $(a^{-1}c)^2$, $(a^{-2}cb)^2$, $[a, c]^2$,
$[ca^{-1}, ba^{-1}b]$, $a^{-1}b^{-1}abc^{-1}a^{-1}bca^{-1}b$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,35,165,757,3427

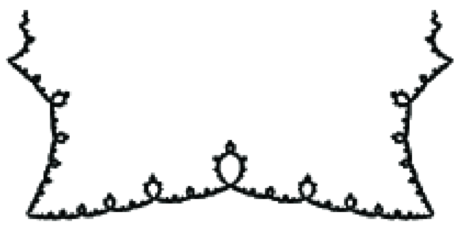


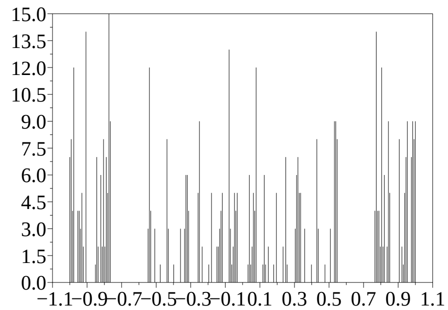


Automaton number 941

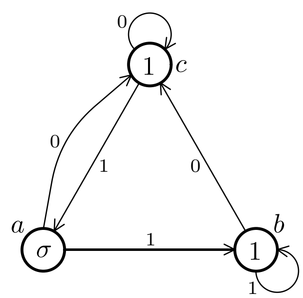


$a = \sigma(b, b)$ Group: $C_2 \ltimes IMG(z^2 - 1)$
$b = (c, b)$ Contracting: *yes*
$c = (c, a)$ Self-replicating: *yes*
Rels: a^2, b^2, c^2, $abcabcacbacb$,
$abcbcabcacbcbacb$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,4,10,22,46,94,184,352,664,1244
Limit space:

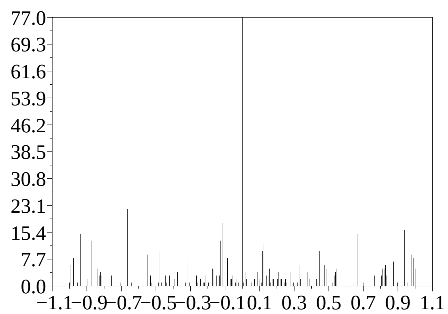




Automaton number 942



$a = \sigma(c, b)$ Group: *Contains Lamplighter group*
$b = (c, b)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $(a^{-1}b)^2$, $(b^{-1}c)^2$, $[a, b]^2$, $[b, c]^2$,
$(a^{-1}c)^4$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,33,143,597,2465



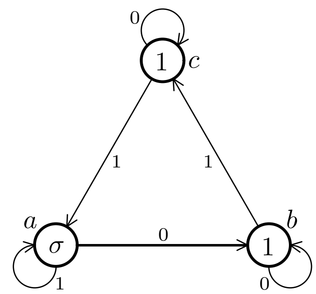
Automaton number 956

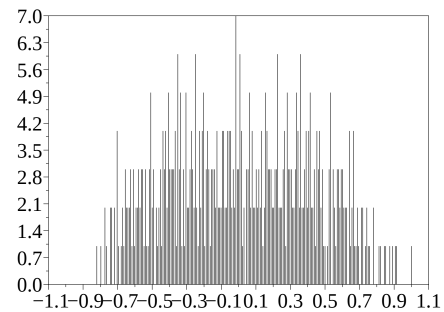
$a = \sigma(b, a)$ Group:
$b = (b, c)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*

Rels: $acba^{-1}b^{-1}ab^{-1}a^{-1}cba^{-1}b^{-1}ab^{-1}aba^{-1}\cdot$
$bab^{-1}c^{-1}a^{-1}ba^{-1}bab^{-1}c^{-1}$,
$acba^{-1}b^{-1}ab^{-1}a^{-1}b^{-1}ca^{-1}caba^{-1}bab^{-1}c^{-1}\cdot$
$a^{-2}bc^{-1}baba^{-1}bab^{-1}c^{-1}a^{-1}b^{-1}cb^{-1}a^{2}cb\cdot$
$a^{-1}b^{-1}ab^{-1}a^{-1}c^{-1}ac^{-1}b$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{90},2^{176}

Gr: 1,7,37,187,937,4687





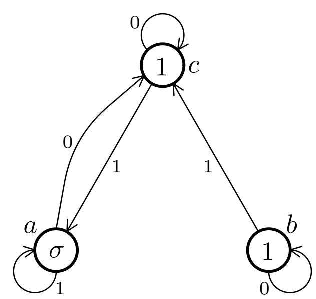
Automaton number 957

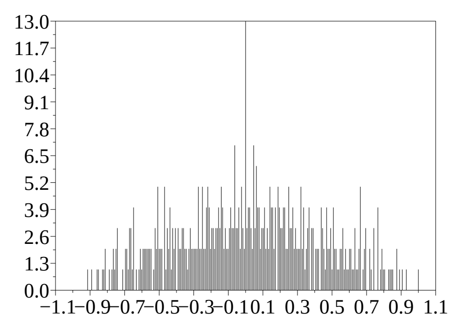
$a = \sigma(c, a)$ Group:
$b = (b, c)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*

Rels: $(a^{-1}c)^2$, $(b^{-1}c)^2$, $[a, c]^2$,
$[b, c]^2$, $(a^{-1}c)^4$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}

Gr: 1,7,33,143,599,2485

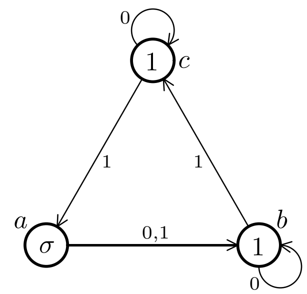


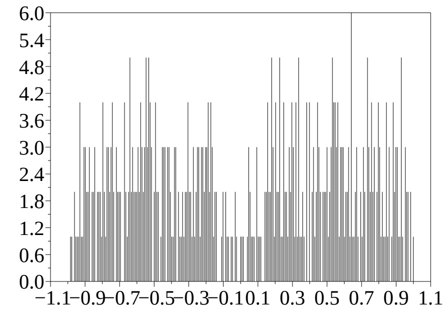


Automaton number 959

$a = \sigma(b, b)$ Group:
$b = (b, c)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*

Rels: a^2, b^2, c^2, $abcabcbabcbacbabab$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,4,10,22,46,94,190,382,766,1525

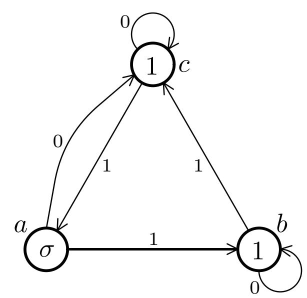


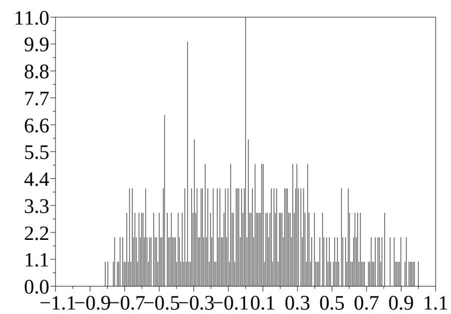


Automaton number 960

$a = \sigma(c, b)$ Group:
$b = (b, c)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*

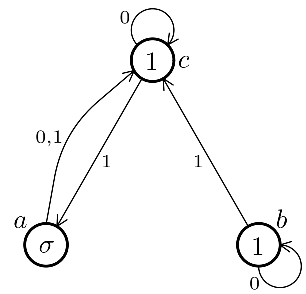
Rels: $(a^{-1}b)^2$, $(a^{-2}bc)^2$, $(a^{-1}c)^4$, $(b^{-1}c)^4$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,35,165,758,3460

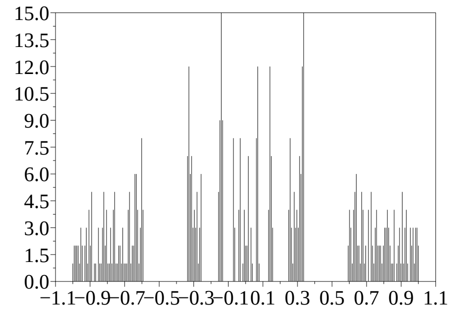




Automaton number 963

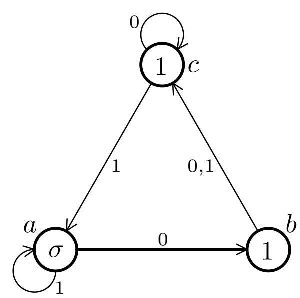
$a = \sigma(c, c)$ Group:
$b = (b, c)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: a^2, b^2, c^2, $acbacacabcabab$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,4,10,22,46,94,190,375,731,
1422,2762,5350,10322

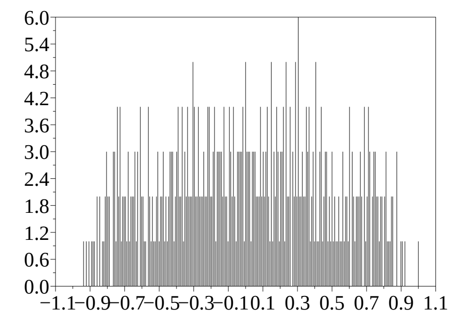




Automaton number 965

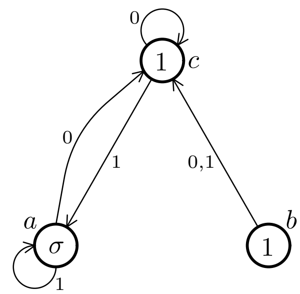
$a = \sigma(b, a)$ Group:
$b = (c, c)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $acb^{-1}a^{-1}cb^{-1}abc^{-1}a^{-1}bc^{-1}$,
$acb^{-1}a^{-1}cac^{-1}b^{-1}cbc^{-2}bca^{-1}c^{-1}$,
$acac^{-1}b^{-1}ca^{-2}cb^{-1}a^2c^{-1}bca^{-1}c^{-1}a^{-1}bc^{-1}$,
$acac^{-1}b^{-1}ca^{-2}cac^{-1}b^{-1}cac^{-1}bca^{-1}c^{-2}bca^{-1}c^{-1}$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,37,187,937,4687

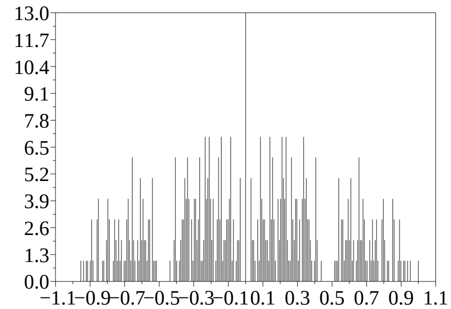




Automaton number 966

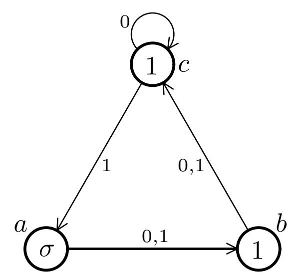
$a = \sigma(c, a)$ Group:
$b = (c, c)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *no*
Rels: $(a^{-1}c)^2$, $(b^{-1}c)^2$, $[ca^{-1}, b]$,
$[a, b]^2$, $(a^{-2}b^2)^2$, $(a^{-1}b)^4$, $[[c^{-1}, a^{-1}], cb^{-1}]$
SF: 2^0,2^1,2^3,2^6,2^9,2^{11},2^{14},2^{16},2^{18}
Gr: 1,7,33,135,495,1725

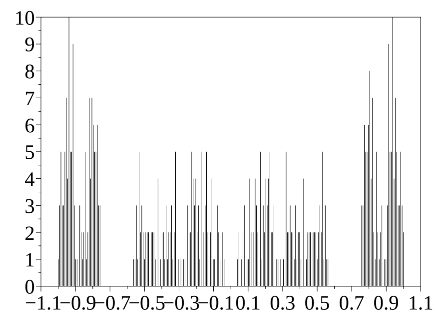




Automaton number 968

$a = \sigma(b, b)$ Group: Virtually $\mathbb{Z}^5$
$b = (c, c)$ Contracting: *yes*
$c = (c, a)$ Self-replicating: *no*
Rels: a^2, b^2, c^2, $(abc)^2(acb)^2$,
$(cbcbaba)^2$, $(cacbcba)^2$,
$(cabacbaba)^2$, $((ac)^4b)^2$
SF: 2^0,2^1,2^3,2^6,2^9,2^{13},2^{17},2^{21},2^{25}
Gr: 1,4,10,22,46,94,184,338,600,1022

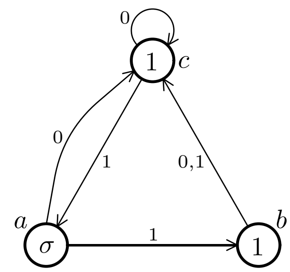


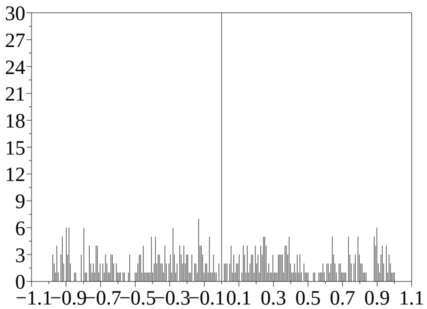


Automaton number 969

$a = \sigma(c, b)$ Group:
$b = (c, c)$ Contracting: *n/a*
$c = (c, a)$ Self-replicating: *yes*

Rels: $a^{-1}c^{-1}bab^{-1}a^{-1}cb^{-1}ab$,
$a^{-1}c^{-1}bac^{-1}a^{-1}cb^{-1}ac$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,37,187,937,4667

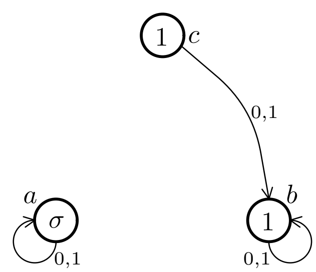


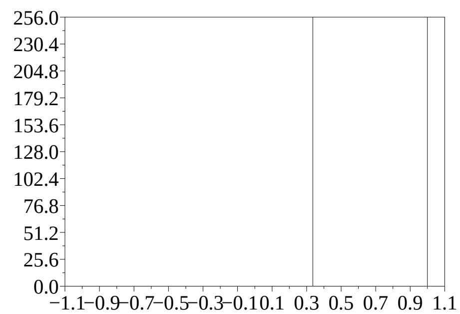


Automaton number 1090

$a = \sigma(a, a)$ Group: C_2
$b = (b, b)$ Contracting: *yes*
$c = (b, b)$ Self-replicating: *no*

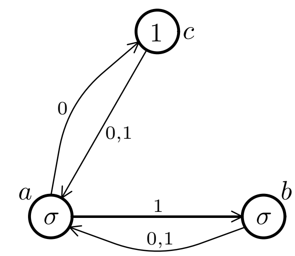
Rels: b, c, a^2
SF: 2^0,2^1,2^1,2^1,2^1,2^1,2^1,2^1,2^1
Gr: 1,2,2,2,2,2,2,2,2,2,2

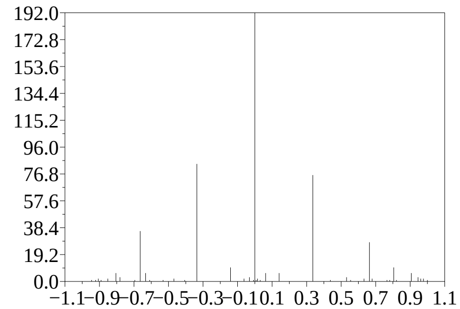




Automaton number 2193

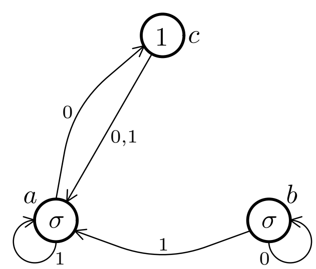
$a = \sigma(c, b)$ Group: *Contains Lamplighter group*
$b = \sigma(a, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*
Rels: $[b, c]$, b^2c^2, a^4, b^4,
$(a^2b)^2$, $(abc)^2$, $(a^2c)^2$
SF: 2^0,2^1,2^3,2^6,2^7,2^9,2^{10},2^{11},2^{12}
Gr: 1,7,27,65,120,204,328,
512,792,1216

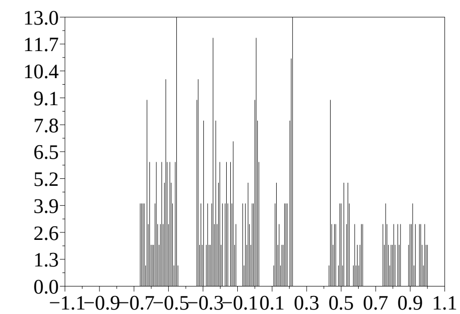




Automaton number 2199

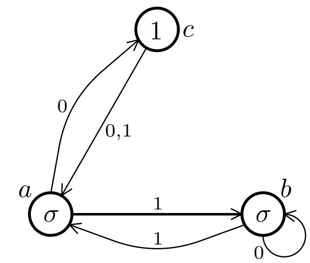
$a = \sigma(c, a)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*
Rels: ca^2, $[a^{-1}b, ab^{-1}]$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,29,115,417,1505

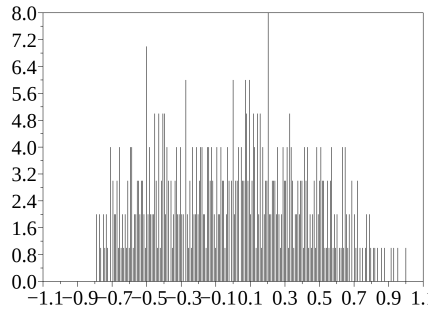




Automaton number 2202

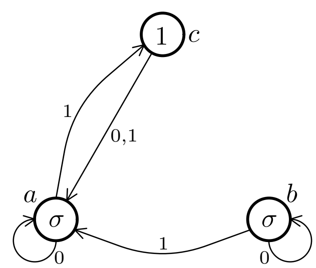
$a = \sigma(c, b)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*
Rels: cab^2a
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,37,177,833,3909

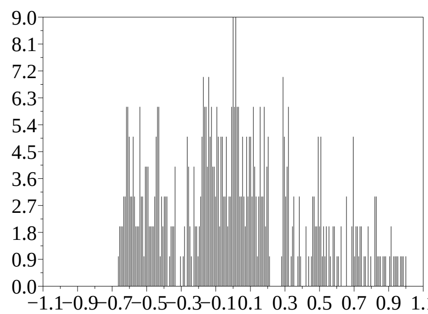




Automaton number 2203

$a = \sigma(a, c)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*
Rels: ca^2, $[c^{-2}ab, bc^{-2}a]$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,29,115,441,1695



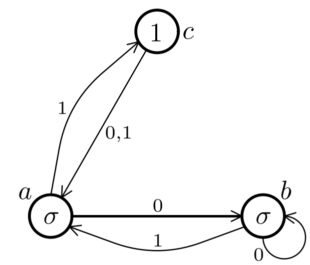


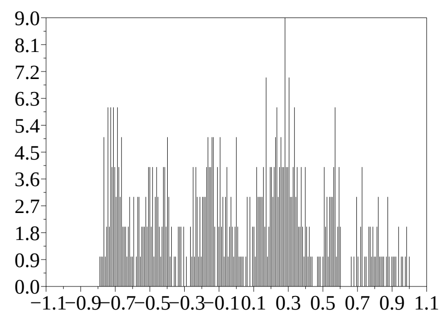
Automaton number 2204

$a = \sigma(b, c)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*

Rels: $bcba^2$, $[b^{-1}a, ba^{-1}]$,
$a^{-1}ba^2ba^{-2}b^{-2}aba^2b^{-1}a^{-2}$

SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,37,177,825,3781

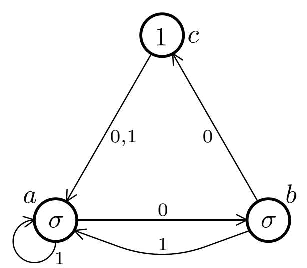


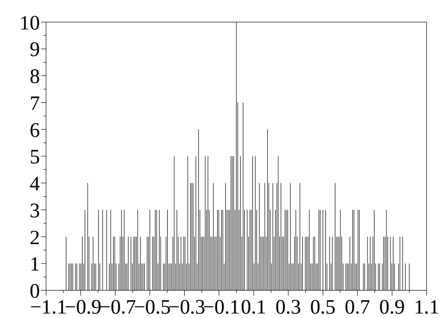


Automaton number 2207

$a = \sigma(b, a)$ Group:
$b = \sigma(c, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*

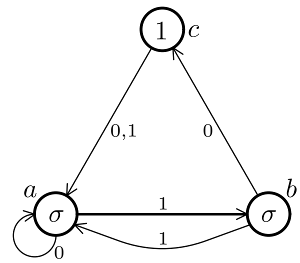
Rels: $[b^{-1}a, ba^{-1}]$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,37,187,929,4599



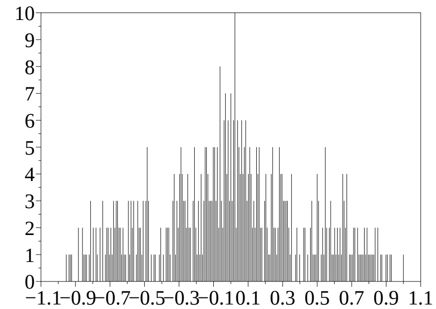


Automaton number 2209

$a = \sigma(a, b)$ Group:
$b = \sigma(c, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*

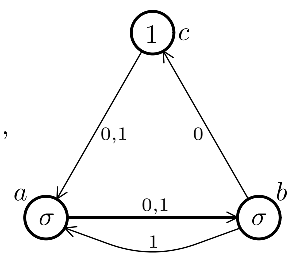


Rels: $aca^{-2}c^{-1}acac^{-1}a^{-2}cac^{-1}$,
$aca^{-2}b^{-1}a^{-1}cacac^{-1}a^{-2}c^{-1}abac^{-1}$,
$aca^{-1}b^{-1}a^{-1}c^{2}a^{-1}c^{-1}ac^{-1}abac^{-1}a^{-2}cac^{-1}$,
$aca^{-1}b^{-1}a^{-1}c^{2}a^{-1}b^{-1}a^{-1}cac^{-1}\cdot$
$abac^{-1}a^{-2}c^{-1}abac^{-1}$,
$bca^{-1}c^{-1}ab^{-1}ca^{-1}c^{-1}aba^{-1}ca\cdot$
$c^{-1}b^{-1}a^{-1}cac^{-1}$
SF: 2^{0},2^{1},2^{3},2^{6},2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,37,187,937,4687

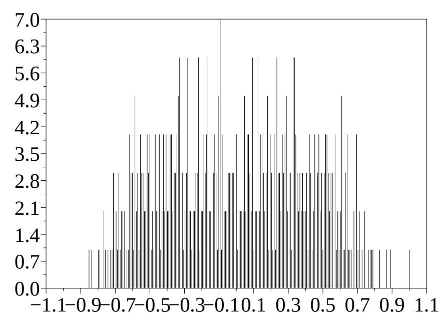


Automaton number 2210

$a = \sigma(b, b)$ Group:
$b = \sigma(c, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*

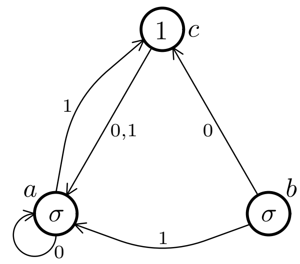


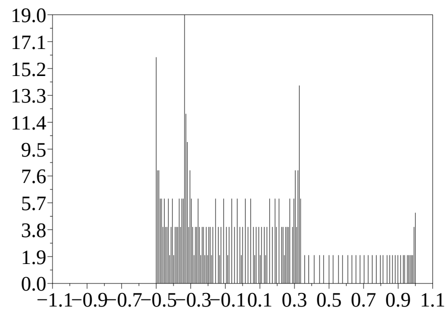
Rels: $acbc^{-1}b^{-1}a^{-1}cbc^{-1}b^{-1}abcb^{-1}c^{-1}a^{-1}bcb^{-1}c^{-1}$,
$bcbc^{-1}b^{-2}cbc^{-1}bcb^{-2}c^{-1}$,
$bcbc^{-1}b^{-2}ca^{-1}b^{-1}cabcb^{-1}c^{-1}a^{-1}c^{-1}bac^{-1}$,
$bca^{-1}b^{-1}cab^{-2}cbc^{-1}ba^{-1}c^{-1}bab^{-1}c^{-1}$,
$bca^{-1}b^{-1}cab^{-2}ca^{-1}b^{-1}caba^{-1}c^{-1}\cdot$
$bac^{-1}a^{-1}c^{-1}bac^{-1}$
SF: 2^{0},2^{1},2^{3},2^{5},2^{8},2^{13},2^{23},2^{42},2^{79}
Gr: 1,7,37,187,937,4687



Automaton number 2212

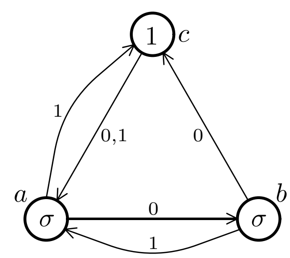
$a = \sigma(a, c)$ Group: *Klein bottle group*
$b = \sigma(c, a)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *no*
Rels: ca^2, cb^2
SF: 2^0,2^1,2^2,2^4,2^6,2^8,2^{10},2^{12},2^{14}
Gr: 1,7,19,37,61,91,127,169,217,271,331

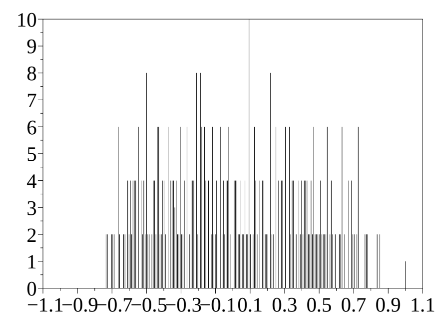




Automaton number 2213

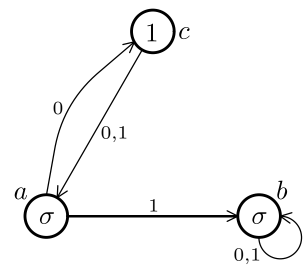
$a = \sigma(b, c)$ Group:
$b = \sigma(c, a)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*
Rels: $bcbc^{-1}b^{-2}cbc^{-1}bcb^{-2}c^{-1}$,
$acbc^{-1}b^{-1}a^{-1}cbc^{-1}b^{-1}abcb^{-1}c^{-1}\cdot$
$a^{-1}bcb^{-1}c^{-1}$,
$acbc^{-1}b^{-1}a^{-1}ba^{-1}c^{-1}b^{2}c^{-1}abcb^{-1}c^{-1}a^{-1}\cdot$
$cb^{-2}cab^{-1}$,
$aba^{-1}c^{-1}b^{2}c^{-1}a^{-1}cbc^{-1}b^{-1}\cdot$
$acb^{-2}cab^{-1}a^{-1}bcb^{-1}c^{-1}$,
SF: 2^0,2^1,2^2,2^3,2^5,2^8,2^{14},2^{25},2^{47}
Gr: 1,7,37,187,937,4687

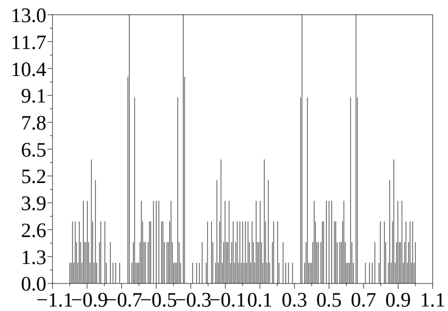




Automaton number 2229

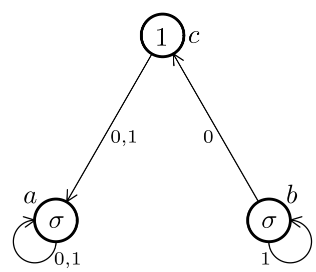
$a = \sigma(c, b)$ Group: $C_4 \ltimes \mathbb{Z}^2$
$b = \sigma(b, b)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *no*
Rels: b^2, $(ab)^2$, $(bc)^2$, a^4, c^4,
$[a, c]^2$, $(a^{-1}c)^4$, $(ac)^4$
SF: 2^0,2^1,2^3,2^6,2^9,2^{11},2^{13},2^{15},2^{17}
Gr: 1,6,20,54,128,270,510,886,1452

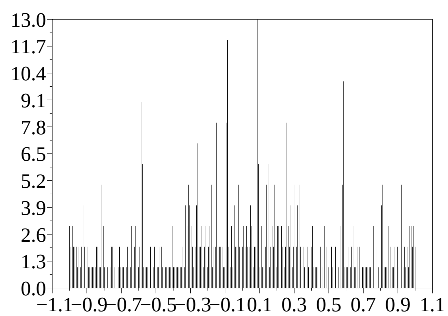




Automaton number 2233

$a = \sigma(a, a)$ Group:
$b = \sigma(c, b)$ Contracting: *yes*
$c = (a, a)$ Self-replicating: *yes*
Rels: a^2, c^2, $abab$, $acac$, $cb^2acbcbcab^2cabcba$
SF: 2^0,2^1,2^3,2^6,2^9,2^{15},2^{26},2^{48},2^{91}
Gr: 1,5,14,32,68,140,284,565,1106





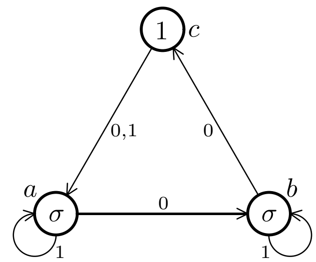
Automaton number 2234

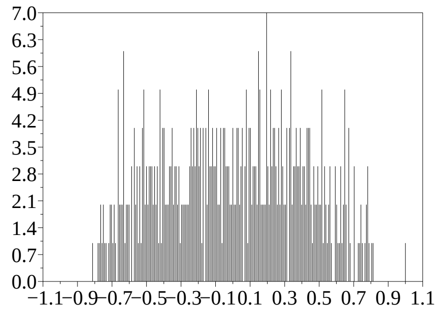
$a = \sigma(b, a)$ Group:
$b = \sigma(c, b)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*

Rels: $ac^{-1}a^2c^{-1}ab^{-1}a^{-1}c^{-1}a^2c^{-1}ab^{-1}ab\cdot$
$a^{-1}ca^{-2}ca^{-1}ba^{-1}ca^{-2}c$,
$ac^{-1}a^2c^{-1}ab^{-1}a^{-1}cbac^{-1}ab^{-1}a^{-1}c^{-1}aba^{-1}\cdot$
$ca^{-1}b^{-1}aba^{-1}ca^{-2}ca^{-1}bac^{-1}ab^{-1}a^{-1}ca\cdot$
$ba^{-1}ca^{-1}b^{-1}c^{-1}$

SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}

Gr: 1,7,37,187,937,4687





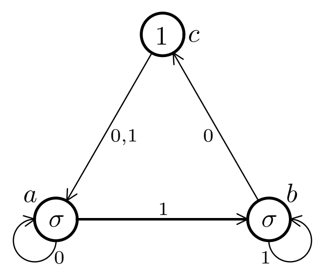
Automaton number 2236

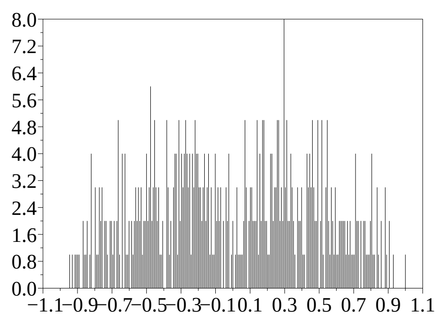
$a = \sigma(a, b)$ Group:
$b = \sigma(c, b)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*

Rels: $[b^{-1}a, ba^{-1}]$, $a^{-1}c^{-1}acb^{-1}ac^{-1}a^{-1}cb$,
$a^{-1}cac^{-1}b^{-1}aca^{-1}c^{-1}b$

SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}

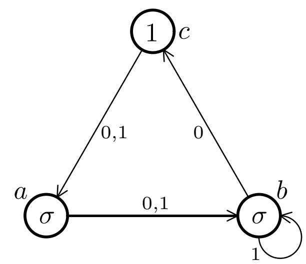
Gr: 1,7,37,187,929,4579

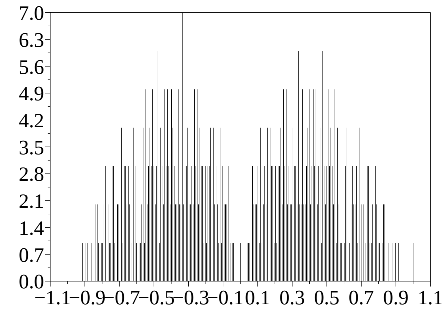




Automaton number 2237

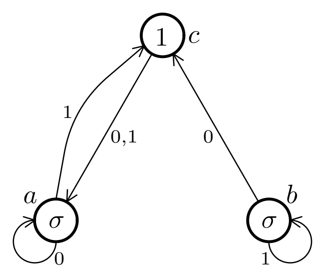
$a = \sigma(b, b)$ Group:
$b = \sigma(c, b)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *no*
Rels: $[b^{-1}a, ba^{-1}]$, $[c^{-1}a, ca^{-1}]$
SF: 2^0,2^1,2^3,2^6,2^9,2^{15},2^{26},2^{45},2^{81}
Gr: 1,7,37,187,921,4511

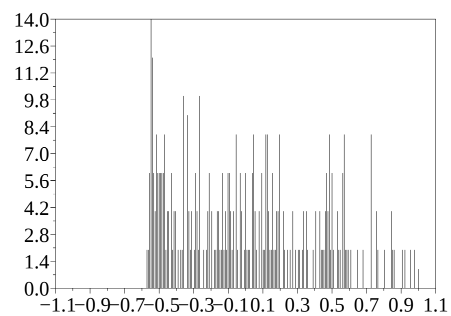




Automaton number 2239

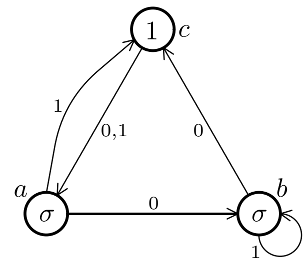
$a = \sigma(a, c)$ Group:
$b = \sigma(c, b)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*
Rels: ca^2, $[ca^{-2}cba^{-1}, a^{-1}ca^{-2}cb]$
SF: 2^0,2^1,2^2,2^3,2^5,2^8,2^{14},2^{25},2^{47}
Gr: 1,7,29,115,441,1695

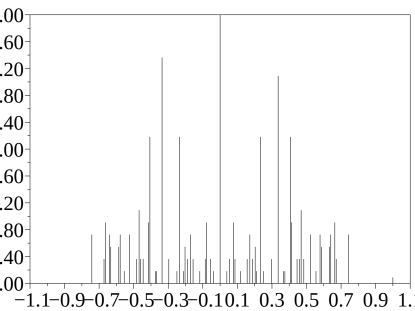




Automaton number 2240

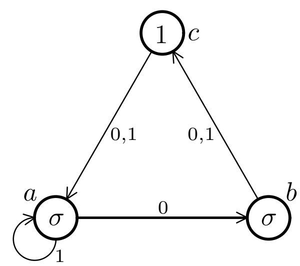
$a = \sigma(b, c)$ Group: F_3
$b = \sigma(c, b)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *no*
Rels:
SF: 2^0,2^1,2^2,2^4,2^7,2^{10},2^{14},2^{21},2^{34}
Gr: 1,7,37,187,937,4687

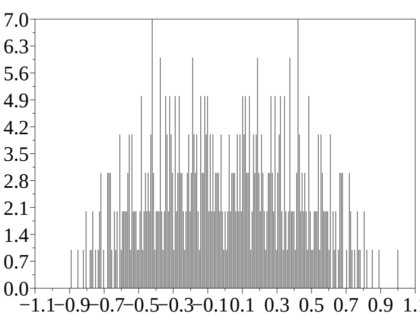




Automaton number 2261

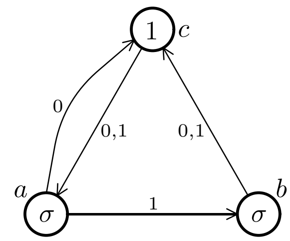
$a = \sigma(b, a)$ Group:
$b = \sigma(c, c)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *yes*
Rels: $acac^{-1}a^{-2}cac^{-1}aca^{-2}c^{-1}$,
$acac^{-1}a^{-2}cba^{-1}c^{-1}aca^{-1}cb^{-1}aca^{-1}c^{-1}$.
$bc^{-1}ac^{-1}a^{-1}cab^{-1}c^{-1}$,
$bcac^{-1}a^{-1}b^{-1}cac^{-1}a^{-1}baca^{-1}c^{-1}b^{-1}aca^{-1}c^{-1}$
SF: 2^0,2^1,2^2,2^4,2^6,2^9,2^{15},2^{26},2^{48}
Gr: 1,7,37,187,937,4687

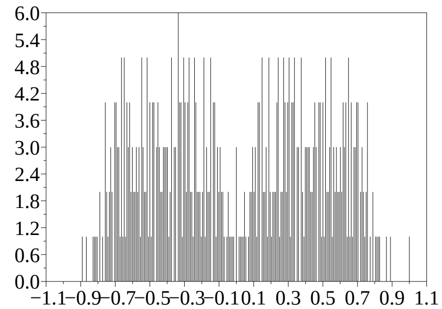




Automaton number 2265

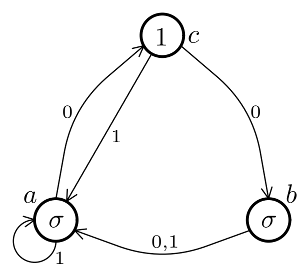
$a = \sigma(c, b)$ Group:
$b = \sigma(c, c)$ Contracting: *no*
$c = (a, a)$ Self-replicating: *no*
Rels: $[b^{-1}a, ba^{-1}]$, $a^{-1}ca^{-1}cb^{-1}ac^{-1}ac^{-1}b$,
$a^{-1}cb^{-1}cb^{-1}ac^{-1}bc^{-1}b$
SF: 2^{0},2^{1},2^{3},2^{6},2^{9},2^{14},2^{22},2^{36},2^{63}
Gr: 1,7,37,187,929,4579,22521

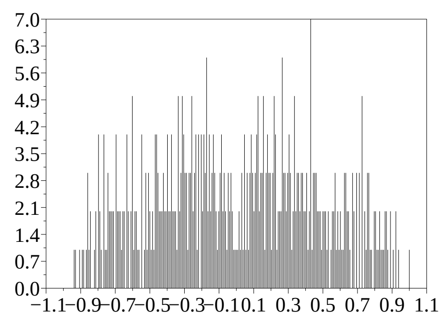




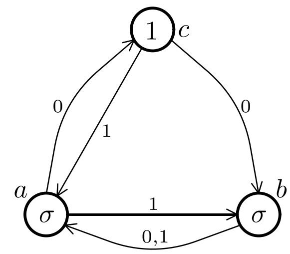
Automaton number 2271

$a = \sigma(c, a)$ Group:
$b = \sigma(a, a)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $[b^{-1}a, ba^{-1}]$, $a^{-1}c^{2}a^{-1}b^{-1}a^{2}c^{-2}b$,
$a^{-1}c^{2}b^{-2}abc^{-2}b$
SF: 2^{0},2^{1},2^{3},2^{7},2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,7,37,187,929,4583

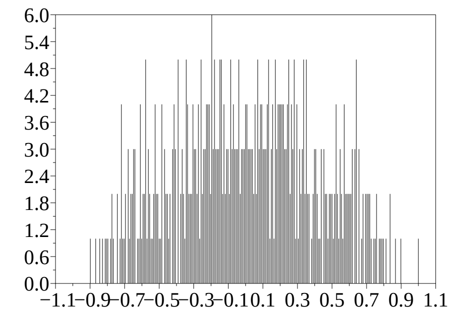




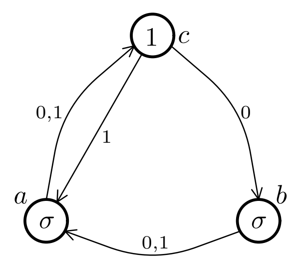
Automaton number 2274



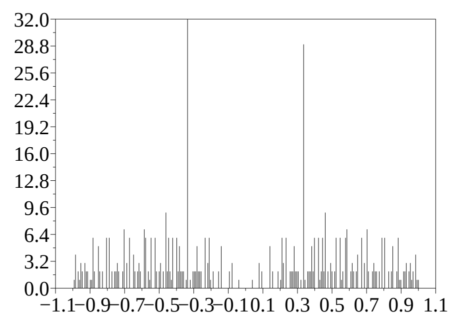
$a = \sigma(c, b)$ Group:
$b = \sigma(a, a)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $ac^3b^{-1}c^{-2}b^3c^{-3}a^{-1}c^3b^{-1}c^{-2}b^3c^{-3}ac^3b^{-3}\cdot$
$c^2bc^{-3}a^{-1}c^3b^{-3}c^2bc^{-3}$,
$ac^3b^{-1}c^{-2}b^3c^{-3}a^{-1}c^2ab^{-2}c^{-1}b^3c^{-3}ac^3b^{-3}\cdot$
$c^2bc^{-3}a^{-1}c^3b^{-3}cb^2a^{-1}c^{-2}$,
$bc^3b^{-1}c^{-2}b^3c^{-3}b^{-1}c^3b^{-1}c^{-2}b^3c^{-3}\cdot$
$bc^3b^{-3}c^2bc^{-3}b^{-1}c^3b^{-3}c^2bc^{-3}$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,7,37,187,937,4687



Automaton number 2277

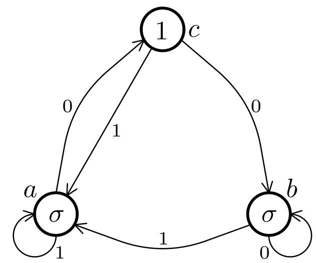


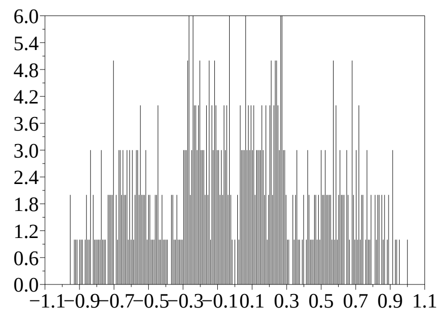
$a = \sigma(c, c)$ Group: $C_2 \ltimes (\mathbb{Z} \times \mathbb{Z})$
$b = \sigma(a, a)$ Contracting: *yes*
$c = (b, a)$ Self-replicating: *yes*
Rels: a^2, b^2, c^2, $(acb)^2$
SF: 2^0,2^1,2^2,2^4,2^5,2^6,2^7,2^8,2^9
Gr: 1,4,10,19,31,46,64,85,109,136,166
Limit space: 2-dimensional sphere S_2



Automaton number 2280

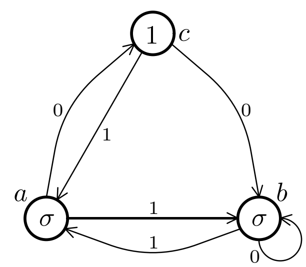
$a = \sigma(c, a)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $(a^{-1}b)^2$, $(b^{-1}c)^2$, $[a, b]^2$, $[b, c]^2$, $(a^{-1}c)^4$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,33,143,597,2465

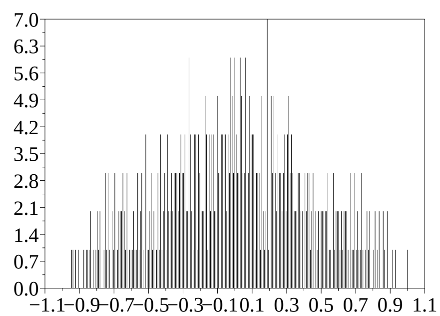




Automaton number 2283

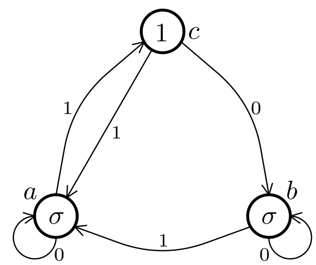
$a = \sigma(c, b)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $(a^{-1}b)^2$, $(b^{-1}c)^2$, $[b, c]^2$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,33,143,604,2534

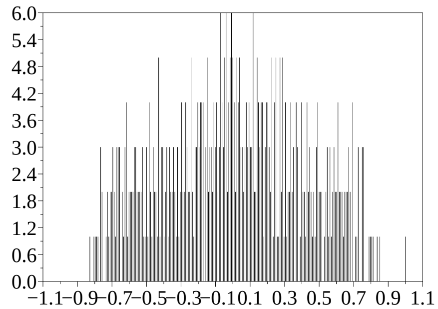




Automaton number 2284

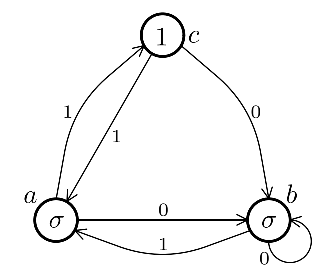
$a = \sigma(a, c)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $(b^{-1}c)^2$, $(a^{-1}b)^4$, $(bc^{-2}a)^2$,
$(a^{-1}c)^4$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,35,165,758,3460

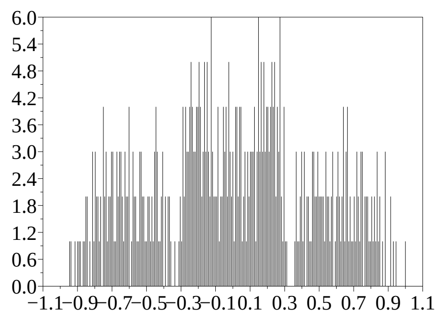




Automaton number 2285

$a = \sigma(b, c)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $(b^{-1}c)^2$, $[b^{-1}a, ba^{-1}]$, $[(c^{-1}a)^2, c^{-1}b]$,
$[(ca^{-1})^2, cb^{-1}]$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,35,165,761,3479





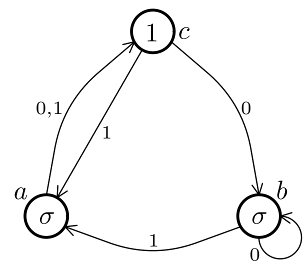
Automaton number 2286

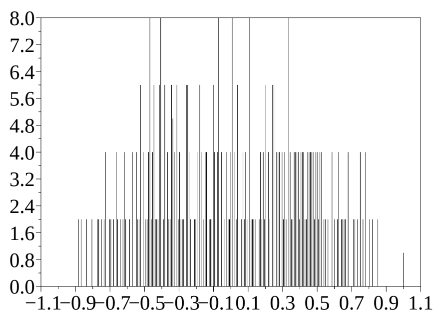
$a = \sigma(c, c)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*

Rels: $(b^{-1}c)^2$, $[a, bc^{-1}]$

SF: 2^0,2^1,2^2,2^3,2^5,2^9,2^{15},2^{27},2^{49}

Gr: 1,7,35,159,705,3107





Automaton number 2287

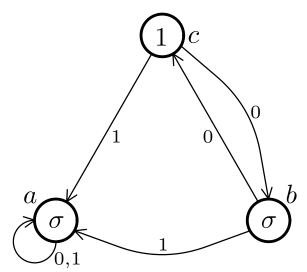
$a = \sigma(a, a)$ Group: $IMG\left(\frac{z^2+2}{1-z^2}\right)$
$b = \sigma(c, a)$ Contracting: *yes*
$c = (b, a)$ Self-replicating: *yes*

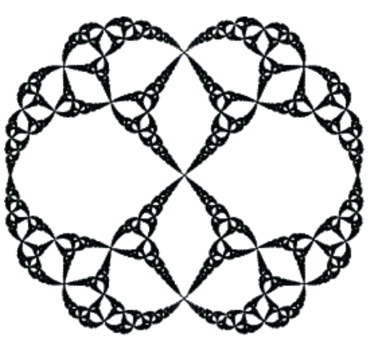
Rels: a^2, $[a, b^2]$, $(b^{-1}ac)^2$, $[ba, c^2]$, $[c^2, aca]$

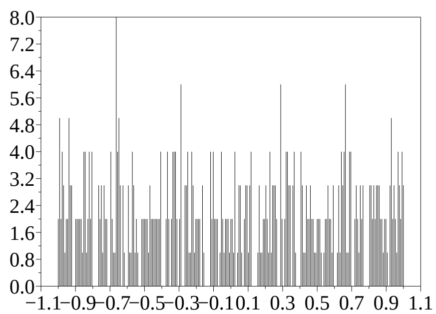
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}

Gr: 1,6,26,100,362,1246

Limit space:





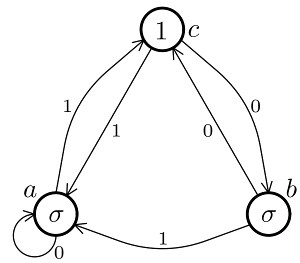


Automaton number 2293

$a = \sigma(a, c)$ Group:

$b = \sigma(c, a)$ Contracting: *no*

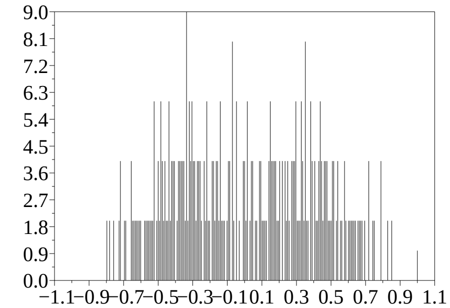
$c = (b, a)$ Self-replicating: *yes*



Rels:
$cb^{-1}a^{-1}ca^{-1}cb^{-1}a^{-1}cac^{-1}abc^{-1}a^{-1}c^{-1}abc^{-1}a$,
$cb^{-1}a^{-1}c^{2}a^{-1}c^{2}b^{-1}a^{-1}c^{2}b^{-1}a^{-1}ca^{-2}c^{-1}a\cdot$
$b^{2}c^{-2}ab^{-1}a^{-1}ca^{2}c^{-1}abc^{-2}abc^{-2}ac^{-1}$,
$ba^{-1}cb^{-1}a^{-1}cab^{-1}a^{-1}cb^{-1}a^{-1}c\cdot$
$aba^{-1}c^{-1}abc^{-1}ab^{-1}a^{-1}c^{-1}abc^{-1}a$

SF: 2^{0},2^{1},2^{2},2^{4},2^{8},2^{13},2^{23},2^{41},2^{76}

Gr: 1,7,37,187,937,4687



Automaton number 2294

$a = \sigma(b, c)$ Group: $BS(1, -3)$

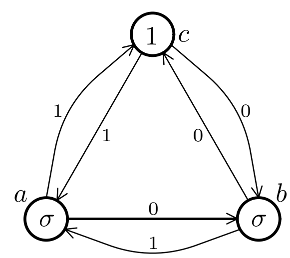
$b = \sigma(c, a)$ Contracting: *no*

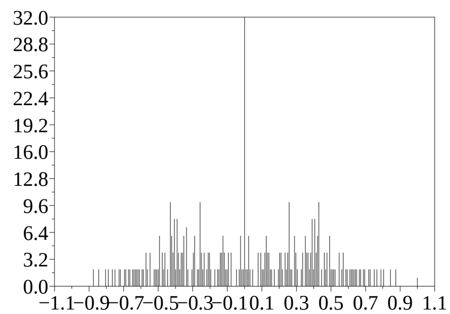
$c = (b, a)$ Self-replicating: *yes*

Rels: $b^{-1}ca^{-1}c$, $(ca^{-1})^{a}(ca^{-1})^{3}$

SF: 2^{0},2^{1},2^{2},2^{4},2^{6},2^{8},2^{10},2^{12},2^{14}

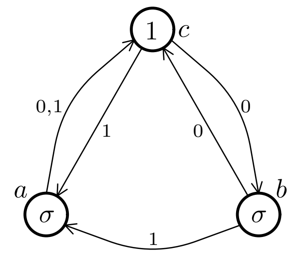
Gr: 1,7,33,127,433,1415

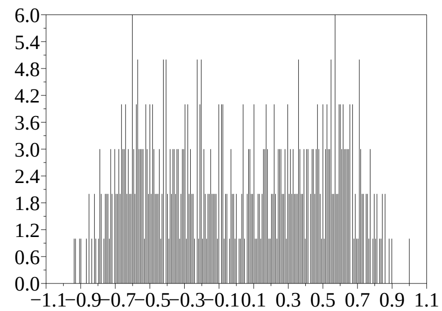




Automaton number 2295

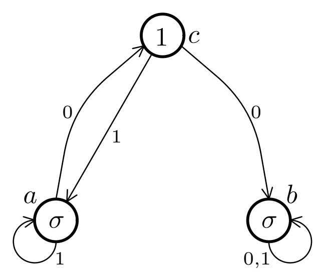
$a = \sigma(c, c)$ Group:
$b = \sigma(c, a)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: $[b^{-1}a, ba^{-1}]$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,7,37,187,929,4599

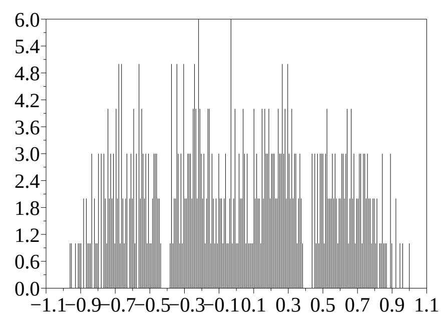




Automaton number 2307

$a = \sigma(c, a)$ Group:
$b = \sigma(b, b)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*
Rels: b^2, $a^{-2}c^{-1}bca^2c^{-1}bc$, $a^{-1}c^{-1}bc^{-2}bcac^2$,
$a^{-1}cbc^{-2}bc^{-1}ac^2$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,6,26,106,426,1681





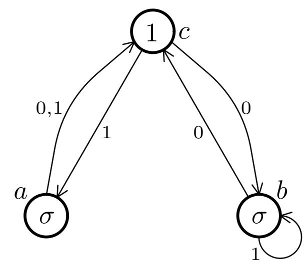
Automaton number 2322

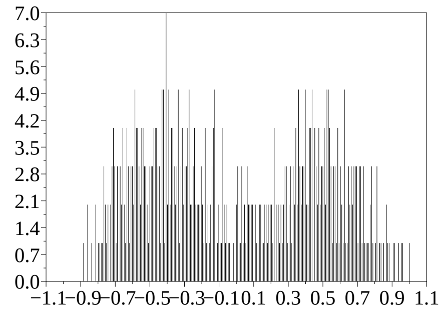
$a = \sigma(c, c)$ Group:
$b = \sigma(c, b)$ Contracting: *no*
$c = (b, a)$ Self-replicating: *yes*

Rels: $[b^{-1}a, ba^{-1}]$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}

Gr: 1,7,37,187,929,4599





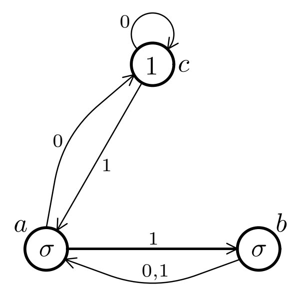
Automaton number 2355

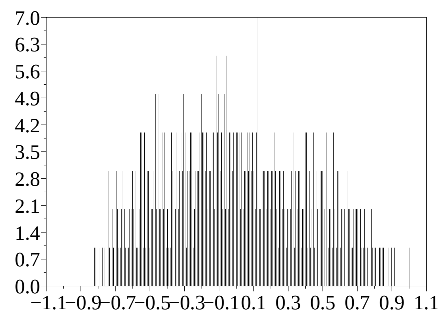
$a = \sigma(c, b)$ Group:
$b = \sigma(a, a)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*

Rels:
$bca^{-2}c^{-1}bcac^{-1}b^{-2}cac^{-1}$,
$aca^{-1}c^{-1}ba^{-1}ca^{-1}c^{-1}bab^{-1}cac^{-1}a^{-1}b^{-1}cac^{-1}$,
$abac^{-1}bc^{-1}b^{-1}a^{-1}ca^{-1}c^{-1}bab\cdot$
$cb^{-1}ca^{-1}b^{-1}a^{-1}b^{-1}cac^{-1}$,
$aca^{-1}c^{-1}ba^{-1}bac^{-1}bc^{-1}b^{-1}a\cdot$
$b^{-1}cac^{-1}a^{-1}bcb^{-1}ca^{-1}b^{-1}$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}

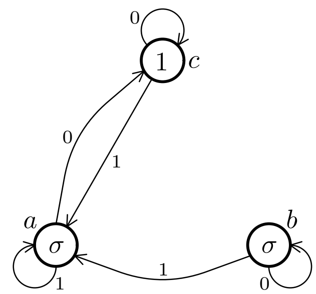
Gr: 1,7,37,187,937,4687

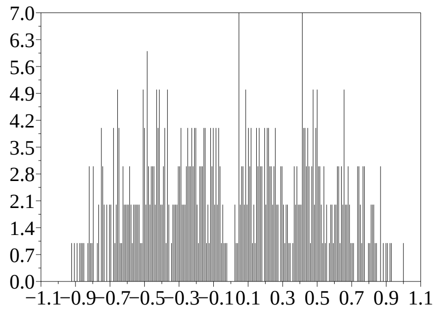




Automaton number 2361

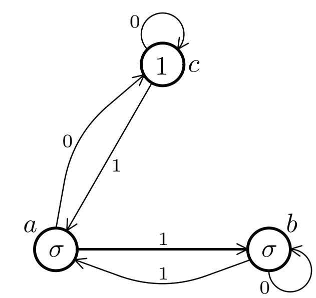
$a = \sigma(c, a)$ Group:
$b = \sigma(b, a)$ Contracting: *n/a*
$c = (c, a)$ Self-replicating: *yes*
Rels: $(a^{-1}c)^2$, $[b^{-1}a, ba^{-1}]$, $[a, c]^2$,
$(b^{-1}a^{-1}c^2)^2$, $[ac^{-1}, bc^{-1}ba^{-1}]$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,35,165,749,3343

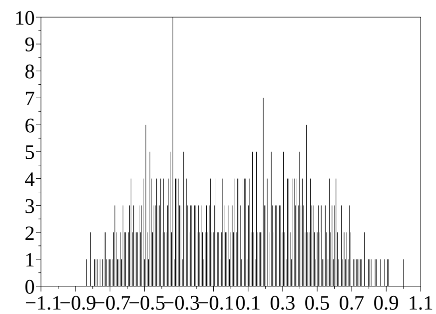




Automaton number 2364

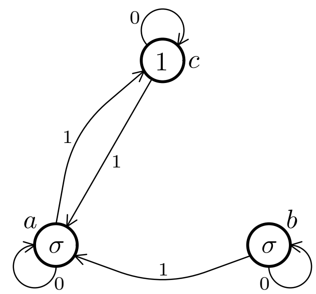
$a = \sigma(c, b)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels:
$aca^{-1}cb^{-1}a^{-1}ca^{-1}cb^{-1}abc^{-1}ac^{-1}a^{-1}bc^{-1}ac^{-1}$,
$bca^{-1}cb^{-2}ca^{-2}ca^{-1}b^3c^{-1}ac^{-1}b^{-2}ac^{-1}a^2c^{-1}$,
$bca^{-2}ca^{-1}ca^{-2}ca^{-1}bac^{-1}a^2c^{-1}b^{-2}ac^{-1}a^2c^{-1}$,
$bca^{-2}ca^{-1}ca^{-1}cb^{-1}ac^{-1}a^2c^{-2}ac^{-1}$,
$bca^{-1}cb^{-2}ca^{-1}cbc^{-1}ac^{-2}ac^{-1}$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{24},2^{46},2^{90},2^{176}
Gr: 1,7,37,187,937,4687

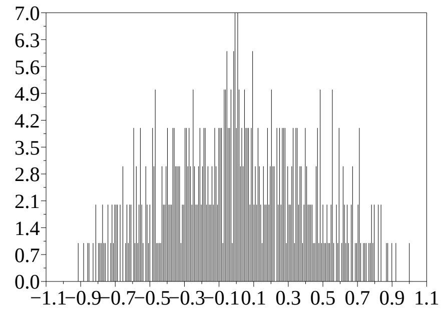




Automaton number 2365

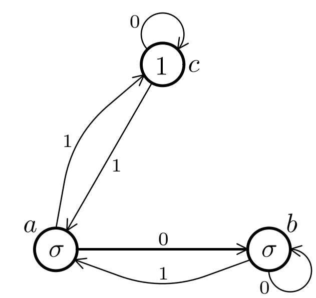
$a = \sigma(a, c)$ Group:
$b = \sigma(b, a)$ Contracting: *n/a*
$c = (c, a)$ Self-replicating: *yes*
Rels: $(a^{-1}b)^2$, $(a^{-1}c)^2$, $[a, c]^2$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,33,143,604,2534

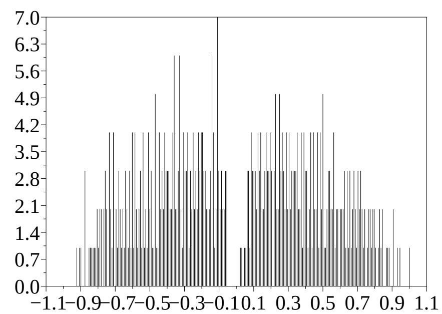




Automaton number 2366

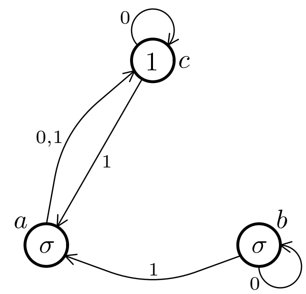
$a = \sigma(b, c)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $[b^{-1}a, ba^{-1}]$, $a^{-1}c^{-1}acb^{-1}ac^{-1}a^{-1}cb$,
$a^{-1}cbc^{-1}b^{-1}acb^{-1}c^{-1}b$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,37,187,929,4579

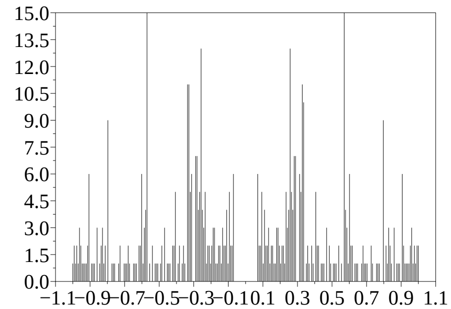




Automaton number 2367

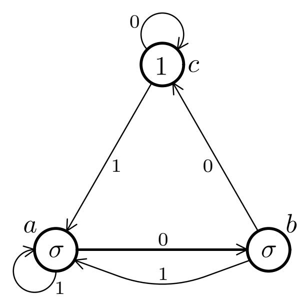
$a = \sigma(c, c)$ Group:
$b = \sigma(b, a)$ Contracting: *yes*
$c = (c, a)$ Self-replicating: *yes*
Rels: a^2, c^2, $b^{-2}cacb^2cac$
SF: 2^0,2^1,2^3,2^5,2^8,2^{14},2^{25},2^{47},2^{90}
Gr: 1,5,17,53,161,480,1422

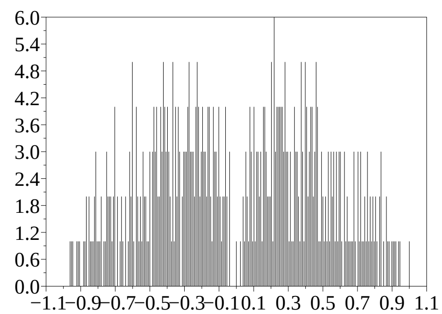




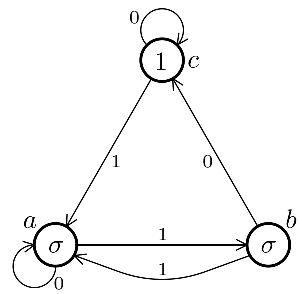
Automaton number 2369

$a = \sigma(b, a)$ Group:
$b = \sigma(c, a)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $(a^{-1}b)^2$, $(b^{-1}c)^2$, $[a, b]^2$, $(a^{-1}c)^4$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,33,143,602,2514

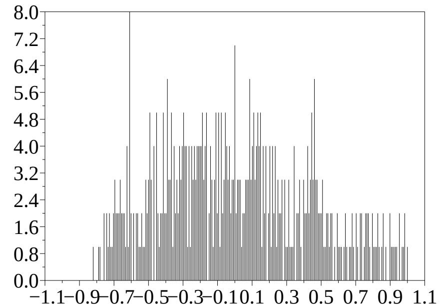




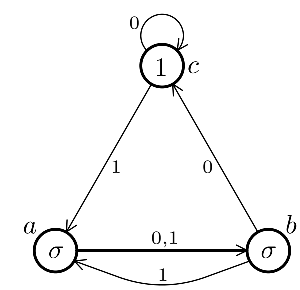
Automaton number 2371



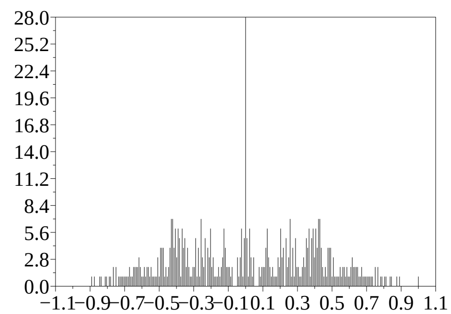
$a = \sigma(a, b)$ Group:
$b = \sigma(c, a)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $(b^{-1}c)^2$, $(a^{-1}b)^4$, $(b^{-1}c^{-1}ac)^2$, $(a^{-1}c)^4$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{25},2^{47},2^{90},2^{176}
Gr: 1,7,35,165,758,3460



Automaton number 2372

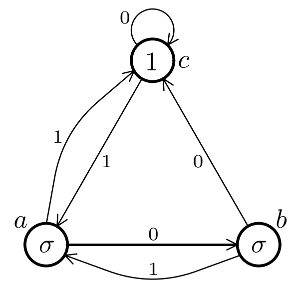


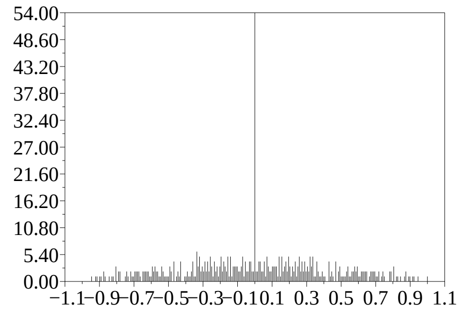
$a = \sigma(b, b)$ Group:
$b = \sigma(c, a)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $(a^{-1}b)^2$, $(b^{-1}c)^2$, $[c, ab^{-1}]$, $[cb^{-1}, a]$, $[c^{-1}, b^{-1}] \cdot [a^{-1}, b^{-1}]$, $[a, c^{-1}] \cdot [b, a^{-1}]$, $[b^{-1}, a^{-1}] \cdot [c^{-1}, a^{-1}]$
SF: 2^0,2^1,2^3,2^5,2^7,2^9,2^{11},2^{13},2^{15}
Gr: 1,7,33,127,433,1415



Automaton number 2375

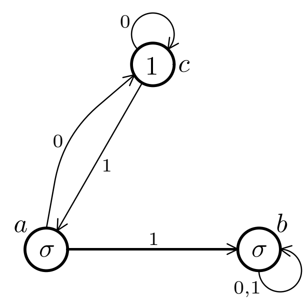
$a = \sigma(b, c)$ Group:
$b = \sigma(c, a)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $(b^{-1}c)^2$
SF: 2^0,2^1,2^3,2^5,2^9,2^{15},2^{26},2^{48},2^{92}
Gr: 1,7,35,165,769,3575

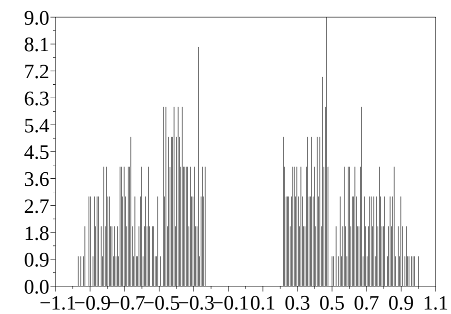




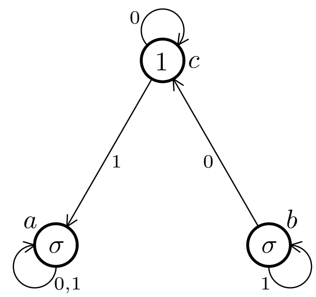
Automaton number 2391

$a = \sigma(c, b)$ Group:
$b = \sigma(b, b)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: b^2, $[a^2, b]$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,6,26,103,399,1538





Automaton number 2395

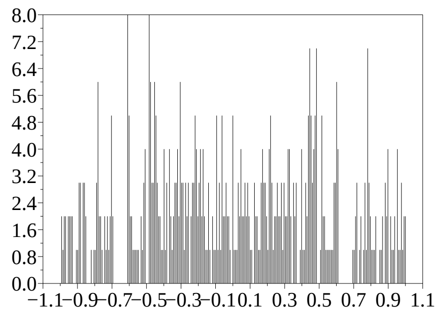


$a = \sigma(a, a)$	Group:
$b = \sigma(c, b)$	Contracting: *no*
$c = (c, a)$	Self-replicating: *yes*

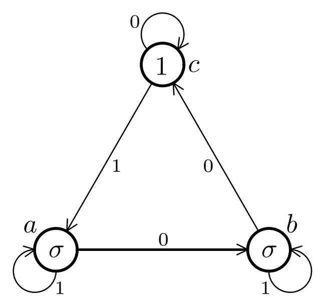
Rels: a^2, c^2, $(acb)^2$, $[b^2, cac]$

SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}

Gr: 1,5,17,50,140,377,995,2605



Automaton number 2396

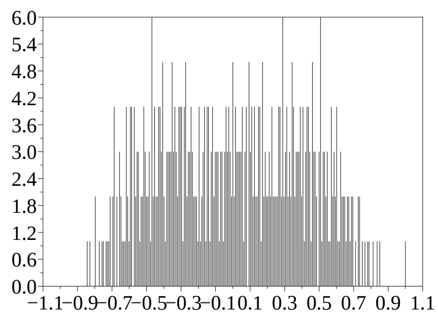


$a = \sigma(b, a)$	Group: *A. Boltenkov group*
$b = \sigma(c, b)$	Contracting: *no*
$c = (c, a)$	Self-replicating: *yes*

Rels: $acb^{-1}ca^{-2}cb^{-1}cac^{-1}bc^{-2}bc^{-1}$,
$acb^{-1}ca^{-2}cb^{-1}a^2c^{-1}b^{-1}a^2c^{-1}bc^{-1}a^{-1}bca^{-2}bc^{-1}$,
$acb^{-1}a^2c^{-1}b^{-1}a^{-1}cb^{-1}cbca^{-2}bc^{-2}bc^{-1}$,
$bcb^{-1}ca^{-1}b^{-1}cb^{-1}a^2c^{-1}ac^{-1}ba^{-2}bc^{-1}$

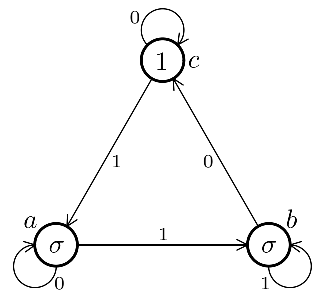
SF: 2^0,2^1,2^3,2^6,2^{12},2^{24},2^{46},2^{90},2^{176}

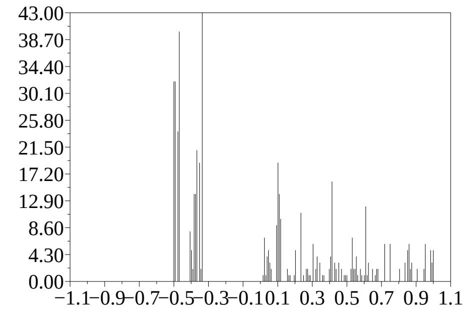
Gr: 1,7,37,187,937,4687



Automaton number 2398

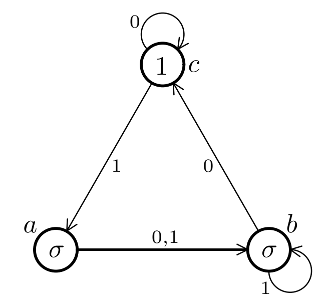
$a = \sigma(a, b)$ Group: *F.Dahmani Group*
$b = \sigma(c, b)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: cba, $b^{-1}a^{-1}b^2a^{-1}b^{-1}a^2$
SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}
Gr: 1,7,31,127,483,1823

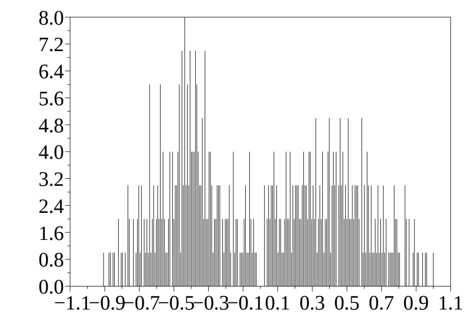




Automaton number 2399

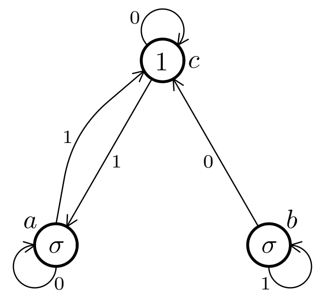
$a = \sigma(b, b)$ Group:
$b = \sigma(c, b)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $[b^{-1}a, ba^{-1}]$
SF: 2^0,2^1,2^3,2^7,2^{13},2^{24},2^{46},2^{89},2^{175}
Gr: 1,7,37,187,929,4599

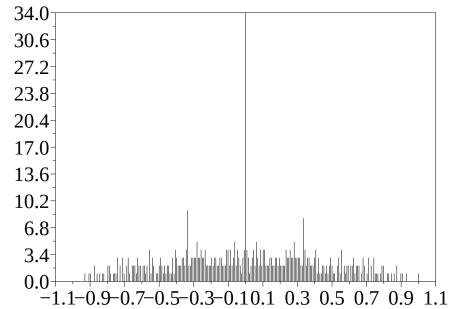




Automaton number 2401

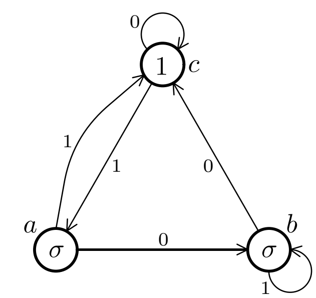
$a = \sigma(a, c)$ Group:
$b = \sigma(c, b)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*
Rels: $(a^{-1}c)^2$, $[a, c]^2$, $(c^{-2}ba)^2$
SF: 2^0,2^1,2^3,2^5,2^9,2^{15},2^{26},2^{48},2^{92}
Gr: 1,7,35,165,757,3447

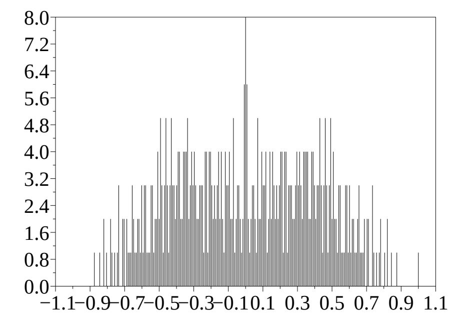




Automaton number 2402

$a = \sigma(b, c)$ Group:
$b = \sigma(c, b)$ Contracting: *n/a*
$c = (c, a)$ Self-replicating: *yes*
Rels: $ac^2b^{-1}a^{-2}c^2b^{-1}abc^{-2}bc^{-2}$,
$ac^2b^{-1}a^{-2}cb^{-2}c^{-1}a^4bc^{-2}a^{-3}cb^2c^{-1}$,
$acb^{-2}c^{-1}ac^2b^{-1}a^{-2}cb^2c^{-1}bc^{-2}$,
$acb^{-2}c^{-1}acb^{-2}c^{-1}acb^2c^{-1}a^{-3}cb^2c^{-1}$
SF: 2^0,2^1,2^3,2^5,2^7,2^{10},2^{15},2^{25},2^{41}
Gr: 1,7,37,187,937,4687





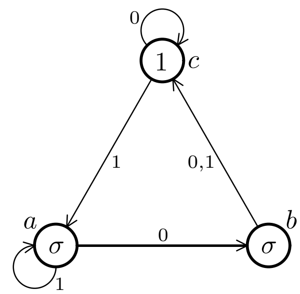
Automaton number 2423

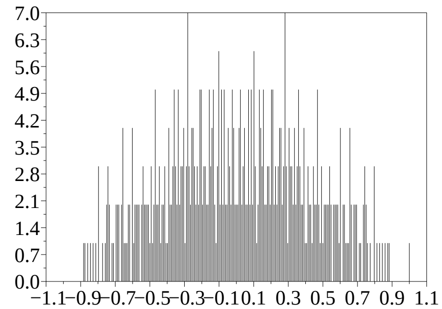
$a = \sigma(b, a)$ Group:
$b = \sigma(c, c)$ Contracting: *no*
$c = (c, a)$ Self-replicating: *yes*

Rels: $ac^{-1}bca^{-2}c^{-1}bcac^{-1}b^{-2}c$,
$ac^{-1}bca^{-1}c^{-1}bac^{-1}b^{-1}a^{2}c^{-1}b^{-1}ca^{-1}b\cdot$
$ca^{-1}b^{-1}ca^{-1}$,
$bc^{-1}bca^{-1}b^{-1}ac^{-1}bac^{-1}ac^{-1}b^{-1}c^{2}a^{-1}\cdot$
$b^{-1}ca^{-1}$,
$bac^{-1}bac^{-1}b^{-2}c^{-1}bca^{-1}b^{2}ca^{-1}\cdot$
$b^{-1}ca^{-1}b^{-1}ac^{-1}b^{-1}c$,
$bac^{-1}bac^{-1}b^{-2}ac^{-1}bac^{-1}bca^{-1}\cdot$
$b^{-1}ca^{-1}ca^{-1}b^{-1}ca^{-1}$

SF: 2^{0},2^{1},2^{3},2^{5},2^{8},2^{14},2^{25},2^{47},2^{90}

Gr: 1,7,37,187,937,4687





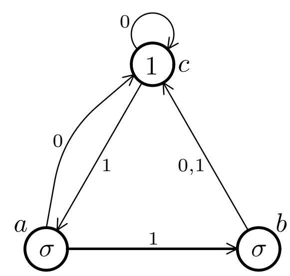
Automaton number 2427

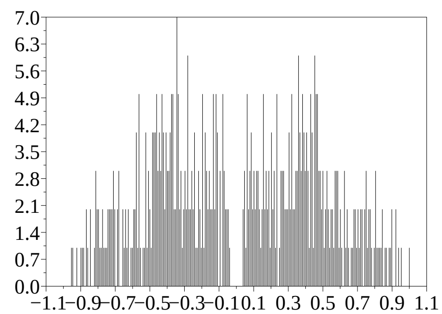
$a = \sigma(c, b)$ Group:
$b = \sigma(c, c)$ Contracting: *n/a*
$c = (c, a)$ Self-replicating: *yes*

Rels: $[b^{-1}a, ba^{-1}]$, $a^{-1}c^{2}a^{-1}b^{-1}a^{2}c^{-2}b$

SF: 2^{0},2^{1},2^{3},2^{7},2^{13},2^{24},2^{46},2^{89},2^{175}

Gr: 1,7,37,187,929,4583





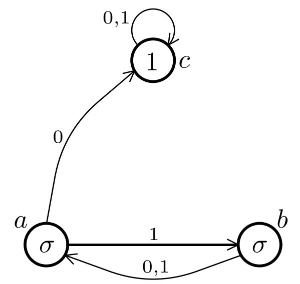
Automaton number 2841

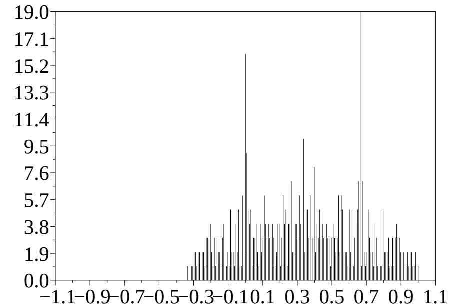
$a = \sigma(c, b)$ Group:
$b = \sigma(a, a)$ Contracting: *no*
$c = (c, c)$ Self-replicating: *yes*

Rels: c, $a^{-1}b^{-1}a^{-2}ba^{-1}b^{-1}aba^2b^{-1}ab$,
$a^{-1}b^{-1}a^{-2}b^{-1}a^{-1}babab^{-2}abab$,
$a^{-1}ba^{-1}b^{-2}a^{-1}ba^{-1}bab^{-1}a^2b^{-1}ab$

SF: 2^0,2^1,2^3,2^5,2^8,2^{13},2^{23},2^{42},2^{79}

Gr: 1,5,17,53,161,485,
1457,4359,12991





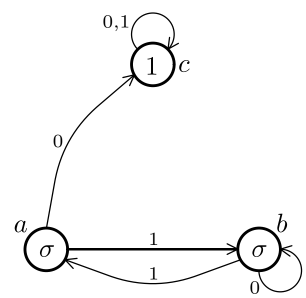
Automaton number 2850

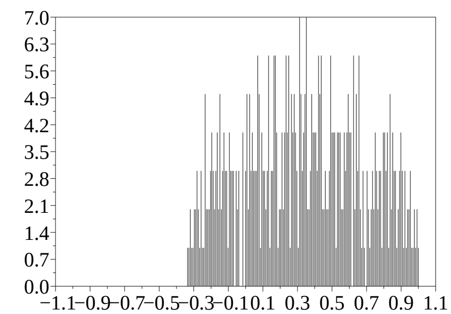
$a = \sigma(c, b)$ Group:
$b = \sigma(b, a)$ Contracting: *no*
$c = (c, c)$ Self-replicating: *yes*

Rels: c, $a^{-4}bab^{-1}a^2b^{-1}ab$

SF: 2^0,2^1,2^3,2^6,2^{12},2^{23},2^{45},2^{88},2^{174}

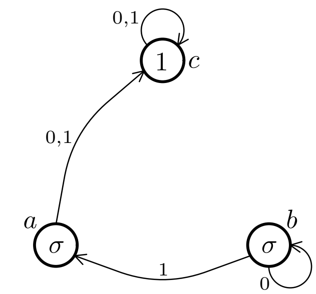
Gr: 1,5,17,53,161,485,1445



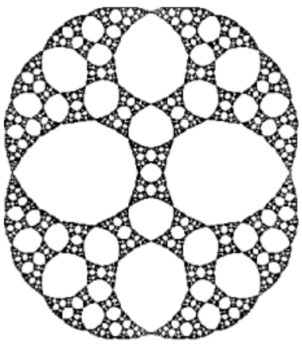


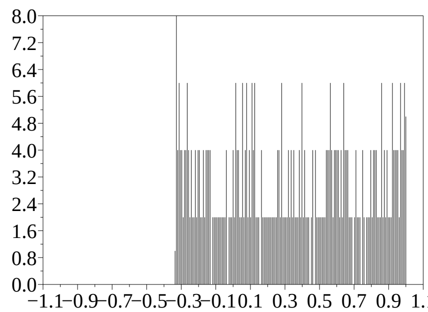
Automaton number 2853

$a = \sigma(c, c)$ Group: $IMG\left(\left(\frac{z-1}{z+1}\right)^2\right)$
$b = \sigma(b, a)$ Contracting: *yes*
$c = (c, c)$ Self-replicating: *yes*



Rels: c, a^2, $ab^{-1}ab^{-2}ab^{-1}abab^2ab$
SF: 2^0,2^1,2^2,2^3,2^5,2^8,2^{14},2^{25},2^{47}
Gr: 1,4,10,22,46,94,190,375,731,
1422,2752,5246,9908
Limit space:





9 Proofs

This section contains proofs of many of the claims contained in the tables in Section 7 and Section 8 and some additional information.

We sometimes encounter one of the following four binary tree automorphisms

$$a = \sigma(1, a), \qquad b = \sigma(b, 1), \qquad c = \sigma(c^{-1}, 1), \qquad d = \sigma(1, d^{-1}).$$

The first one is the binary adding machine, the second is its inverse, and all are conjugate to the adding machine and therefore act level transitively on the binary tree and have infinite order.

We freely use the known classification of groups generated by 2-state automata over a 2-letter alphabet.

Theorem 7 ([GNS00]). *Up to isomorphism, there are six $(2,2)$-automaton groups: the trivial group, the cyclic group of order 2 (we denote it by C_2), Klein group $C_2 \times C_2$ of order 4, the infinite cyclic group $\mathbb{Z}$, the infinite dihedral group D_∞ and the Lamplighter group $\mathbb{Z} \wr C_2$.*

In particular the sixteen 2-state automata for which both states are inactive generate the trivial group, and the sixteen 2-state automata in which both states are active generate C_2 (since both states in that case describe the mirror automorphism $\mu = \sigma(\mu, \mu)$ of order 2.

The automata given by either of the wreath recursions

$$\begin{aligned} a &= \sigma(a, a), \qquad b = (a, a), \\ a &= \sigma(b, b), \qquad b = (a, a), \end{aligned}$$

generate the Klein group $C_2 \times C_2$.

The automata given by the wreath recursions

$$\begin{aligned} a &= \sigma(a, a), \qquad b = (a, b), \\ a &= \sigma(a, a), \qquad b = (b, a), \\ a &= \sigma(b, b), \qquad b = (a, b), \\ a &= \sigma(b, b), \qquad b = (b, a), \end{aligned}$$

generate the infinite dihedral group D_∞.

The automata given by the wreath recursions

$$\begin{aligned} a &= \sigma(a, a), \qquad b = (b, b), \\ a &= \sigma(b, b), \qquad b = (b, b), \end{aligned}$$

generate the cyclic group C_2.

The automata given by the wreath recursions

$$\begin{aligned} a &= \sigma(a, b), \qquad b = (a, a), \\ a &= \sigma(b, a), \qquad b = (a, a), \\ a &= \sigma(a, b), \qquad b = (b, b), \\ a &= \sigma(b, a), \qquad b = (b, a), \end{aligned}$$

generate the infinite cyclic group $\mathbb{Z}$. Moreover, in the first two cases we have $b = a^{-2}$, in the fourth case $b = 1$ and a is the adding machine, and in the third case $b = 1$ and a is the inverse of the adding machine.

The automata given by the wreath recursions

$$\begin{aligned} a &= \sigma(a, b), & b &= (a, b), \\ a &= \sigma(a, b), & b &= (b, a), \\ a &= \sigma(b, a), & b &= (a, b), \\ a &= \sigma(b, a), & b &= (b, a), \end{aligned}$$

generate the Lamplighter group $\mathbb{Z} \wr C_2 = \mathbb{Z} \ltimes (\oplus_{\mathbb{Z}} C_2)$.

The results on the next few pages concern the existence of elements of infinite order and the level transitivity of the action. They are used in some of the proofs that follow.

Lemma 1 ([BGK+a])**.** *Let G be a group generated by an automaton $\mathcal{A}$ over a 2-letter alphabet. Assume that the set of states S of $\mathcal{A}$ splits into two nonempty parts P and Q such that*

(i) *one of the parts consists of the active states (those with nontrivial vertex permutation) and the other consists of the inactive states;*

(ii) *for each state from P, both arrows go to states in the same part (either both to P or both to Q);*

(iii) *for each state from Q, one arrow goes to a state in P and the other to a state in Q.*

Then any element of the group that can be written as a product of odd number of active generators or their inverses and odd number of inactive generators and their inverses, in any order, has infinite order. In particular, the group G is not a torsion group.

Proof. Denote by D the set of elements in G that can be represented as a product of odd number of active generators or their inverses and odd number of inactive generators and their inverses, in any order.

We note that if $g \in D$ then both sections of g^2 are in D. Indeed, for such an element, $g = \sigma(g_0, g_1)$ and $g^2 = (g_1 g_0, g_0 g_1)$. Both sections of g^2 are products (in some order) of the first level sections of the generators (and/or their inverses) used to express g as an element in D. By assumption, among these generators, there are odd number of active and odd number of inactive ones. The generators from P, by condition (ii), produce even number of active and even number of inactive sections on level 1, while the generators from Q, by condition (iii), produce odd number of active sections and odd number of inactive sections. Thus both sections of g are in D.

By way of contradiction, assume that h is an element of D of finite order 2^n, for some $n \geq 0$. If $n > 0$ the sections of h^2 are elements in D of order 2^{n-1}. Thus, continuing in this fashion, we reach an element in D that is trivial. This is contradiction since all elements in D act nontrivially on level 1. $\square$

There is a simple criterion that determines whether a given element of a self-similar group generated by a finite automaton over the 2-letter alphabet $X = \{0, 1\}$ acts level transitively on the tree. The criterion is based on the image of the given element in the abelianization of $\mathrm{Aut}(X^*)$, which is isomorphic to the infinite Cartesian product $\prod_{i=0}^{\infty} C_2$. The canonical isomorphism sends $g \in G$ to $(a_i \mod 2)_{i=0}^{\infty}$, where a_i is the number of active sections of g at level i. We also make use of the ring structure on $\prod_{i=0}^{\infty} C_2$ obtained by identifying $(b_i)_{i=0}^{\infty}$ with $\sum_{i=0}^{\infty} b_i t^i$ in the ring of formal power series $C_2[[t]]$. It is known that a binary tree automorphism g acts level transitively on X^* if and only if $\bar{g} = (1, 1, 1, \ldots)$, where $\bar{g}$ be the image of g in the abelianization $\prod_{i=0}^{\infty} C_2$ of $\mathrm{Aut}(X^*)$.

Lemma 2 (Element transitivity, [BGK$^+$a])**.** *Let G be a group generated by an automaton $\mathcal{A}$ over a 2-letter alphabet. There exists an algorithm that decides if g acts level transitively on X^*.*

Proof. Let $g = \sigma^i(g_0, g_1)$, where $i \in \{0, 1\}$. Then

$$\overline{g} = i + t \cdot (\overline{g_0} + \overline{g_1}).$$

Similar equations hold for all sections of g. Since G is generated by a finite automaton, g has only finitely many different sections, say k. Therefore we obtain a linear system of k equations over the k variables $\{g_v, v \in X^*\}$. The solution of this system expresses $\bar{g}$ as a rational function $P(t)/Q(t)$, where P an Q are polynomials of degree not higher than k. The element g acts level transitively if and only if $\bar{g} = \frac{1}{1-t}$. □

We often need to show that a given group of tree automorphisms is level transitive. Here is a very convenient necessary and sufficient condition for this in the case of a binary tree.

Lemma 3 (Group transitivity, [BGK$^+$a])**.** *A self-similar group of binary tree automorphisms is level transitive if and only if it is infinite.*

Proof. Let G be a self-similar group acting on a binary tree.

If G acts level transitively then G must be infinite (since the size of the levels is not bounded).

Assume now that the group G is infinite.

We first prove that all level stabilizers $\mathrm{Stab}_G(n)$ are different. Note that, since all level stabilizers have finite index in G and G is infinite, all level stabilizers are infinite. In particular, each contains a nontrivial element.

Let $n > 0$ and $g \in \mathrm{Stab}_G(n-1)$ be an arbitrary nontrivial element. Let $v = x_1 \ldots x_k$ be a word of shortest length such that $g(v) \neq v$. Since $g \in \mathrm{Stab}_G(n-1)$, we must have $k \geq n$. The section $h = g_{x_1 x_2 \ldots x_{k-n}}$ is an element of G by the self-similarity of G. The minimality of the word v implies that $g \in \mathrm{Stab}_G(k-1)$, and therefore $h \in \mathrm{Stab}_G(n-1)$. On the other hand h acts nontrivially on $x_{k-n+1} \ldots x_k$ and we conclude that $h \in \mathrm{Stab}_G(n-1) \setminus \mathrm{Stab}_G(n)$. Thus all level stabilizers are different.

We now prove level transitivity by induction on the level.

The existence of elements in $\mathrm{Stab}_G(0) \setminus \mathrm{Stab}_G(1)$ shows that G acts transitively on level 1.

Assume that G acts transitively on level n. Select an arbitrary element $h \in \mathrm{Stab}_G(n) \setminus \mathrm{Stab}_G(n+1)$ and let $w =\in X^n$ be a word of length n such that $h(w1) = w0$.

Let u be an arbitrary word of length n and let x be a letter in $X = \{0, 1\}$. We will prove that ux is mapped to $w0$ by some element of G, proving the transitivity of the action at level $n+1$. By the inductive assumption there exists $f \in G$ such that $f(u) = w$. If $f(ux) = w0$ we are done. Otherwise, $hf(ux) = h(w1) = w0$ and we are done again. □

Consider the infinitely iterated permutational wreath product $\wr_{i\geq 1} C_d$, consisting of the automorphisms of the d-ary tree for which the activity at every vertex is a power of some fixed cycle of length d. The last proof works, mutatis mutandis, for the self-similar subgroups of $\wr_{i\geq 1} C_d$ and may be easily adapted in other situations.

The following lemma is used often when we want to prove that some automaton group is not free.

Lemma 4. *If a self-similar group contains two nontrivial elements of the form* $(1, u), (v, 1)$*, then the group is not free.*

Proof. Suppose $a = (1, u), b = (v, 1)$ are two nontrivial elements of a self-similar group G and G is free. Obviously $[a, b] = 1$, hence a and b are powers of some element $x \in G$: $a = x^m$, $b = x^n$. Then $a^n = b^m$, so $a^n = (1, u^n) = b^m = (v^m, 1)$. This implies that $u^n = v^m = 1$, which is a contradiction, since u and v are nontrivial elements of a free group. □

In most case when the corresponding group is finite we do not offer a full proof. In all such cases the proof can be easily done by direct calculations. As an example, a detailed proof is given in the case of the automaton [748].

We now proceeds to individual analysis of the properties of the automaton groups in our classification.

1. Trivial group.

730. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(a, a)$, $b = (a, a)$, $c = (a, a)$.

The claim follows from the relations $b = c$, $a^2 = b^2 = abab = 1$.

731 $\cong \mathbb{Z}$. Wreath recursion: $a = \sigma(b, a)$, $b = (a, a)$, $c = (a, a)$.

We have $c = b$ and $b = a^{-2}$. The states a and b form a 2-state automaton generating $\mathbb{Z}$ (see Theorem 7).

734 $\cong G_{730}$. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(b, b)$, $b = (a, a)$, $c = (a, a)$.

The claim follows from the relations $b = c$, $a^2 = b^2 = abab = 1$.

739 $\cong C_2 \ltimes (C_2 \wr \mathbb{Z})$. Wreath recursion: $a = \sigma(a, a)$, $b = (b, a)$, $c = (a, a)$.

All generators have order 2. The elements $u = acba = (1, ba)$ and $v = bc = (ba, 1)$ generate $\mathbb{Z}^2$. This is clear since $ba = \sigma(1, ba)$ is the adding machine and therefore has infinite order. Further, we have $ac = \sigma$ and $\langle u, v\rangle$ is normal in $H = \langle u, v, \sigma\rangle$, since $u^\sigma = v$ and $v^\sigma = u$. Thus $H \cong C_2 \ltimes (\mathbb{Z} \times \mathbb{Z}) = C_2 \wr \mathbb{Z}$.

We have $G_{739} = \langle H, a\rangle$ and H is normal in G_{739}, since it has index 2. Moreover, $u^a = v^{-1}$, $v^a = u^{-1}$ and $\sigma^a = \sigma$. Thus $G_{739} = C_2 \ltimes (C_2 \wr \mathbb{Z})$, where the action of C_2 on H is specified above.

740. Wreath recursion: $a = \sigma(b, a)$, $b = (b, a)$, $c = (a, a)$.

The states a, b form a 2-state automaton generating the Lamplighter group (see Theorem 7). Thus G_{740} has exponential growth and is neither torsion nor contracting.

Since $c = (a, a)$ we obtain that G_{740} can be embedded into the wreath product $C_2 \wr (\mathbb{Z} \wr \mathbb{C}_2)$. Thus G_{740} is solvable.

741. Wreath recursion: $a = \sigma(c, a)$, $b = (b, a)$, $c = (a, a)$.

The states a and c form a 2-state automaton generating the infinite cyclic group $\mathbb{Z}$ in which $c = a^{-2}$ (see Theorem 7).

Since $b = (b, a)$, we see that b has infinite order and that G_{741} is not contracting).

We have $c = a^-2$ and $b^{-1}a^{-3}b^{-1}ababa = 1$. Since a and b do not commute the group is not free.

743 $\cong G_{739} \cong C_2 \ltimes (C_2 \wr \mathbb{Z})$. Wreath recursion: $a = \sigma(b, b)$, $b = (b, a)$, $c = (a, a)$.

All generators have order 2. The elements $u = acba = (1, ba)$ and $v = bc = (ba, 1)$ generate $\mathbb{Z}^2$ because $ba = \sigma(ab, 1)$ is conjugate to the adding machine and has infinite order. Further, we have $babc = \sigma$ and $\langle u, v\rangle$ is normal in $H = \langle u, v, \sigma\rangle$ because $u^\sigma = v$ and $v^\sigma = u$. In other words, $H \cong C_2 \ltimes (\mathbb{Z} \times \mathbb{Z}) = C_2 \wr \mathbb{Z}$.

Furthermore, $G_{743} = \langle H, a\rangle$ and H is normal in G_{743} because $u^a = v^{-1}$, $v^a = u^{-1}$ and $\sigma^a = \sigma$. Thus $G_{743} = C_2 \ltimes (C_2 \wr \mathbb{Z})$, where the action of C_2 on H is specified above and coincides with the one in G_{739}. Therefore $G_{743} \cong G_{739}$.

744. Wreath recursion: $a = \sigma(c, b)$, $b = (b, a)$, $c = (a, a)$.

Since $(a^{-1}c)^2 = (c^{-1}ab^{-1}a, b^{-1}ac^{-1}a)$ and $c^{-1}ab^{-1}a = ((c^{-1}ab^{-1}a)^{-1}, a^{-1}c)$, the element $(a^{-1}c)^2$ fixes the vertex 01 and its section at this vertex is equal to $a^{-1}c$. Hence, $a^{-1}c$ has infinite order.

The element $c^{-1}ab^{-1}a$ also has infinite order, fixes the vertex 00 and its section at this vertex is equal to $c^{-1}ab^{-1}a$. Therefore G_{744} is not contracting.

We have $b^{-1}c^{-1}ba^{-1}ca = (1, a^{-1}c^{-1}ac)$, $ab^{-1}c^{-1}ba^{-1}c = (ca^{-1}c^{-1}a, 1)$, hence by Lemma 4 the group is not free.

747 $\cong G_{739} \cong C_2 \ltimes (C_2 \wr \mathbb{Z})$. Wreath recursion: $a = \sigma(c, c)$, $b = (b, a)$, $c = (a, a)$.

All generators have order 2 and a commutes with c. Conjugating this group by the automorphism $\gamma = (\gamma, c\gamma)$ yields an isomorphic group generated by automaton $a' = \sigma$, $b' = (b', a')$ and $c' = (a', a')$. On the other hand we obtain the same automaton after conjugating G_{739} by $\mu = (\mu, a\mu)$ (here a denotes the generator of G_{739}).

748 $\cong D_4 \times C_2$. Wreath recursion: $a = \sigma(a, a)$, $b = (c, a)$, $c = (a, a)$.

Since a is nontrivial and b and c have a as a section, none of the generators is trivial. All generators have order 2. Indeed, we have $a^2 = (a^2, a^2)$, $b^2 = (c^2, a^2)$, $c^2 = (a^2, a^2)$, showing that a^2, b^2 and c^2 generate a self-similar group in which no element is active. Therefore $a^2 = b^2 = c^2 = 1$. Since $ac = \sigma$ we have that $(ac)^2 = 1$. Therefore a and c commute. Since $(bc)^2 = ((ca)^2, 1) = 1$, we see that b and c also commute. Further, the relations $(ab)^2 = (ac, 1) = (\sigma, 1) \neq 1$ and $(ab)^4 = 1$ show that a and b generate the dihedral group D_4. It remains to be

shown that $c \notin \langle a, b\rangle$. Clearly c could only be equal to one of the four elements 1, b, aba, and $abab$ in D_4 that stabilize level 1. However, c is nontrivial, differs from b at 0 (the section $b|_0 = c$ is not active, while $c|_0 = a$ is active), differs from aba at 1 (the section $(aba)|_1 = aca$ is not active, while $c|_1 = a$ is active), and differs from $abab$ at 1 (the section of $abab$ at 1 is trivial). This completes the proof.

749. Wreath recursion: $a = \sigma(b, a)$, $b = (c, a)$, $c = (a, a)$.

The element $(a^{-1}c)^4$ stabilizes the vertex 000 and its section at this vertex is equal to $a^{-1}c$. Hence, $a^{-1}c$ has infinite order.

We have $ac^{-1} = \sigma(ba^{-1}, 1)$, $ba^{-1} = \sigma(1, cb^{-1})$, $cb^{-1} = (ac^{-1}, 1)$, Thus the subgroup generated by these elements is isomorphic to $IMG(1 - \frac{1}{z^2})$ (see [BN06]).

We have $c^{-1}b = (a^{-1}c, 1)$, $ac^{-1}ba^{-1} = (1, ca^{-1})$. Thus, by Lemma 4 the group is not free.

748 $\cong G_{848} \cong C_2 \wr \mathbb{Z}$. Wreath recursion:$a = \sigma(c, a)$, $b = (c, a)$, $c = (a, a)$.

It is proven below that $G_{848} \cong G_{2190}$ and for G_{2190} we have $a = \sigma(c, a)$, $b = \sigma(a, a)$, $c = (a, a)$. Therefore $G_{2190} = \langle a, b, c\rangle = \langle a, c, c^{-1}b = \sigma\rangle = \langle a = (c, a)\sigma, c = (a, a), a\sigma = (c, a)\rangle = G_{750}$.

752. Wreath recursion: $a = \sigma(b, b)$, $b = (c, a)$, $c = (a, a)$.

The group G_{752} is a contracting group with nucleus consisting of 41 elements. It is a virtually abelian group, containing $\mathbb{Z}^3$ as a subgroup of index 4.

All generators have order 2.

Let $x = ca$, $y = babc$, and $K = \langle x, y\rangle$. Since $xy = ((cbab)^{ca}, abcb) = ((y^{-1})^x, abcb)$ and $yx = (cbab, abcb) = (y^{-1}, abcb)$ the elements x and y commute. Conjugating by $\gamma = (\gamma, bc\gamma)$ yields the self-similar copy K' of K generated by $x' = \sigma((y')^{-1}, (x')^{-1})$ and $y' = \sigma((y')^{-1}x', 1)$, where $x' = x^\gamma$ and $y' = y^\gamma$. Since $(x')^2 = ((x')^{-1}(y')^{-1}, (y')^{-1}(x')^{-1})$ and $(y')^2 = ((y')^{-1}x', (y')^{-1}x')$, the virtual endomorphism of K' is given by

$$A = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}.$$

The eigenvalues $\lambda = -\frac{1}{2} \pm \frac{1}{2}i$ of this matrix are not algebraic integers, and therefore, by the results in [NS04], the group $K' \cong K$ is free abelian of rank 2.

Let $H = \langle ba, cb\rangle$. The index of $\mathrm{Stab}_H(1)$ in G is 4, since the index of $\mathrm{Stab}_H(1)$ in H is 2 and the index of H in G is 2 (the generators have order 2). We have $\mathrm{Stab}_H(1) = \langle cb, cb^{ba}, (ba)^2\rangle$. If we conjugate the generators of $\mathrm{Stab}_H)(1)$ by $g = (1, b)$, we obtain

$$\begin{aligned} g_1 &= (cb)^g &&= (x^{-1}, 1), \\ g_2 &= \left((cb)^{ba}\right)^g &&= (1, \quad x), \\ g_3 &= \left((ba)^2\right)^g &&= (y^{-1}, y). \end{aligned}$$

Therefore, g_1, g_2, and g_3 commute. If $g_1^{n_1}g_2^{n_2}g_3^{n_3} = 1$, then we must have $x^{-n_1}y^{-n_3} = x^{n_2}y^{n_3} = 1$. Since K is free abelian, this implies $n_1 = n_2 = n_3 = 0$. Thus, $\mathrm{Stab}_H(1)$ is a free abelian group of rank 3.

753. Wreath recursion: $a = \sigma(c, b)$, $b = (c, a)$, $c = (a, a)$.

Since $ab^{-1} = \sigma(1, ba^{-1})$, this element is conjugate to the adding machine.

For a word w in $w \in \{a^{\pm 1}, b^{\pm 1}, c^{\pm 1}\}^*$, let $|w|_a$, $|w|_b$ and $|w|_c$ denote the sum of the exponents of a, b and c in w. Let w represents the element $g \in G$. If $|w|_a$ and $|w|_b$ are odd, then g acts transitively on the first level, and $g^2|_0$ is represented by a word w_0, which is the product (in some order) of all first level sections of all generators appearing in w. Hence, $|w_0|_a = |w|_b + 2|w|_c$ and $|w_0|_b = |w|_a$ are odd again. Therefore, similarly to Lemma 1, any such element has infinite order.

In particular c^2ba has infinite order. Since $a^4 = (caca, a^4, acac, a^4)$ and $caca = (baca, c^2ba, bac^2, caba)$, the element a^4 has infinite order (and so does a). Since a^4 fixes the vertex 01 and its section at that vertex is equal to a^4, the group G_{753} is not contracting.

We have $cb^{-1} = (ac^{-1}, 1), acb^{-1}a^{-1} = (1, bac^{-1}b^{-1})$, hence by Lemma 4 the group is not free.

756 $\cong G_{748} \cong D_4 \times C_2$. Wreath recursion: $a = \sigma(c, c)$, $b = (c, a)$, $c = (a, a)$.

All generators have order 2. The generator c commutes with both a and b. Since $(ab)^2 = (ca, ca)$ the order of ca is 4 and the group is isomorphic to $D_4 \times C_2$

766 $\cong G_{730}$. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(a, a)$, $b = (b, b)$, $c = (a, a)$.

The state b is trivial. The states a and c form a 2-state automaton generating $C_2 \times C_2$ (see Theorem 7).

767 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(1, a)$, $b = (b, b)$, $c = (a, a) = a^2$.

The state b is trivial. The automorphism a is the binary adding machine.

768 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(c, a)$, $b = (b, b)$, $c = (a, a)$.

The states a and c form a 2-state automaton generating $\mathbb{Z}$ (see Theorem 7) in which $c = a^{-2}$.

770 $\cong G_{730}$. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(b, b)$, $b = (b, b)$, $c = (a, a)$.

The state b is trivial. The states a and c form a 2-state automaton generating $C_2 \times C_2$ (see Theorem 7).

771 $\cong \mathbb{Z}^2$. Wreath recursion: $a = \sigma(c, b)$, $b = (b, b)$, $c = (a, a)$.

The group G_{771} is finitely generated, abelian, and self-replicating. Therefore, it is free [NS04]. Since $b = 1$ the rank is 1 or 2. We prove that the rank is 2, by showing that $c^n \neq a^m$, unless $n = m = 0$. By way of contradiction, let $c^n = a^m$ for some integer n and m and choose such integers with minimal $|n| + |m|$. Since c^n stabilizes level 1, m must be even and we have $(a^n, a^n) = c^n = a^m = (c^{m/2}, c^{m/2})$, implying $a^n = c^{m/2}$. By the minimality assumption, m must be 0, which then implies that n must be 0 as well.

774 $\cong G_{730}$. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(c, c)$, $b = (b, b)$, $c = (a, a)$.

The state b is trivial. The states a and c form a 2-state automaton generating $C_2 \times C_2$ (see Theorem 7).

775 $\cong C_2 \ltimes IMG\left(\left(\frac{z-1}{z+1}\right)^2\right)$. Wreath recursion: $a = \sigma(a, a)$, $b = (c, b)$, $c = (a, a)$.

All generators have order 2. Further, $ac = ca = \sigma(1,1)$ and $ba = \sigma(ba, ca)$. Hence, for the subgroup $H = \langle ba, ca\rangle \cong G_{2853} \cong IMG\left(\left(\frac{z-1}{z+1}\right)^2\right)$.

Since the generators have order 2, H is normal subgroup of index 2 in G_{775}. Moreover $(ba)^a = (ba)^{-1}$ and $(ca)^a = ca$. Therefore $G \cong C_2 \ltimes H$, where C_2 is generated by a and the action of a on H is given above.

Conjugating the generators by $g = \sigma(g, g)$ we obtain the wreath recursion

$$a' = \sigma(a', a'), \qquad b' = (b', c'), \qquad c' = (a', a'),$$

where $a' = a^g$, $b' = b^g$ and $c' = c^g$. This is the wreath recursion defining G_{793}. Denote G_{793} by G and its generators by a, b, and c (we continue working only with G_{793}). Thus

$$a = \sigma(a, a), \qquad b = (b, c), \qquad c = (a, a).$$

The generators have order 2. Moreover $ac = ca$ and $\langle a, c\rangle = C_2 \times C_2$ is the Klein group. Denote $A = \langle a, c\rangle$.

The element $x = ba$ has infinite order, since x^2 fixes 00, and has itself as a section at 00. Note that

$$x = ba = (b, c)\sigma(a, a) = \sigma(ca, ba) = \sigma(\sigma, x).$$

and, therefore, $x^2 = (x\sigma, \sigma x) = (x, \sigma, \sigma, x)$.

Proposition 1. *The subgroup $H = \langle x, y\rangle$ of G, where $x = ba$ and $y = cabc$ is torsion free.*

Proof. The first level decompositions of $x^{\pm 1}$ and $y^{\pm 1}$ and the second level decompositions of x and y are given by

$$\begin{aligned}
&x = \sigma(\sigma, x)\\
&y = cabc = \sigma aaba\sigma = \sigma ba\sigma = x^\sigma = \sigma(x, \sigma)\\
&x^{-1} = \sigma(x^{-1}, \sigma)\\
&y^{-1} = \sigma(\sigma, x^{-1})\\
&x = \sigma(\sigma(1,1), \sigma(\sigma, x)) = \mu(1, 1, \sigma, x)\\
&y = x^\sigma = \mu(\sigma, x, 1, 1),
\end{aligned}$$

where $\mu = \sigma(\sigma, \sigma)$ permutes the first two levels of the tree as $00 \leftrightarrow 11,\ 10 \leftrightarrow 01$. We encode this as the permutation $\mu = (03)(12)$.

For a word w over $\{x^{\pm 1}, \sigma\}$, denote by $\#_x(w)$ and $\#_\sigma(w)$ the total number of appearances of x and x^{-1} and the number of appearances of σ in w, respectively.

Note that x and x^{-1} act as the permutation $(03)(12)$ on the second level, and σ acts as the permutation $(02)(13)$. These permutations have order 2, commute, and their product is $(01)(23)$, which is not trivial. Thus, a tree automorphisms represented by a word w over $\{x^{\pm 1}, \sigma\}$ cannot be trivial unless both $\#_x(w)$ and $\#_\sigma(w)$ are even.

Let g be an element of H that can be written as $g = z_1 z_2 \dots z_n$, for some $z_i \in \{x^{\pm 1}, y^{\pm 1}\}$, $i = 1, \dots, n$.

If n is odd, the element g cannot have order 2. By way of contradiction assume otherwise. For z in $\{x^{\pm 1}, y^{\pm 1}\}$ denote $z' = \sigma z$. Thus, for instance $x' = (\sigma, x)$ and $y' = (x, \sigma)$. Note that

$$g^2 = (z_1 z_2 \dots z_n)^2 = (z_1')^\sigma z_2' (z_3')^\sigma z_4' \dots (z_n')^\sigma z_1' (z_2')^\sigma \dots z_n' = (w_0, w_1),$$

where the words w_i over $\{x^{\pm 1}, \sigma\}$ are such that

$$\#_x(w_i) = \#_\sigma(w_i) = n, \tag{8}$$

for $i = 1, 2$. The last claim holds because exactly one of z_i' and $(z_i')^\sigma$ contributes $x^{\pm 1}$ to w_0 and σ to w_1, respectively, while the other contributes the same letters to w_1 and w_0, respectively. Since n is odd, (8) shows that neither w_0 nor w_1 can be 1 and therefore g^2 cannot be 1.

Assume that H contains an element of finite order. In particular, this implies that H must contain an element of order 2. Let $g = z_1 z_2 \dots z_n$ be such an element of the shortest possible length, where $z_i \in \{x^{\pm 1}, y^{\pm 1}\}$, $i = 1, \dots, n$.

Note that n must be even. Therefore,

$$g = z_1 z_2 \dots z_n = (z_1')^\sigma z_2' \dots (z_{n-1}')^\sigma z_n' = (w_0, w_1),$$

where w_0 and w_1 are words over $\{x^{\pm 1}, \sigma\}$. Moreover, as elements in H, the orders of w_0 and w_1 divide 2 and the order of at least one of them is 2. We claim that

$$\#_x(w_0) \equiv \#_\sigma(w_0) \equiv \#_x(w_1) \equiv \#_\sigma(w_0) \mod 2. \tag{9}$$

The congruence $\#_x(w_i) \equiv \#_\sigma(w_i) \mod 2$ holds because $\#_x(w_i) + \#_\sigma(w_i) = n$ is even. For the other congruences, observe that whenever z_i' or $(z_i')^\sigma$ contributes $x^{\pm 1}$ or σ to w_0, respectively, it contributes σ or $x^{\pm 1}$ to w_1, respectively. Therefore $\#_x(w_0) = \#_\sigma(w_1)$ and $\#_\sigma(w_0) = \#_x(w_1)$.

If the numbers in (9) are even, then w_0 and w_1 represent elements in H and can be rewritten as words over $\{x^{\pm 1}, y^{\pm 1}\}$ of lengths at most $\#_x(w_0) = n - \#_\sigma(w_0)$ and $\#_x(w_1) = n - \#_\sigma(w_1)$, respectively. If both of these lengths are shorter than n then none of them can represent an element of order 2 in H. Otherwise, one of the words w_i is a power of x and the other is trivial. Sice x has infinite order this shows that g cannot have order 2.

If the numbers in (9) are odd, then, for $i = 1, 2$, w_i can be rewritten as σu_i, where u_i are words of odd length over $\{x^{\pm 1}, y^{\pm 1}\}$. Let $w_0 = \sigma t_1 \dots t_m$, where m is odd, and t_j are letters in $\{x^{\pm 1}, y^{\pm 1}\}$, $j = 1, \dots, m$. We have

$$w_0 = t_1' (t_2')^\sigma \dots (t_{m-1}')^\sigma t_m' = (w_{00}, w_{01}),$$

where w_{00} and w_{01} are words of odd length m over $\{x^{\pm 1}, \sigma\}$. Moreover, exactly one of the words w_{00} and w_{01} has even number of σ's and this word can be rewritten as a word over $\{x^{\pm 1}, y^{\pm 1}\}$ of odd length. However, an element in H represented by such a word cannot have order dividing 2. This completes the proof. $\square$

Since

$$\begin{aligned} x^a &= abaa = ab = x^{-1}, & y^a &= acabca = cbac = y^{-1}, \\ x^b &= bbab = ab = x^{-1}, & y^b &= bcabcb = bacbacab = xy^{-1}x^{-1}, \\ x^c &= cbac = y^{-1}, & y^c &= ccabcc = ab = x^{-1}, \end{aligned}$$

we see that H is the normal closure of x in G. Further, $G = \{x, y, a, c\}$ and $G = AH$. It follows from Proposition 1 that $A \cap H = 1$ (since A is finite) and therefore $G = A \ltimes H$.

Proposition 2. *The group G is a regular, weakly branch group, branching over H''.*

Proof. The group G is infinite self-similar group acting on a binary three. Therefore it is level transitive by Lemma 3.

Since

$$x^2 = (x, \sigma, \sigma, x)$$
$$y^{-1}x^2y = (y, x^{-1}\sigma x, \sigma, x)$$

we have that

$$H'' \times \langle \sigma, x^{-1}\sigma x\rangle'' \times \langle \sigma\rangle'' \times \langle x\rangle'' \preceq H''.$$

On the other hand, $\langle \sigma, x^{-1}\sigma x\rangle$ is metabelian (in fact dihedral, since the generators have order 2) and $\langle \sigma\rangle$ and $\langle x\rangle$ are abelian (cyclic). Therefore

$$H'' \times 1 \times 1 \times 1 \preceq H''.$$

The group H'' is normal in G, since it is characteristic in the normal subgroup H. Finally, H'' is not trivial. For instance it is easy to show that $[[x, y], [x, y^{-1}]] \neq 1$ (see [BGK$^+$b]). $\square$

776. Wreath recursion: $a = \sigma(b, a)$, $b = (c, b)$, $c = (a, a)$.

The element $(b^{-1}a)^4$ stabilizes the vertex 00 and its section at this vertex is equal to $(b^{-1}a)^{-1}$. Hence, $b^{-1}a$ has infinite order. Furthermore, by Lemma 1 ab has infinite order, which yields that a,c and b also have infinite order, because $a^2 = (ab, ba)$. Since $b^n = (c^n, b^n)$ we obtain that b^n belong to the nucleus for all $n \geq 1$. Thus G_{776} is not contracting.

We have $a^{-1}ba^{-1}c = (1, b^{-1}c), ba^{-1}ca^{-1} = (cb^{-1}, 1)$, hence by Lemma 4 the group is not free.

777. Wreath recursion: $a = \sigma(c, a)$, $b = (c, b)$, $c = (a, a)$.

The states a, c form the 2-state automaton generating $\mathbb{Z}$ (see Theorem 7). So the group is not torsion and $G_{777} = \langle a, b\rangle$. Since c has infinite order, so has b. Therefore the relation $b^n = (c^n, b^n)$ implies that b^n belong to the nucleus for all $n \geq 1$. Thus G_{777} is not contracting.

Also we have $ab^{-1} = \sigma(1, ab^{-1})$ is the adding machine. Since $a^{-3} = \sigma(1, a^3)$ elements ab^{-1} and a^{-3} generate the Brunner-Sidki-Vierra group (see [BSV99]).

779. Wreath recursion: $a = \sigma(b, b)$, $b = (c, b)$, $c = (a, a)$.

The element $(ab^{-1})^2$ stabilizes the vertex 01 and its section at this vertex is equal to $(ab^{-1})^{-1}$. Hence, ab^{-1} has infinite order.

780. Wreath recursion: $a = \sigma(c, b)$, $b = (c, b)$, $c = (a, a)$.

The element $(c^{-1}a)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $c^{-1}a$. Hence, $c^{-1}a$ has infinite order. Since $[c, a]\big|_{100} = (c^{-1}a)^a$ and 100 is fixed under the action of $[c, a]$ we obtain that $[c, a]$ also has infinite order. Finally, $[c, a]$ stabilizes the vertex 00 and its section at this vertex is $[c, a]$. Therefore G_{780} is not contracting.

783 $\cong G_{775} \cong C_2 \ltimes IMG\left(\left(\frac{z-1}{z+1}\right)^2\right)$. Wreath recursion: $a = \sigma(c, c)$, $b = (c, b)$, $c = (a, a)$.

All generators have order 2 and $ac = ca$. If we conjugate the generators of this group by the automorphism $\gamma = (c\gamma, \gamma)$ we obtain the wreath recursion

$$a' = \sigma(1, 1), \qquad b' = (c', b'), \qquad c' = (a', a'),$$

where $a' = a^\gamma$, $b' = b^\gamma$, and $c' = c^\gamma$. The same wreath recursion is obtained after conjugating G_{775} by $\mu = (a\mu, \mu)$ (where a denotes the generator of G_{775}).

Since $bca = \sigma(bca, a)$, $G_{783} = \langle acb, a, c\rangle \cong G_{2205}$.

802 $\cong C_2 \times C_2 \times C_2$. Wreath recursion: $a = \sigma(a, a)$, $b = (c, c)$, $c = (a, a)$.

Direct calculation.

803 $\cong G_{771} \cong \mathbb{Z}^2$. Wreath recursion: $a = \sigma(b, a)$, $b = (c, c)$, $c = (a, a)$.

The group G_{771} is finitely generated, abelian, and self-replicating. Therefore, it is free abelian [NS04]. Let $\phi : \mathrm{Stab}_{G_{803}}(1) \to G_{803}$ be the $\frac{1}{2}$-endomorphism associated to the vertex 0, given by $\phi(g) = h$, provided $g = (h, *)$. The matrix of the linear map $\mathbb{C}^3 \to \mathbb{C}^3$ induced by ϕ with to the basis corresponding to the triple $\{a, b, c\}$ is given by

$$A = \begin{pmatrix} \frac{1}{2} & 0 & 1 \\ \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

The eigenvalues are $\lambda_1 = 1$, $\lambda_2 = -\frac{1}{4} - \frac{1}{4}i\sqrt{7}$ and $\lambda_3 = -\frac{1}{4} + \frac{1}{4}i\sqrt{7}$. Let v_i, $i = 1, 2, 3$, be eigenvectors corresponding to the eigenvalues λ_i, $i = 1, 2, 3$. Note that v_1 may be selected to be equal to $v_1 = (2, 1, 1)$. This shows that $a^2bc = 1$ in G_{803} and the rank of $G_{803} = \langle a, c\rangle$ is at most 2. We will prove that $a^{2m}c^n \neq 1$ (except when $m = n = 0$) by proving that iterations of the action of A eventually push the vector $v = (2m, 0, n)$ out of the set $D = \{(2i, j, k), i, j, k \in \mathbb{Z}\}$ corresponding to the first level stabilizer.

Let $v = a_1v_1 + a_2v_2 + a_3v_3$. The vector v is not a scalar multiple of v_1. Therefore either $a_2 \neq 0$ or $a_3 \neq 0$. Since $|\lambda_2| = |\lambda_3| < 1$, we have $A^t(v) = a_1v_1 + \lambda_2^t a_2v_2 + \lambda_3^t a_3v_3 \to a_1v_1$, as $t \to \infty$. Note that, since $a_2 \neq 0$ or $a_3 \neq 0$, $A^t(v)$is never equal to a_1v_1. Choose a neighborhood U of a_1v_1 that does not contain vectors from D, except possibly the vector a_1v_1. For t large enough t, the vector $A^t(v)$ is in U and is therefore outside of D.

Thus the rank of G_{803} is 2.

804 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(c, a)$, $b = (c, c)$, $c = (a, a)$.

Indeed, the states a and c form a 2-state automaton generating the cyclic group $\mathbb{Z}$ (see Theorem 7). Since $b = a^4$ we are done.

806 $\cong G_{802} \cong C_2 \times C_2 \times C_2$. Wreath recursion: $a = \sigma(b, b)$, $b = (c, c)$, $c = (a, a)$.

Direct calculation.

807 $\cong G_{771} \cong \mathbb{Z}^2$. Wreath recursion: $a = \sigma(c, b)$, $b = (c, c)$, $c = (a, a)$.

The same arguments as in the case of G_{771} show that G_{807} is free abelian. It has a relation $c^2ba^2 = 1$ and, hence, it has either rank 1 or rank 2. Analogically to G_{803} we consider a $\frac{1}{2}$-endomorphism $\phi : \mathrm{Stab}_{G_{807}}(1) \to G_{807}$, and a linear map $A : \mathbb{C}^3 \to \mathbb{C}^3$ induced by ϕ. It has the following matrix representation with respect to the basis corresponding to the triple $\{a, b, c\}$:

$$A = \begin{pmatrix} 0 & 0 & 1 \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 1 & 0 \end{pmatrix}.$$

Its characteristic polynomial $\chi_A(\lambda) = -\lambda^3 + \frac{1}{2}\lambda + \frac{1}{2}$ has three distinct complex roots $\lambda_1 = 1$, $\lambda_2 = -\frac{1}{2} - \frac{1}{2}i$ and $\lambda_3 = -\frac{1}{2} + \frac{1}{2}i$. Analogically for $v = (2m, 0, n)$ we get that $A^t(v)$ will be pushed out from the domain corresponding to $\mathrm{Stab}_{G_{807}}(1)$. Thus $c^n a^{2m} \neq 1$ in G_{807} and $G_{807} \cong \mathbb{Z}^2$.

810 $\cong G_{802} \cong C_2 \times C_2 \times C_2$. Wreath recursion: $a = \sigma(c, c)$, $b = (c, c)$, $c = (a, a)$.

Direct calculation.

820 $\cong D_\infty$. Wreath recursion: $a = \sigma(a, a)$, $b = (b, a)$, $c = (b, a)$.

The states a and b form a 2-state automaton generating D_∞ (see Theorem 7) and $c = b$.

821. Lamplighter group $\mathbb{Z} \wr C_2$. Wreath recursion: $a = \sigma(b, a)$, $b = (b, a)$, $c = (b, a)$.

The states a and b form a 2-state automaton generating the Lamplighter group (see Theorem 7) and $c = b$.

824 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(a, a)$, $b = (b, a)$, $c = (b, a)$.

The states a and b form a 2-state automaton generating D_∞ (see Theorem 7) and $c = b$.

838 $\cong C_2 \ltimes \langle s, t \mid s^2 = t^2 \rangle$. Wreath recursion: $a = \sigma(a, a)$, $b = \sigma(a, b)$, $c = (b, a)$.

All generators have order 2. Consider the subgroup $H = \langle ba = \sigma(ba, 1), ca = \sigma(1, ab) \rangle \cong G_{2860} = \langle s, t \mid s^2 = t^2 \rangle$. This subgroup is normal in G_{838} because the generators have order 2. Since $G_{838} = \langle H, a \rangle$, it has a structure of a semidirect product $\langle a \rangle \ltimes H = C_2 \ltimes \langle s, t \mid s^2 = t^2 \rangle$ with the action of a on H as $(ba)^b = (ba)^{-1}$ and $(ca)^b = (ca)^{-1}$.

839 $\cong G_{821}$. Lamplighter group $\mathbb{Z} \wr C_2$. Wreath recursion: $a = \sigma(b, a)$, $b = (a, b)$, $c = (b, a)$.

The states a and b form a 2-state automaton generating the Lamplighter group (see Theorem 7). Since $b^{-1}a = \sigma = ac^{-1}$, we see that $c = a^{-1}ba$ and $G = \langle a, b \rangle$.

840. Wreath recursion: $a = \sigma(c, a)$, $b = (a, b)$, $c = (b, a)$.

The element $(b^{-1}a)^2$ stabilizes the vertex 01 and its section at this vertex is equal to $b^{-1}a$. Hence, $b^{-1}a$ has infinite order.

The element $(c^{-1}b)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $(c^{-1}b)^{-1}$. Hence, $c^{-1}b$ has infinite order. Since

$(b^{-1}a^{-1}b^{-1}cba)^2\big|_{00000000} = c^{-1}b$ and the vertex 00000000 is fixed under the action of $(b^{-1}a^{-1}b^{-1}cba)^2$ we obtain that $b^{-1}a^{-1}b^{-1}cba$ also has infinite order. Finally, $b^{-1}a^{-1}b^{-1}cba$ stabilizes the vertex 0001 and has itself as a section at this vertex. Therefore G_{840} is not contracting.

We have $b^{-1}a^{-1}ca = (1, b^{-1}c^{-1}bc), ab^{-1}a^{-1}c = (cb^{-1}c^{-1}b, 1)$, hence by Lemma 4 the group is not free.

842 $\cong G_{838} \cong C_2 \ltimes \langle s,t \mid s^2 = t^2\rangle$. Wreath recursion: $a = \sigma(b,b)$, $b = \sigma(a,b)$, $c = (b,a)$.

All generators have order 2. Consider the subgroup $H = \langle u = ba = \sigma(1,ba) = \sigma(1,u^{-1}), v = ca = \sigma(ab,1) = \sigma(u^{-1},1)\rangle$. Let us prove that $H \cong W = \langle s,t \mid s^2 = t^2\rangle$. Indeed, the relation $u^2 = v^2$ is satisfied, so H is a homomorphic image of W with respect to the homomorphism induced by $s \mapsto u$ and $t \mapsto v$. Each element of W can be written in its normal form $t^r(st)^l s^n$, $n \in \mathbb{Z}, l \geq 0, r \in \{0,1\}$. It suffices to prove that images of these words (except for the identity word, of course) represent nonidentity elements in H.

We have $u^{2n} = (u^{-n}, u^{-n})$, $u^{2n+1} = \sigma(a^{-n}, a^{-n-1})$ for any integer n; $(uv)^l = (u^{2l}, 1)$ for any integer l. Thus

$$(uv)^l u^{2n} = (u^{-2l-n}, u^{-n}) \neq 1$$

in G if $n \neq 0$ or $l \neq 0$ since u has infinite order, as it is conjugate to the adding machine.

Furthermore,

$$v(uv)^l u^{2n} = \sigma(u^{-2l-n-1}, u^{-n}) \neq 1,$$
$$(uv)^l u^{2n+1} = \sigma(u^{-n}, u^{-2l-n-1}) \neq 1$$

since they act nontrivially on the first level of the tree.

Finally, $v(uv)^l u^{2n+1} = (u^{-2l-n-2}, u^{-n}) = 1$ if and only if $n = 0$ and $l = -1$, which is not the case, because l must be nonnegative. Thus $H \cong W$.

The subgroup H is normal in G_{842} because generators are of order 2. Since $G_{842} = \langle H, a\rangle$, it has a structure of a semidirect product $\langle a\rangle \ltimes H = C_2 \ltimes \langle s,t \mid s^2 = t^2\rangle$ with the action of a on H as $(ba)^b = (ba)^{-1}$ and $(ca)^b = (ca)^{-1}$. Therefore it has the same structure as G_{838}.

843. Wreath recursion: $a = \sigma(c,b)$, $b = (a,b)$, $c = (b,a)$.

The element $c^{-1}a = \sigma(a^{-1}c, 1)$ is a conjugate of the adding machine. Therefore, it acts transitively on the level of the tree and has infinite order.

Since $(c^{-1}ab^{-1}a)^2$ fixes the vertex 000 and its section at this vertex is equal to $c^{-1}a$, we obtain that $c^{-1}ab^{-1}a$ has infinite order. Since the element $c^{-1}ab^{-1}a$ fixes the vertex 10 and has itself as a section at this vertex, G_{843} is not contracting.

We have $c^{-1}a^{-1}ba = (1, a^{-1}c^{-1}ac)$, $ac^{-1}a^{-1}b = (ca^{-1}c^{-1}a, 1)$, hence by Lemma 4 the group is not free.

846 $\cong C_2 * C_2 * C_2$. Wreath recursion: $a = \sigma(c,c)$, $b = (a,b)$, $c = (b,a)$.

The automaton [846] was studied during the Advanced Course on Automata Groups in Bellaterra, Spain, in the summer of 2004 and is since called the Bellaterra automaton. We present here a proof that $G_{846} = C_2 * C_2 * C_2$,

based on the concept of dual automata. A different proof, still based on dual automata, is given in [Nek05].

Let $\mathcal{A} = (Q, X, \pi, \tau)$ be a finite automaton. Its *dual* automaton, by definition, is $\mathcal{A}' = (X, Q, \pi', \tau')$, where $\pi'(x, q) = \tau(q, x)$, and $\tau'(x, q) = \pi(q, x)$. Thus the dual automaton is obtained by exchanging the roles of the states and the alphabet (and the roles of the transition and output function) in a given automaton. The notion od dual automata is not new, but there is a recent renewed interest based on the new results and applications in [MNS00, GM05, BŠ06, VV05].

If in addition to $\mathcal{A}$, both $\mathcal{A}'$ and $(\mathcal{A}^{-1})'$ are invertible, the automaton $\mathcal{A}$ is called *fully invertible* (or *bi-reversible*). Examples of such automata are the automaton 2240 generating a free group with three generators [VV05], Bellaterra automaton [846], and various automata constructed in [GM05], generating free groups of various ranks.

We now consider the automaton [846] and its dual more closely. Since the generators a, b, and c have order 2, in order to prove that $G_{846} \cong C_2 * C_2 * C_2$ we need to show that no word in $w \in R_n$, $n \geq 1$, is trivial in G_{846}, where R_n is the set of reduced words over $\{a, b, c\}$ of length n (here a word is reduced if it does not contain aa, bb, or cc). For every $n > 0$, the set of words in R_n that are nontrivial in G_{846} is nonempty, since the word $r_n = acbcbcb\cdots$ of length n acts nontrivially on level 1. If we prove that the dual automaton acts transitively on the sets R_n, $n \geq 1$, this would mean that r_n is a section of every element of G_{846} that can be represented as a reduced word of length n. Therefore, every word in R_n would represent a nontrivial element in G_{846} and our proof would be complete.

The automaton dual to 846 is the invertible automaton defined by the wreath recursion

$$\begin{array}{rcl} A & = & (acb)(B, A, A), \\ B & = & (ac)(A, B, B), \end{array} \tag{10}$$

where the three coordinates in the recursion represent the sections at a, b, and c, respectively. Denote $D = \langle A, B\rangle$. The set $R = \bigcup_{n\geq 0} R_n$ of all reduced words over $\{a, b, c\}$ is a subtree of the ternary tree $\{a, b, c\}^*$ and this subtree R is invariant under the action of D (this is because the set $\{aa, bb, cc\}$ is invariant under the action of D). The structure of R is as follows. The root of R has three children a, b and c, each of which is a root of a binary tree. We want to understand the actio of D on the subtree R. It is given by

$$\begin{array}{rcl} A & = & (acb)(B_a, A_b, A_c) \\ B & = & (ac)(A_a, B_b, B_c) \end{array} \tag{11}$$

where $A_a, A_b, A_c, B_a, B_b, B_c$ are automorphisms of the binary trees hanging down from the vertices a, b and c. After identification of these three trees with the binary tree $\{0, 1\}^*$, the action of $A_a, A_b, \ldots, B_c$ is defined by

$$\begin{array}{rcl} A_a & = & (A_b, A_c), \\ A_b & = & \sigma(B_a, A_c), \\ A_c & = & \sigma(B_a, A_b), \\ B_a & = & \sigma(B_b, B_c), \\ B_b & = & \sigma(A_a, B_c), \\ B_c & = & \sigma(A_a, B_b). \end{array} \tag{12}$$

Using Lemma 2 one can verify that B_b acts level transitively on the binary tree. This is sufficient to show that D acts transitively on R, since it acts transitively on the first level, B stabilizes the vertex b, and its section at b is B_b.

The fact that G_{846} is not contracting follows now from the result of Nekrashevych [Nek07a], that a contracting group can not have free subgroups. Alternatively, it is sufficient to observe that aba has infinite order, stabilizes the vertex 01 and has itself as a section at this vertex.

847 $\cong D_4$. Wreath recursion: $a = \sigma(a, a)$, $b = (b, b)$, $c = (b, a)$.

The state b is trivial. The states a and c form a 2-state automaton generating D_4 (see Theorem 7).

848 $\cong C_2 \wr \mathbb{Z}$. Wreath recursion: $a = \sigma(b, a)$, $b = (b, b)$, $c = (b, a)$.

The state b is trivial and a is the adding machine. Every element $g \in G_{848}$ has the form $g = \sigma^i(a^n, a^m)$. On the other hand, $c = (1, a), c^{ac^{-1}} = (a, 1)$, so $\mathrm{Stab}_G(1) = \{(a^n, a^m)\} \cong \mathbb{Z}^2$. Since $ac^{-1} = \sigma$ we see that $G \cong C_2 \wr \mathbb{Z}$.

849. Wreath recursion: $a = \sigma(c, a)$, b=(b,b), $c = (b, a)$.

The state b is trivial. The element $a^2c = (ac, ca^2)$ is nontrivial because its section at 0 is ac, and ac acts nontrivially on level 1. The automorphism $(a^2c)^2$ fixes the vertex 00 and its section at this vertex is equal to a^2c. Therefore a^2c has infinite order. Further, the section of a^2c at 100 coincides with a^2c, implying that G_{849} is not contracting.

The group G_{849} is regular weakly branch group over its commutator G'_{849}. This is clear since the group is self-replicating and $[a^{-1}, c] \cdot [c, a] = ([a, c], 1)$.

Conjugation of the generators of G_{849} by $\mu = \sigma(\mu, c^{-1}\mu)$ yields the wreath recursion

$$x = \sigma(yx, 1), \qquad y = (x, 1),$$

where $x = a^\mu$ and $y = c^\mu$. Further, we have

$$x = \sigma(yx, 1), \qquad yx = \sigma(yx, x),$$

and the last wreath recursion coincides with the one defining the automaton 2852. Therefore $G_{849} \cong G_{2852}$ (see G_{2852} for more information on this group).

851 $\cong G_{847} \cong D_4$. Wreath recursion: $a = \sigma(b, b)$, b=(b,b), $c = (b, a)$.

Direct calculation.

852. Basilica group $\mathcal{B} = IMG(z^2 - 1)$. Wreath recursion: $a = \sigma(c, b)$, $b = (b, b)$, $c = (b, a)$.

This group was studied in [GŻ02a], where it is shown that $\mathcal{B}$ is not a sub-exponentially amenable group, it does not contain free subgroups of rank 2, and that the monoid generated by a and b is free. Some spectral considerations are provided in [GŻ02b]. Bartholdi and Virág showed in [BV05] that $\mathcal{B}$ is amenable, distinguishing the Basilica group as the first example of an amenable group that is not sub-exponentially amenable.

855 $\cong G_{847} \cong D_4$. Wreath recursion: $a = \sigma(c, c)$, b=(b,b), $c = (b, a)$.

Direct calculation.

856 $\cong C_2 \ltimes G_{2850}$. Wreath recursion: $a = \sigma(a, a)$, $b = (c, b)$, $c = (b, a)$.

All generators have order 2, hence $H = \langle ba, ca \rangle$ is normal in G_{856}. Furthermore, $ba = \sigma(ba, ca)$, $ca = \sigma(1, ba)$, and therefore $H = G_{2850}$. Thus $G_{856} = \langle a \rangle \ltimes H \cong C_2 \ltimes G_{2850}$, where $(ba)^a = (ba)^{-1}$ and $(ca)^a = (ca)^{-1}$. The group is not contracting since G_{2850} is not contracting.

857. Wreath recursion: $a = \sigma(b, a)$, $b = (c, b)$, $c = (b, a)$.

By using the approach used for G_{875}, we can show that the forward orbit of 10^∞ under the action of a is infinite, and therefore a has infinite order.

Since $c = (b, a)$ and $b = (c, b)$, both b and c have infinite order and G_{857} is not a contracting group.

858. Wreath recursion: $a = \sigma(c, a)$, $b = (c, b)$, $c = (b, a)$.

The element $ab^{-1} = \sigma(1, ab^{-1})$ is the adding machine.

By using the approach used for G_{875}, we can show that the forward orbit of 10^∞ under the action of a is infinite, and therefore a has infinite order.

Since $c = (b, a)$ and $b = (c, b)$, both b and c have infinite order and G_{857} is not a contracting group.

We have $c^{-1}b^{-1}aba^{-1}b = (1, a^{-1}b^{-1}aca^{-1}b)$, $a^{-1}c^{-1}b^{-1}aba^{-1}ba = (a^{-2}b^{-1}aca^{-1}ba, 1)$, hence by Lemma 4 the group is not free.

860. Wreath recursion: $a = \sigma(b, b)$, $b = (c, b)$, $c = (b, a)$.

The element $(ba^{-1})^2$ stabilizes the vertex 11 and its section at this vertex is equal to $(ba^{-1})^{-1}$. Hence, ba^{-1} has infinite order.

Furthermore, $bc^{-1} = (cb^{-1}, ba^{-1})$ implies that the order of bc^{-1} is infinite. Since this element stabilizes vertex 00 and its section at this vertex is equal to bc^{-1}, all its powers belong to the nucleus. Thus, G_{860} is not contracting.

861. Wreath recursion: $a = \sigma(b, b)$, $b = (a, a)$, $c = (b, a)$.

The element $a^{-1}c = \sigma(1, c^{-1}a)$ is conjugate to the adding machine and has infinite order.

864. Wreath recursion: $a = \sigma(c, c)$, $b = (c, b)$, $c = (b, a)$.

The element $(ab^{-1})^2$ stabilizes the vertex 11 and its section at this vertex is equal to ab^{-1}. Hence, ab^{-1} has infinite order.

Furthermore, $cb^{-1} = (bc^{-1}, ab^{-1})$ implies that the order of cb^{-1} is infinite. Since this element stabilizes vertex 00 and its section at this vertex is equal to cb^{-1}, G_{864} is not contracting.

865 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(a, a), b = (a, c), c = (b, a)$.

All generators have order 2. Since $abac = (acab, 1)$ and $acab = (1, abac)$, we see that $c = aba$ and $G_{865} = \langle a, b \rangle$. The section of $(ba)^2$ at the vertex 0 is $(ba)^{-1}$, so ba has infinite order and $G_{865} \cong D_\infty$.

Note that the group is conjugate to G_{932} by the automorphism $\delta = (a\delta, \delta)$.

866. Wreath recursion: $a = \sigma(b, a)$, $b = (a, c)$, $c = (b, a)$.

The element $(c^{-1}b)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $c^{-1}b$, which is nontrivial. Hence, $c^{-1}b$ has infinite order.

The element $(b^{-1}a)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $b^{-1}a$. Hence, $b^{-1}a$ has infinite order. Since $b^{-1}c^{-1}ba^{-1}ba\big|_{10} = (b^{-1}a)^b$ and vertex 10 is fixed under the action of $b^{-1}c^{-1}ba^{-1}ba$ we obtain that $b^{-1}c^{-1}ba^{-1}ba$ also has infinite order. Finally, $b^{-1}c^{-1}ba^{-1}ba$ stabilizes the vertex 00 and has itself as a section at this vertex. Therefore G_{866} is not contracting.

869. Wreath recursion: $a = \sigma(b,b)$, $b = (a,c)$, $c = (b,a)$.

All generators have order 2. By Lemma 1 ab has infinite order, which implies that $babcba$ also has infinite order, because it fixes the vertex 000 and its section at this vertex is equal to ab. But $babcba$ fixes 10 and has itself as a section at this vertex. Thus, G_{869} is not contracting.

870: Baumslag-Solitar group $BS(1,3)$. Wreath recursion: $a = \sigma(c,b)$, $b = (a,c)$, $c = (b,a)$.

The automaton satisfies the conditions of Lemma 1. In particular ab has infinite order. Since $bc = (ab, ca)$, $a^2 = (bc, cb)$, we obtain that bc and a have infinite order. Since $b = (a,c)$, b also has infinite order. Since b has infinite order, fixes the vertex 10 and has itself as a section at this vertex, G_{870} is not contracting.

The element $\mu = b^{-1}a = \sigma(1, a^{-1}b) = \sigma(1, \mu^{-1})$ is conjugate to the adding machine and therefore has infinite order. Since $a^{-1}c = \sigma(1, c^{-1}a)$ we see that $a^{-1}c = \mu$. Therefore $c = ab^{-1}a$ and $G_{870} = \langle a, b\rangle = \langle \mu, b\rangle$.

We claim that $b^{-1}\mu b = \mu^3$. Since $c = ab^{-1}a$, we have

$$ab^{-1}ab^{-1}ab^{-1}a^{-1}b = (ba^{-1}bc^{-1}b^{-1}a, ca^{-1}ba^{-1}) = (ba^{-1}ba^{-1}ba^{-1}b^{-1}a, 1).$$

But $ba^{-1}ba^{-1}ba^{-1}b^{-1}a$ is a conjugate of the inverse of $ab^{-1}ab^{-1}ab^{-1}a^{-1}b$, which shows that $ab^{-1}ab^{-1}ab^{-1}a^{-1}b = 1$, and the last relation is equivalent to $b^{-1}\mu b = \mu^3$.

Since b and μ have infinite order, $G_{870} \cong BS(1,3)$.

See [BŠ06] for realizations of $BS(1,m)$ for any value of m, $m \neq \pm 1$.

874 $\cong C_2 \ltimes G_{2852}$. Wreath recursion: $a = \sigma(a,a)$, $b = (b,c)$, $c = (b,a)$.

All the generators have order 2, hence $H = \langle ba, ca\rangle$ is normal in G_{874}. Furthermore, $ba = \sigma(ca, ba)$, $ca = \sigma(1, ba)$, therefore $H = G_{2852}$. Thus $G_{874} = \langle a\rangle \ltimes H \cong C_2 \ltimes G_{2852}$, where $(ba)^a = (ba)^{-1}$ and $(ca)^a = (ca)^{-1}$. In particular, G_{874} is not contracting and has exponential growth.

875. Wreath recursion: $a = \sigma(b,a)$, $b = (b,c)$, $c = (b,a)$.

The equalities

$$a(10^\infty) = 010^\infty, \qquad b(10^\infty) = 10^\infty, \qquad c(10^\infty) = 110^\infty,$$

show that all members of the forward orbit of 10^∞ under the action of a have only finitely many 1's and that the position of the rightmost 1 cannot decrease under the action of a. Since $a(10^\infty) = 010^\infty$, the forward orbit of 10^∞ under the action of a can never return to 10^∞ and a has infinite order.

Note that the above equalities also show that no nonempty words w over $\{a,b,c\}$ satisfies a relation of the form $w = 1$ in G_{875}. First note that $c = (b,a)$ and $b = (b,c)$, implying that b and c have infinite order. Thus $b^n \neq 1$, for $n > 0$. On the other hand, for any word w that contains a or c, $w(10^\infty) \neq 10^\infty$ (again, since the position of the rightmost 1 moves to the right and never decreases).

Since b has infinite order and $b = (b,c)$, G_{875} is not contracting.

876. Wreath recursion: $a = \sigma(c,a)$, $b = (b,c)$, $c = (b,a)$.

By Lemma 2 the elements ba and acb^2a^2cb act transitively on the levels of the tree and, hence, have infinite order. Since $(b^8)\big|_{1100001100} = acb^2a^2cb$ and

vertex 1100001100 is fixed under the action of b^8 we obtain that b also has infinite order. Finally, b stabilizes the vertex 0 and has itself as a section at this vertex. Therefore G_{876} is not contracting.

We have $c^{-1}b = (1, a^{-1}c), ac^{-1}ba^{-1} = (ca^{-1}, 1)$, hence by Lemma 4 the group is not free.

878 $\cong C_2 \ltimes IMG(1 - \frac{1}{z^2})$. Wreath recursion: $a = \sigma(b, b)$, $b = (b, c)$, $c = (b, a)$.

Let $x = bc$ and $y = ca$. Since all generators have order 2, the index of the subgroup $H = \langle x, y\rangle$ in G_{878} is 2, H is normal and $G_{878} \cong C_2 \ltimes H$, where C_2 is generated by c. The action of C_2 on H is given by $x^c = x^{-1}$ and $y^c = y^{-1}$. We have $x = bc = (1, ca) = (1, y)$ and $y = ca = \sigma(ab, 1) = \sigma(y^{-1}x^{-1}, 1)$. An isomorphic copy of H is obtained by exchanging the letters 0 and 1, yielding the wreath recursion $x = (y, 1)$ and $y = \sigma(1, y^{-1}x^{-1})$. The last recursion defines $IMG(1 - \frac{1}{z^2})$ [BN06]. Thus, $G_{878} \cong C_2 \ltimes IMG(1 - \frac{1}{z^2})$.

879. Wreath recursion: $a = \sigma(c, b)$, $b = (b, c)$, $c = (b, a)$.

The element $c^{-1}a = \sigma(a^{-1}c, 1)$ is conjugate to the adding machine and has infinite order.

By Lemma 2 the element ca acts transitively on the levels of the tree and, hence, has infinite order. Since $(b^2)\big|_{1101} = ca$ and vertex 1101 is fixed under the action of b^2 we obtain that b also has infinite order. Finally, b stabilizes the vertex 0 and has itself as a section at this vertex. Therefore G_{879} is not contracting.

882. Wreath recursion: $a = \sigma(c, c)$, $b = (b, c)$, $c = (b, a)$.

The element $(ca^{-1}cb^{-1})^2$ stabilizes the vertex 00 and its section at this vertex is equal to $ca^{-1}cb^{-1}$. Hence, $ca^{-1}cb^{-1}$ has infinite order.

883 $\cong C_2 \ltimes G_{2841}$. Wreath recursion: $a = \sigma(a, a)$, $b = (c, c)$, $c = (b, a)$.

All generators have order 2, hence $H = \langle ba, ca\rangle$ is normal in G_{883}. Furthermore, $ba = \sigma(ca, ca)$, $ca = \sigma(1, ba)$, therefore $H = G_{2841}$. Thus $G_{883} = \langle a\rangle \ltimes H \cong C_2 \ltimes G_{2841}$, where $(ba)^a = (ba)^{-1}$ and $(ca)^a = (ca)^{-1}$. In particular, G_{883} is not contracting and has exponential growth.

884. Wreath recursion: $a = \sigma(b, a)$, $b = (c, c)$, $c = (b, a)$.

The element $(b^{-1}ca^{-1}c)^2$ stabilizes the vertex 0 and its section at this vertex is equal to $(b^{-1}ca^{-1}c)^{-1}$. Hence, $b^{-1}ca^{-1}c$ has infinite order. Since $[b, a]^2\big|_{0100} = (b^{-1}ca^{-1}c)^c$ and 0100 is fixed under the action of $[b, a]^2$ we obtain that $[b, a]$ also has infinite order. Finally, $[b, a]$ stabilizes the vertex 00 and its section at this vertex is $[b, c] = [b, a]$. Therefore G_{884} is not contracting.

885. Wreath recursion: $a = \sigma(c, a)$, $b = (c, c)$, $c = (b, a)$.

The element $(c^{-1}b)^2$ stabilizes the vertex 10 and its section at this vertex is equal to $c^{-1}b$. Hence, $c^{-1}b$ has infinite order. Furthermore, $c^{-1}b$ stabilizes the vertex 00 and has itself as a section at this vertex. Therefore G_{885} is not contracting.

We have $b^{-1}aba^{-1} = (1, c^{-1}aca^{-1}), a^{-1}b^{-1}ab = (a^{-1}c^{-1}ac, 1)$, hence by Lemma 4 the group is not free.

887. Wreath recursion: $a = \sigma(b, b)$, $b = (c, c)$, $c = (b, a)$.

The element $(ac^{-1})^4$ stabilizes the vertex 001 and its section at this vertex is equal to $(ac^{-1})^2$, which is nontrivial. Hence, ac^{-1} has infinite order.

888. Wreath recursion: $a = \sigma(c, b)$, $b = (c, c)$, $c = (b, a)$.

The element $a^{-1}c = \sigma(1, c^{-1}a)$ is conjugate to the adding machine and has infinite order. Since $c^{-1}b\big|_1 = a^{-1}c$ and vertex 1 is fixed under the action of $c^{-1}b$ we obtain that $c^{-1}b$ also has infinite order. Finally, $c^{-1}b$ stabilizes the vertex 00 and has itself as a section at this vertex. Therefore G_{888} is not contracting.

We have $c^{-1}ab^{-1}a = (1, a^{-1}b)$, $ac^{-1}ab^{-1} = (ca^{-1}bc^{-1}, 1)$, hence by Lemma 4 the group is not free.

891 $\cong C_2 \ltimes (\mathbb{Z} \wr C_2)$. Wreath recursion: $a = \sigma(c, c)$, $b = (c, c)$, $c = (b, a)$.

Let $x = ac$ and $y = cb$. Since all generators have order 2, the index of the subgroup $H = \langle x, y\rangle$ in G_{891} is 2, H is normal and $G_{891} \cong C_2 \ltimes H$, where C_2 is generated by c. The action of C_2 on H is given by $x^c = x^{-1}$ and $y^c = y^{-1}$.

In fact, to support the claim that H has index 2 in G_{891} we need to prove that $c \notin H$. We will prove a little bit more than that. Let $w = 1$ be a relation in G_{891}, where w is a word over $\{a, b, c\}$. The number of occurrences of a in w must be even (otherwise w would act nontrivially on level 1). Similarly, the number of occurrences of c in w is even. Indeed, if it were odd, then exactly one of the words w_0 and w_1 in the decomposition $w = (w_0, w_1)$ would have odd number of occurrences of the letter a, and the action of w would be nontrivial on level 2. Finally, we claim that the number of occurrences of b in w is also even. Otherwise the number of c's in both w_0 and w_1 would be odd and the action of w would be nontrivial on level 3. Thus every word over $\{a, b, c\}$ representing 1 must have even number of occurrences of each of the three letters. Note that this implies that the abelianization of G_{891} is $C_2 \times C_2 \times C_2$.

We now prove that H is isomorphic to the Lamplighter group $\mathbb{Z} \wr C_2$. The group H is self-similar, which can be seen from

$$x = ac = \sigma(cb, ca) = \sigma(y, x^{-1}), \qquad y = cb = (bc, ac) = (y^{-1}, x).$$

Consider the elements $s_n = \sigma^{y^n} = y^{-n}xy^{n+1}$, $n \in \mathbb{Z}$ (note that $xy = \sigma$). For $n > 0$, we have $s_0 s_1 \cdots s_{n-1} = x^n y^n$ and $s_{-n} s_{-n+1} \cdots s_{-1} = y^n x^n$. On the other hand, $s_n = y^{-n}\sigma y^n = \sigma(x^{-n}y^{-n}, y^n x^n)$ and $s_{-n} = y^n \sigma y^{-n} = \sigma(x^n y^n, y^{-n}x^{-n})$, implying

$$s_n = \sigma(s_{-1}s_{-2}\cdots s_{-n}, s_{-n}\cdots s_{-2}s_{-1})$$

and

$$s_{-n} = \sigma(s_0 s_1 \cdots s_{n-1}, s_{n-1}\cdots s_1 s_0).$$

By induction on n we obtain that the depth of s_n is $2n + 1$ for $n \geq 0$ and the depth of s_{-n} is $2n$ for $n > 0$ (*depth* of a finitary element is the lowest level at which all sections of the element are trivial). This implies that all s_i, $i \in \mathbb{Z}$ are different, have order 2 (they are conjugates of σ), and commute (for each i and each level m all sections of s_i at level m are equal). Therefore y has infinite order and $H = \langle x, y\rangle = \langle y, \sigma\rangle \cong \mathbb{Z} \wr C_2$.

Since y has infinite order, stabilizes the vertex 00 and has itself as a section at this vertex, G_{891} is not contracting.

919 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(a, a), b = (a, b), c = (c, a)$.

The states a, b form a 2-state automaton generating D_∞ (see Theorem 7) and $c = aba$.

920. Wreath recursion: $a = \sigma(b, a)$, $b = (a, b)$, $c = (c, a)$.

The element $(ac^{-1})^2$ stabilizes the vertex 00 and its section at this vertex is equal to ac^{-1}. Hence, ba^{-1} has infinite order.

923. Wreath recursion: $a = \sigma(b, b)$, $b = (a, b)$, $c = (c, a)$.

The states a and b form a 2-state automaton generating D_∞ (see Theorem 7).

924 $\cong G_{870}$. Baumslag-Solitar group $BS(1, 3)$. Wreath recursion: $a = \sigma(c, b)$, $b = (a, b)$, $c = (c, a)$.

This fact is proved in [BŠ06].

928 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(a, a)$, $b = (b, b)$, $c = (c, a)$.

The states a and c form a 2-state automaton generating D_∞ (see Theorem 7) and b is trivial.

929 $\cong G_{2851}$. Wreath recursion: $a = \sigma(b, a)$, $b = (b, b)$, $c = (c, a)$.

See G_{2851} for an isomorphism (in fact the groups coincide as subgroups of $\mathrm{Aut}(X^*)$).

930 $\cong G_{821}$. Lamplighter group $\mathbb{Z} \wr C_2$. Wreath recursion: $a = \sigma(c, a)$, $b = (b, b)$, $c = (c, a)$.

The states a and c form a 2-state automaton generating the Lamplighter group (see Theorem 7) and b is trivial.

932 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(b, b)$, $b = (b, b)$, $c = (c, a)$.

We have $b = 1$ and $a^2 = c^2 = 1$. The element $ac = \sigma(c, a)$ is clearly nontrivial. Since $(ac)^2 = (ac, ca)$, this element has infinite order. Thus $G \cong D_\infty$.

933 $\cong G_{849}$. Wreath recursion: $a = \sigma(c, b)$, $b = (b, b)$, $c = (c, a)$.

See G_{2852} for an isomorphism between G_{933} and G_{2852} and G_{849} for an isomorphism between G_{2852} and G_{849}.

936 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(c, c)$, $b = (b, b)$, $c = (c, a)$.

The states a and c form a 2-state automaton generating D_∞ (see Theorem 7) and b is trivial.

937 $\cong C_2 \ltimes G_{929}$. Wreath recursion: $a = \sigma(a, a)$, $b = (c, b)$, $c = (c, a)$.

All generators have order 2, hence $H = \langle ca, ba \rangle = \langle ca, caba \rangle$ is normal in G_{937}. Furthermore, $ca = \sigma(1, ca)$, $caba = \sigma(caba, ca)$, therefore $H = G_{929}$. Thus $G_{937} = \langle a \rangle \ltimes H \cong C_2 \ltimes G_{929}$, where $(ba)^a = (ba)^{-1}$ and $(ca)^a = (ca)^{-1}$. In particular, G_{937} is regular weakly branch over H', has exponential growth and is not contracting.

938. Wreath recursion: $a = \sigma(b, a)$, $b = (c, b)$, $c = (c, a)$.

The element $(b^{-1}a^{-1}ca)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $\left((b^{-1}a^{-1}ca)^{-1}\right)^{a^{-1}c}$. Hence, $b^{-1}a^{-1}ca$ has infinite order. Furthermore, $b^{-1}a^{-1}ca$ stabilizes the vertex 1 and has itself as a section at this vertex. Therefore G_{938} is not contracting.

We have $c^{-1}b = (1, a^{-1}b), a^{-1}c^{-1}ba = (a^{-2}ba, 1)$, hence by Lemma 4 the group is not free.

939. Wreath recursion: $a = \sigma(c, a)$, $b = (c, b)$, $c = (c, a)$.

The states a and c form a 2-state automaton generating the Lamplighter group (see Theorem 7). Hence, G_{939} is neither torsion, nor contracting, and has

exponential growth.
941. Wreath recursion: $a = \sigma(b, b)$, $b = (c, b)$, $c = (c, a)$.
The second iteration of the wreath recursion is

$$a = (02)(13)(c, b, c, b), \qquad b = (c, a, c, b), \qquad c = (23)(c, a, b, b).$$

Conjugation by $g = (cg, g, g, bg)$ gives the wreath recursion

$$a' = (02)(13), \qquad b = (c', a', c', b'), \qquad c = (23)(c', a', 1, 1),$$

where $a' = a^g$, $b' = b^g$, and $c' = c^g$. The last recursion coincides with the second iteration of the recursion

$$\alpha = \sigma, \qquad \beta = (\gamma, \beta), \qquad \gamma = (\gamma, \alpha).$$

Conjugating the last recursion by $h = (\gamma h, h)$ yields the recursion defining G_{945}. Thus, $G_{941} \cong G_{945} \cong C_2 \ltimes IMG(z^2 - 1)$ (see G_{945}). The limit space is half of the Basilica.
942. Wreath recursion: $a = \sigma(c, b)$, $b = (c, b)$, $c = (c, a)$.
The Lamplighter group $L = \mathbb{Z} \wr C_2$ can be defined as the group generated by a' and b' given by the wreath recursion (see Theorem 7)

$$\begin{aligned} a' &= \sigma(a', b'), \\ b' &= (a', b'). \end{aligned}$$

Let $H = \langle a, b \rangle \leq G_{942}$. We will show that H and L are isomorphic. Let Y^* be the subtree of X^* consisting of all words over the alphabet $Y = \{01, 11\}$. The element b fixes the letter in Y, while a swaps them. Since $a_{01} = b_{01} = a$, $a_{11} = b_{11} = b$, the tree Y^* is invariant under the action of H. Moreover, the action of H on Y^* coincides with the action of the Lamplighter group $L = \langle a', b' \rangle$ on X^* (after the identification $01 \leftrightarrow 0$, $11 \leftrightarrow 1$). This implies that the map $\phi : H \to L$ given by $a \mapsto a'$, $b \mapsto b'$ can be extended to a homomorphism. We claim that this homomorphism is in fact an isomorphism. Let $w = w(a, b)$ be a group word representing an element of the kernel of ϕ. Since $w(a', b')$ represents the identity in the lamplighter group L, the total exponent of a in w must be even and the total exponent ε of both a and b in w must be 0. Therefore the element $g = w(a, b)$ stabilizes the top two levels of the tree X^* and can be decomposed as

$$g = (c^{\varepsilon}, *, c^{\varepsilon}, *),$$

where the $*$'s are words over a and b representing the identity in H (these words correspond precisely to the first level sections of $w(a', b')$ in L). Since $\varepsilon = 0$, we see that $g = 1$ and the kernel of ϕ is trivial.

Thus, the Lamplighter group is a subgroup of G_{942}, which shows that G_{942} is not a torsion group, it is not free, and has exponential growth. Since $b = (c, b)$ and b has infinite order, G_{942} is not a contracting group.
945 $\cong G_{941} \cong C_2 \ltimes IMG(z^2 - 1)$. Wreath recursion: $a = \sigma(c, c)$, $b = (c, b)$, $c = (c, a)$.

All generators have order 2. Since $ab = \sigma(1, cb)$ and $cb = (1, ab)$ we see that $H = \langle ab, cb\rangle \cong G_{852} = IMG(z^2 - 1)$. This subgroup is normal in G_{945} because the generators have order 2. Since $G_{945} = \langle H, b\rangle$, it has a structure of a semidirect product $\langle b\rangle \ltimes H = C_2 \ltimes IMG(z^2-1)$ with the action of b on H given by $(ab)^b = (ab)^{-1}$ and $(cb)^b = (cb)^{-1}$. It follows that G_{945} is regular weakly branch over H' and has exponential growth. See G_{941} for an isomorphism.

955 $\cong G_{937} \cong C_2 \ltimes G_{929}$. Wreath recursion: $a = \sigma(a, a)$, $b = (b, c)$, $c = (c, a)$.

All generators have order 2. Consider the subgroup $H = \langle ba = \sigma(ca, ba), ca = \sigma(1, ca)\rangle \cong G_{929}$. This subgroup is normal in G_{955} because all generators have order 2. Since $G_{955} = \langle H, a\rangle$, it has a structure of a semidirect product $\langle a\rangle \ltimes H = C_2 \ltimes G_{929}$ with the action of a on H given by $(ba)^b = (ba)^{-1}$ and $(ca)^b = (ca)^{-1}$. It is proved above that G_{937} has the same structure. It follows that G_{955} is regular weakly branch over H' and has exponential growth.

956. Wreath recursion: $a = \sigma(b, a)$, $b = (b, c)$, $c = (c, a)$.

The element $(c^{-1}b)^2$ stabilizes the vertex 10 and its section at this vertex is equal to $(c^{-1}b)^{-1}$. Hence, $c^{-1}b$ has infinite order. Furthermore, $c^{-1}b$ stabilizes the vertex 0 and has itself as a section at this vertex. Therefore G_{956} is not contracting.

We have $c^{-1}b^{-1}aba^{-1}b = (1, a^{-1}c^{-1}aba^{-1}c), a^{-1}c^{-1}b^{-1}aba^{-1}ba = (a^{-2}c^{-1}aba^{-1}ca, 1)$, hence by Lemma 4 the group is not free.

957. Wreath recursion: $a = \sigma(c, a)$, $b = (b, c)$, $c = (c, a)$.

The states a, c form a 2-state automaton generating the Lamplighter group (see Theorem 7). Hence, G_{957} is neither torsion, nor contracting and has exponential growth.

959. Wreath recursion: $a = \sigma(b, b)$, $b = (b, c)$, $c = (c, a)$.

The element $(a^{-1}c)^4$ stabilizes the vertex 00 and its section at this vertex is equal to $(a^{-1}c)^{-1}$. Hence, $a^{-1}c$ has infinite order.

Furthermore, since $c^{-1}b = (c^{-1}b, a^{-1}c)$, this element also has infinite order. Thus, G_{959} is not contracting.

960. Wreath recursion: $a = \sigma(c, b)$, $b = (b, c)$, $c = (c, a)$.

Define $x = ac^{-1}$, $y = ba^{-1}$ and $z = cb^{-1}$. Then $x = \sigma(1, y)$, $y = \sigma(z, z^{-1})$ and $z = (z, x)$.

The element $(zxy)^8$ stabilizes the vertex 001010 and its section at this vertex is equal to $xy^{-1}z = xyz = (zxy)^{z^{-1}}$ (since $y^2 = 1$). Hence, zxy has infinite order.

Denote $t = (b^{-1}c)^4(b^{-1}a)(c^{-1}a)^5(b^{-1}c)$. Then t^2 stabilizes the vertex 00 and $t^2\big|_{00} = t^{b^{-1}c}$. Hence, t has infinite order. Let $s = c^{-2}b^2$. Since $s^{32}\big|_{111000000100} = t^c$ and s^{32} fixes 111000000100, we obtain that s also has infinite order. Finally, s stabilizes the vertex 00 and has itself as a section at this vertex. Therefore G_{960} is not contracting.

963. Wreath recursion: $a = \sigma(c, c)$, $b = (b, c)$, $c = (c, a)$.

All generators have order 2. The element $ac = \sigma(1, ca)$ is conjugate to the adding machine and has infinite order.

Furthermore, since $cb = (cb, ac)$, this element also has infinite order. Thus, G_{963} is not contracting.

964 $\cong G_{739} \cong C_2 \ltimes (C_2 \wr \mathbb{Z})$. Wreath recursion: $a = \sigma(a, a)$, $b = (c, c)$, $c = (c, a)$.

All generators have order 2. The elements $u = acba = (ca, 1)$ and $v = bc = (1, ca)$ generate $\mathbb{Z}^2$ because $ca = \sigma(1, ca)$ is the adding machine and has infinite order. We have $cacb = \sigma$ and $\langle u, v\rangle$ is normal in $H = \langle u, v, \sigma\rangle$ because $u^\sigma = v$ and $v^\sigma = u$. In other words, $H \cong C_2 \ltimes (\mathbb{Z} \times \mathbb{Z}) = C_2 \wr \mathbb{Z}$.

Furthermore, $G_{964} = \langle H, a\rangle$ and H is normal in G_{972} because $u^a = v^{-1}$, $v^a = u^{-1}$ and $\sigma^a = \sigma$. Thus $G_{964} = C_2 \ltimes (C_2 \wr \mathbb{Z})$, where the action of C_2 on H is specified above and coincides with the one in G_{739}. Therefore $G_{964} \cong G_{739}$.

965. Wreath recursion: $a = \sigma(b, a)$, $b = (c, c)$, $c = (c, a)$.

The element $(ac^{-1})^2$ stabilizes the vertex 01 and its section at this vertex is equal to $(ac^{-1})^{-1}$. Hence, ac^{-1} has infinite order.

By Lemma 2 the element a acts transitively on the levels of the tree and, hence, has infinite order. Since $c = (c, a)$ we obtain that c also has infinite order. Therefore G_{965} is not contracting.

We have $bc^{-1} = (1, ca^{-1}), a^{-1}bc^{-1}a = (a^{-1}c, 1)$, hence by Lemma 4 the group is not free.

966. Wreath recursion: $a = \sigma(c, a)$, $b = (c, c)$, $c = (c, a)$.

The states a and c form a 2-state automaton generating the Lamplighter group (see Theorem 7). Hence, G_{966} is neither torsion, nor contracting, and has exponential growth.

Since $b = (c, c)$ we obtain that G_{966} can be embedded into the wreath product $C_2 \wr (\mathbb{Z} \wr \mathbb{C}_2)$. This shows that G_{966} is solvable.

968. Wreath recursion: $a = \sigma(b, b)$, $b = (c, c)$, $c = (c, a)$.

We will show that this group contains $\mathbb{Z}^5$ as a subgroup of index 16. It is a contracting group, with nucleus consisting of 73 elements (the self-similar closure of the nucleus consists of 77 elements).

All generators have order 2. Let $x = (ac)^2$, $y = bcba$, and $K = \langle x, y\rangle$. Conjugating x and y by $\gamma = (b\gamma, a\gamma)$ yields the self-similar copy K' of K generated by $x' = ((y')^{-1}, (y')^{-1})$ and $y = \sigma(x', y')$, where $x' = x^\gamma$ and $y' = y^\gamma$. Since $[x', y'] = ([x', y']^{(y')^{-1}}, 1)$ K' is abelian. The matrix of the corresponding virtual endomorphism is given by

$$A = \begin{pmatrix} 0 & \frac{1}{2} \\ -1 & \frac{1}{2} \end{pmatrix}.$$

The eigenvalues $\lambda = \frac{1}{4} \pm \frac{1}{4}\sqrt{7}i$ of this matrix are not algebraic integers. Therefore K' (ad therefore K as well) is free abelian of rank 2, by the results in [NS04].

The subgroup $H = \langle ab, bc\rangle$ has index 2 in G_{968} (the generators of G_{968} have order 2). The second level stabilizer $\mathrm{Stab}_H(2)$ has index 8 in H (the quotient group is isomorphic to the dihedral group D_4). The stabilizer $\mathrm{Stab}_H(2)$, is generated by $(bc)^2$, $\left((bc)^2\right)^{ba}$, $(ab)^2$, $\left((ab)^2\right)^{bc}$, $\left((ab)^2\right)^{(bc)^{ba}}$, and $\left((ab)^2\right)^{bc(bc)^{ba}}$.

Conjugating these elements by $g = (b, c, b, 1)$ gives

$$\begin{array}{llllll}
g_1 = \left((bc)^2\right)^g & = (bcbc)^g = & (1, & 1, & y, & y^{-1}), \\
g_2 = \left((bc)^2\right)^{bag} & = (acbcba)^g = & (y, & y, & 1, & 1), \\
g_3 = \left((ab)^2\right)^{bcg} & = (cbabac)^g = & (1, & x, & x, & 1), \\
g_4 = \left((ab)^2\right)^g & = (abab)^g = & (1, & x, & 1, & x^{-1}), \\
g_5 = \left((ab)^2\right)^{(bc)^{ba}g} & = (abcbabacba)^g = & (x, & 1, & 1, & x^{-1}), \\
g_6 = \left((ab)^2\right)^{bc(bc)^{ba}g} & = (abcacbabacacba)^g = & (x, & 1, & x, & 1).
\end{array}$$

Therefore, $\mathrm{Stab}_H(2)$ is abelian and $g_6 = g_5 g_3 g_4^{-1}$. If $\prod_{i=1}^5 g_i^{n_i} = 1$, then $x^{n_5}y^{n_2} = x^{n_3+n_4}y^{n_2} = x^{n_3}y^{n_1} = x^{n_4+n_5}y^{n_1} = 1$. Since K is free abelian, we obtain $n_i = 0$, $i = 1, \dots, 5$. Therefore $\mathrm{Stab}_H(2)$ is a free abelian group of rank 5.

969. Wreath recursion: $a = \sigma(c, b)$, $b = (c, c)$, $c = (c, a)$.

The element $(cb^{-1})^4$ stabilizes the vertex 100 and its section at this vertex is equal to cb^{-1}. Hence, cb^{-1} has infinite order.

We have $bc^{-1} = (1, ca^{-1})$, $ca^{-1} = \sigma(ab^{-1}, 1)$, $ab^{-1} = \sigma(1, bc^{-1})$, hence the subgroup generated by these elements is isomorphic to $IMG(1 - \frac{1}{z^2})$ (see [BN06]).

We also have $c^{-1}b = (1, a^{-1}c)$, $a^{-1}c^{-1}ba = (b^{-1}a^{-1}cb, 1)$, hence by Lemma 4 the group is not free.

972 $\cong G_{739} \cong C_2 \ltimes (C_2 \wr \mathbb{Z})$. Wreath recursion : $a = \sigma(c, c)$, $b = (c, c)$, $c = (c, a)$.

All generators have order 2. The elements $u = acba = (ca, 1)$ and $v = bc = (1, ac)$ generate $\mathbb{Z}^2$ because $ca = \sigma(ac, 1)$ is conjugate to the adding machine and has infinite order. Also we have $ba = \sigma$ and $\langle u, v\rangle$ is normal in $H = \langle u, v, \sigma\rangle$ because $u^\sigma = v$ and $v^\sigma = u$. In other words, $H \cong C_2 \ltimes (\mathbb{Z} \times \mathbb{Z}) = C_2 \wr \mathbb{Z}$.

Furthermore, $G_{972} = \langle H, a\rangle$ and H is normal in G_{972} because $u^a = v^{-1}$, $v^a = u^{-1}$ and $\sigma^a = \sigma$. Thus $G_{972} = C_2 \ltimes (C_2 \wr \mathbb{Z})$, where the action of C_2 on H is specified above and coincides with the one in G_{739}. Therefore $G_{972} \cong G_{739}$.

1090 $\cong C_2$. Wreath recursion: $a = \sigma(a, a)$, $b = (b, b)$, $c = (b, b)$.

Both b and c are trivial and $a^2 = 1$.

1091 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(b, a)$, $b = (b, b)$, $c = (b, b)$.

Both b and c are trivial and a is the adding machine.

1094 $\cong G_{1090} \cong C_2$. Wreath recursion: $a = \sigma(b, b)$, $b = (b, b)$, $c = (b, b)$.

Both b and c are trivial and $a^2 = 1$.

2190 $\cong G_{848} \cong C_2 \wr \mathbb{Z}$. Wreath recursion: $a = \sigma(c, a)$, $b = \sigma(a, a)$, $c = (a, a)$.

First note that $c = a^{-2}$. Therefore $G = \langle a, b\rangle$, where $a = \sigma(a^{-2}, a)$, and $b = \sigma(a, a)$. Also, a has infinite order.

Consider the subgroup $H = \langle ba, ab\rangle < G$. The generators of H commute since $ba = (a^{-1}, a^2)$ and $ab = (a^2, a^{-1})$. Furthermore, $(ba)^n(ab)^m = (a^{-n+2m}, a^{2n-m}) = 1$ if and only if $m = n = 0$. Therefore $H \cong \mathbb{Z}^2$.

Consider the element $ba^2 = bc^{-1} = \sigma$. This element does not belong to H, since H stabilizes the first level of the tree. On the other hand $a = (ba)^{-1}ba^2 = (ba)^{-1}\sigma$ and $b = a^{-1}(ab)$ so $G = \langle \sigma, H\rangle$. Finally, $(ba)^\sigma = ab$ and $(ab)^\sigma = ba$ implies that H is normal in G and $G = C_2 \wr H \cong C_2 \wr \mathbb{Z} \cong G_{848}$.

Also note that $\langle a, a^b \rangle = G_{2212} \cong \mathbb{Z} *_{2\mathbb{Z}} \mathbb{Z}$.
2193. Wreath recursion: $a = \sigma(c, b)$, $b = \sigma(a, a)$, $c = (a, a)$.

Let $x = ca^{-1}$ and $y = ab^{-1}$. Then $x = \sigma(ab^{-1}, ac^{-1}) = \sigma(y, x^{-1})$ and $y = (ba^{-1}, ca^{-1}) = (y^{-1}, x)$. It is already shown (see G_{891}), that $\langle x, y \rangle$ is not contracting and is isomorphic to the Lamplighter group. Therefore G_{2193} is not a torsion group, it is not contracting, and has exponential growth.
2196 $\cong G_{802} \cong C_2 \times C_2 \times C_2$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(a, a)$, $c = (a, a)$.

Direct calculation.
2199. Wreath recursion: $a = \sigma(c, a)$, $b = \sigma(b, a)$, $c = (a, a)$.

By Lemma 2 the element ac acts transitively on the levels of the tree and, hence, has infinite order. Since $ba = (ac, ba)$ we obtain that ba also has infinite order. Therefore G_{2199} is not contracting.

We have $b^{-2}abcba = b^{-2}aba^{-2}ba = 1$, and a and b do not commute, hence the group is not free.
2202. Wreath recursion: $a = \sigma(c, b)$, $b = \sigma(b, a)$, $c = (a, a)$.

The element $(b^{-1}a)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $b^{-1}a$. Hence, $b^{-1}a$ has infinite order. Furthermore, $b^{-1}a$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2202} is not contracting.

We have $cb^{-1}c^{-1}b = (1, ab^{-1}a^{-1}b), bcb^{-1}c^{-1} = (bab^{-1}a^{-1}, 1)$, hence by Lemma 4 the group is not free.
2203. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(b, a)$, $c = (a, a)$.

The states a and c form a 2-state automaton generating the infinite cyclic group $\mathbb{Z}$ in which $c = a^{-2}$ (see Theorem 7).

Since $b^{-1}a\big|_1 = a^{-1}c$ and vertex 1 is fixed under the action of $b^{-1}a$ we obtain that $b^{-1}a$ also has infinite order. Finally, $b^{-1}a$ stabilizes the vertex 0 and has itself as a section at this vertex. Therefore G_{2203} is not contracting.

We have $c^{-2}ab = (1, a^{-2}cb), bc^{-2}a = (ba^{-2}c, 1)$, hence by Lemma 4 the group is not free.
2204. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(b, a)$, $c = (a, a)$.

The element $(b^{-1}ac^{-1}a)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $b^{-1}ac^{-1}a$. Hence, $b^{-1}ac^{-1}a$ has infinite order. Since $[c, a]^2\big|_{000} = (b^{-1}ac^{-1}a)^{a^{-1}cb}$ and 000 is fixed under the action of $[c, a]^2$ we obtain that $[c, a]$ also has infinite order. Finally, $[c, a]$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2204} is not contracting.

We have $ab^{-1} = (1, ca^{-1}), b^{-1}a = (a^{-1}c, 1)$, hence by Lemma 4 the group is not free.
2205 $\cong G_{775} \cong C_2 \ltimes IMG\left(\left(\frac{z-1}{z+1}\right)^2\right)$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(b, a)$, $c = (a, a)$.

See G_{783} for an isomorphism between G_{783} and G_{2205}.
2206 $\cong G_{748} \cong D_4 \times C_2$. Wreath recursion: $a = \sigma(a, a)$, $b = \sigma(c, a)$, $c = (a, a)$.

Direct calculation.
2207. Wreath recursion: $a = \sigma(b, a)$, $b = \sigma(c, a)$, $c = (a, a)$.

The element $(c^{-1}a)^4$ stabilizes the vertex 000 and its section at this vertex is equal to $c^{-1}a$. Hence, $c^{-1}a$ has infinite order.

Since $b^{-1}a^{-1}b^{-1}aba\big|_{001} = (c^{-1}a)^a$ and the vertex 001 is fixed under the action of $b^{-1}a^{-1}b^{-1}aba$ we obtain that $b^{-1}a^{-1}b^{-1}aba$ also has infinite order. Finally, $b^{-1}a^{-1}b^{-1}aba$ stabilizes the vertex 000 and has itself as a section at this vertex. Therefore G_{2207} is not contracting.

We have $a^{-2}bab^{-2}ab = 1$, and a and b do not commute, hence the group is not free.

2209. Wreath recursion: $a = \sigma(a, b)$, $b = \sigma(c, a)$, $c = (a, a)$.

The element $(b^{-1}a)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $(b^{-1}a)^{-1}$. Hence, $b^{-1}a$ has infinite order. Furthermore, $b^{-1}a$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2209} is not contracting.

We have $aca^{-2}c^{-1}acac^{-1}a^{-2}cac^{-1} = 1$, and a and c do not commute, hence the group is not free.

2210. Wreath recursion: $a = \sigma(b, b)$, $b = \sigma(c, a)$, $c = (a, a)$.

The element $(a^{-1}c)^2$ stabilizes the vertex 000 and its section at this vertex is equal to $a^{-1}c$. Hence, $a^{-1}c$ has infinite order. Since $(b^{-1}a)^2\big|_{00} = a^{-1}c$ and 00 is fixed under the action of $b^{-1}a$ we obtain that $b^{-1}a$ also has infinite order. Finally, $b^{-1}a$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2210} is not contracting.

We have $c^{-1}b^{-1}cb = (1, a^{-1}c^{-1}ac), bc^{-1}b^{-1}c = (ca^{-1}c^{-1}a, 1)$, hence by Lemma 4 the group is not free.

2212. Klein bottle group, $\langle a, b \mid a^2 = b^2 \rangle$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(c, a)$, $c = (a, a)$.

The states a and c form a 2-state automaton generating the infinite cyclic group $\mathbb{Z}$ in which $c = a^{-2}$ (see Theorem 7).

We have $a = \sigma(a, a^{-2})$, $b = \sigma(a^{-2}, a)$, and $x = ab^{-1} = (a^{-3}, a^3)$. Finally, since $x^a = b^{-1}a = (a^3, a^{-3}) = x^{-1}$, we have $G_{2212} = \langle x, a \mid x^a = x^{-1} \rangle$ and G_{2212} is the Klein bottle group. Tietze transformations yield the presentation $G_{2212} = \langle a, b \mid a^2 = b^2 \rangle$ in terms of the generators a and b.

2213. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(c, a)$, $c = (a, a)$.

By Lemma 2 the element cb acts transitively on the levels of the tree and, hence, has infinite order. Since $(ba)\big|_{100} = cb$ and the vertex 100 is fixed under the action of ba we obtain that ba also has infinite order. Finally, ba stabilizes the vertex 01 and has itself as a section at this vertex. Therefore G_{2213} is not contracting.

We have $c^{-1}b^{-1}cb = (1, a^{-1}c^{-1}ac), bc^{-1}b^{-1}c = (ca^{-1}c^{-1}a, 1)$, hence by Lemma 4 the group is not free.

2214 $\cong G_{748} \cong D_4 \times C_2$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, a)$, $c = (a, a)$.

Direct calculation.

2226 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(c, a)$, $b = \sigma(b, b)$, and $c = (a, a)$.

We have $ba = (bc, ba)$, $bc = \sigma(ba, ba)$, and $b = \sigma(b, b)$. Therefore x, y and b satisfy the wreath recursion defining the automaton $\mathcal{A}_{2394}$. Thus $G_{2226} = G_{2394} \cong G_{820}$.

2229. Wreath recursion: $a = \sigma(c, b)$, $b = \sigma(b, b)$, $c = (a, a)$.

Note that b is of order 2. Post-conjugating the recursion by $(1, b)$ (which is equivalent to conjugating by the tree automorphism $g = (g, bg)$ in $\mathrm{Aut}(X^*)$ gives a copy of G_{2229} defined by

$$a = \sigma(bc, 1), \qquad b = \sigma, \qquad c = (a, bab)$$

The stabilizer of the first level is generated by

$$a^2 = (bc, bc), \qquad c = (a, bab), \qquad ba = (bc, 1), \qquad bcb = (bab, a).$$

Its projection on the first level is generated by

$$bc = \sigma(a, bab), \qquad a = \sigma(bc, 1), \qquad bab = \sigma(1, bc).$$

Furthermore,

$$bcbc = (baba, abab), \qquad abab = (1, bcbc), \qquad baba = (bcbc, 1),$$

which implies that bc is of order 2 and $a^{-1} = bab$. Hence, the projection of the stabilizer on the first level is generated by the recursion

$$a = \sigma(bc, 1), \qquad bc = \sigma(a, a^{-1}).$$

Post-conjugating by $(1, a)$, we obtain the recursion

$$a = \sigma(a^{-1} \cdot bc, a), \qquad bc = \sigma,$$

which is the group $C_4 \ltimes \mathbb{Z}^2$ of all orientation preserving automorphisms of the integer lattice (see [BN06]). Note that the nucleus of G_{2229} consists of 52 elements.

2232 $\cong G_{730}$. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(b, b)$, $c = (a, a)$.

Direct calculation.

2233. Wreath recursion: $a = \sigma(a, a)$, $b = \sigma(c, b)$, $c = (a, a)$.

Therefore, $\langle ba = (ba, ca), ca = \sigma \rangle = G_{932} \cong D_\infty$.

Conjugating by $g = (ag, g)$, we obtain the recursion

$$\alpha = \sigma, \qquad \beta = \sigma(\gamma\beta, \alpha\beta), \qquad \gamma = (\alpha, \alpha),$$

where $\alpha = a^g$, $\beta = b^g$, and $\gamma = c^g$. Therefore

$$\alpha = \sigma, \qquad \alpha\beta = (\gamma\alpha, \alpha\beta), \qquad \gamma\alpha = \sigma(\alpha, \alpha),$$

and the last wreath recursion defines a bounded automaton (see Section 3 for a definition). It follows from [BKN] that G_{2233} is amenable.

2234. Wreath recursion: $a = \sigma(b, a)$, $b = \sigma(c, b)$, $c = (a, a)$.

The element $(c^{-1}b)^4$ stabilizes the vertex 00 and its section at this vertex is equal to $(c^{-1}b)^{-1}$. Hence, $c^{-1}b$ has infinite order. Since $(b^{-1}a)\big|_0 = c^{-1}b$ and

0 is fixed under the action of $b^{-1}a$ we obtain that $b^{-1}a$ also has infinite order. Finally, $b^{-1}a$ stabilizes the vertex 1 and has itself as a section at this vertex. Therefore G_{2234} is not contracting.

We have $c^{-1}b^{-1}ac^{-1}a^2 = (1, a^{-1}c^{-1}b^2), ac^{-1}b^{-1}ac^{-1}a = (ba^{-1}c^{-1}b, 1)$, hence by Lemma 4 the group is not free.

2236. Wreath recursion: $a = \sigma(a, b)$, $b = \sigma(c, b)$, $c = (a, a)$.

By Lemma 2 the element b acts transitively on the levels of the tree and, hence, has infinite order.

By Lemma 2 the element cb acts transitively on the levels of the tree and, hence, has infinite order. Since $ba = (ba, cb)$ we obtain that ba also has infinite order. Since ba has itself as a section at 0 the group is not contracting.

We have $a^{-2}bab^{-2}ab = 1$, and a and b do not commute, hence the group is not free.

2237. Wreath recursion: $a = \sigma(b, b)$, $b = \sigma(c, b)$, $c = (a, a)$.

By Lemma 2 the elements b and $(bc)^3$ acts transitively on the levels of the tree and, hence, have infinite order.

Since $(cba)^2\big|_{00000} = (bc)^3$ and 00000 is fixed under the action of $(cba)^2$ we obtain that cba also has infinite order. Finally, cba stabilizes the vertex 101 and has itself as a section at this vertex. Therefore G_{2237} is not contracting.

We have $a^{-2}bab^{-2}ab = 1$, and a and b do not commute, hence the group is not free.

2239. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(c, b)$, $c = (a, a)$.

The group contains elements of infinite order by Lemma 1. In particular, ca has infinite order. Since $(ba)\big|_{100} = ca$ and the vertex 100 is fixed under the action of ba we obtain that ba also has infinite order. Finally, ba stabilizes the vertex 1 and has itself as a section at this vertex. Therefore G_{2239} is not contracting.

We have $ca^{-2}cba^{-1} = (1, c^{-1}abc^{-1}), a^{-1}ca^{-2}cb = (c^{-2}ab, 1)$, hence by Lemma 4 the group is not free.

We can also simplify the wreath recursion in the following way. Since $c = a^{-2}$ we have

$$a = \sigma(a, a^{-2}), \qquad b = \sigma(a^{-2}, b).$$

Therefore

$$ab = (a^{-4}, ab), \qquad a = \sigma(a, a^{-2}),$$

which can be written as

$$ab = (a^{-4}, ab), \qquad a = \sigma(1, a^{-1}),$$

which is a subgroup of

$$\beta = (a, \beta), \qquad a = \sigma(1, a^{-1}).$$

2240. Free group of rank 3. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(c, b)$, $c = (a, a)$.

The automaton appeared for the first time in [Ale83]. The fact that G_{2240} is free group of rank 3 with basis $\{a, b, c\}$ is proved in [VV05]. This is the

smallest automaton among all automata over a 2-letter alphabet generating a free nonabelian group.

The fact that G_{2240} is not contracting follows now from the result of Nekrashevych [Nek07a], that a contracting group cannot have free subgroups. Alternatively, $b^{-1}ca$ has infinite order, stabilizes the vertex 11 and has itself as a section at this vertex. Hence, the group is not contracting.

2241 $\cong G_{739} \cong C_2 \ltimes (C_2 \wr \mathbb{Z})$. Wreath recursion: $a = \sigma(c,c)$, $b = \sigma(c,b)$, $c = (a,a)$.

Consider G_{747}. Its wreath recursion is given by $a = \sigma(c,c)$, $b = (b,a)$, $c = (a,a)$. All generators have order 2 and a commutes with c. Therefore $acb = \sigma(cab,c) = \sigma(acb,c)$ and wa have $G_{747} = \langle a, acb, c\rangle = G_{2241}$. Thus $G_{2241} = G_{747} \cong G_{739}$.

2260 $\cong G_{802} \cong C_2 \times C_2 \times C_2$. Wreath recursion: $a = \sigma(a,a)$, $b = (c,c)$, $c = (a,a)$.

Direct calculation.

2261. Wreath recursion: $a = \sigma(b,a)$, $b = \sigma(c,c)$, $c = (a,a)$.

The element $(ac^{-1})^2$ stabilizes the vertex 00 and its section at this vertex is equal to $(ac^{-1})^{-1}$. Hence, ac^{-1} and $c^{-1}a$ have infinite order.

Since $b^{-1}c^{-1}ac^{-1}ba\big|_{001} = \left((c^{-1}a)^2\right)^a$ and the vertex 001 is fixed under the action of $b^{-1}c^{-1}ac^{-1}ba$ we obtain that $b^{-1}c^{-1}ac^{-1}ba$ also has infinite order. Finally, $b^{-1}c^{-1}ac^{-1}ba$ stabilizes the vertex 000 and has itself as a section at this vertex. Therefore G_{2261} is not contracting.

We have $acac^{-1}a^{-2}cac^{-1}aca^{-2}c^{-1} = 1$, and a and c do not commute, hence the group is not free.

2262 $\cong G_{848} \cong C_2 \wr \mathbb{Z}$. Wreath recursion: $a = \sigma(c,a)$, $b = \sigma(c,c)$, $c = (a,a)$.

The states a and c form a 2-state automaton (see Theorem 7). Moreover, $c = a^{-2}$ and a has infinite order.

Thus $a = \sigma(a^{-2},a)$, $b = \sigma(a^{-2},a^{-2})$ and $G_{2262} = \langle a,b\rangle$. Further, $b^{-1}a = (1,a^3)$ and $a^{-3} = \sigma(1,a^3)$, yielding $a^{-4}b = \sigma$. Therefore $G = \langle a,\sigma\rangle$. Since $\langle a, a^\sigma\rangle = \mathbb{Z}^2$, we obtain that $G_{2262} \cong C_2 \wr Z^2 \cong G_{848}$.

2264 $\cong G_{730}$. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(b,b)$, $b = \sigma(c,c)$, $c = (a,a)$.

Direct calculation.

2265. Wreath recursion: $a = \sigma(c,b)$, $b = \sigma(c,c)$, $c = (a,a)$.

The element $(c^{-1}b)^4$ stabilizes the vertex 0000 and its section at this vertex is equal to $\left((c^{-1}b)^{-1}\right)^{c^{-1}a}$. Hence, $c^{-1}b$ has infinite order. Since $[c,a]\big|_{10} = (c^{-1}b)^c$ and 10 is fixed under the action of $[c,a]$ we obtain that $[c,a]$ also has infinite order. Finally, $[c,a]$ stabilizes the vertex 00 and has itself as a section at this vertex. Therefore G_{2265} is not contracting.

We have $a^{-2}bab^{-2}ab = 1$, and a and b do not commute, hence the group is not free.

2271. Wreath recursion: $a = \sigma(c,a)$, $b = \sigma(a,a)$, $c = (b,a)$.

The element $(ac^{-1})^4$ stabilizes the vertex 001 and its section at this vertex is equal to ac^{-1}. Hence, ac^{-1} has infinite order.

The element $(a^{-1}b)^4$ stabilizes the vertex 000 and its section at this vertex is equal to $a^{-1}b$. Hence, $a^{-1}b$ has infinite order. Since $b^{-1}c^{-1}ac^{-1}a^2\big|_{001}= (a^{-1}b)^a$ and the vertex 001 is fixed under the action of $b^{-1}c^{-1}ac^{-1}a^2$ we obtain that $b^{-1}c^{-1}ac^{-1}a^2$ also has infinite order. Finally, $b^{-1}c^{-1}ac^{-1}a^2$ stabilizes the vertex 000 and has itself as a section at this vertex. Therefore G_{2271} is not contracting.

We have $a^{-2}bab^{-2}ab = 1$, and a and b do not commute, hence the group is not free.

2274. Wreath recursion: $a = \sigma(c, b)$, $b = \sigma(a, a)$, $c = (b, a)$.

The element $a^{-1}c = \sigma(1, c^{-1}a)$ is conjugate to the adding machine and has infinite order. Since $(b^{-1}a)\big|_0= a^{-1}c$ and 0 is fixed under the action of $b^{-1}a$ we obtain that $b^{-1}a$ also has infinite order. Finally, $b^{-1}a$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2274} is not contracting.

We have $bc^{-2}b = (1, ab^{-2}a), b^2c^{-2} = (a^2b^{-2}, 1)$, hence by Lemma 4 the group is not free.

2277 $\cong C_2 \ltimes (\mathbb{Z} \times \mathbb{Z})$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(a, a)$, $c = (b, a)$.

All generators have order 2. Let $x = cb$, $y = ab$ and $H = \langle x, y\rangle$. We have $x = \sigma(1, y^{-1})$ and $y = (xy^{-1}, xy^{-1})$. The elements x and y commute and the matrix of the associated virtual endomorphism is given by

$$A = \begin{pmatrix} 0 & 1 \\ -1/2 & -1 \end{pmatrix}.$$

The eigenvalues $-\frac{1}{2} \pm \frac{1}{2}i$ are not algebraic integers, and therefore, according to [NS04], H is free abelian of rank 2.

The subgroup H is normal of index 2 in G_{2277}. Therefore $G_{2277} = \langle H, b\rangle = C_2 \ltimes (\mathbb{Z} \times \mathbb{Z})$, where C_2 is generated by b, which acts on H is inversion of the generators.

2280. Wreath recursion: $a = \sigma(c, a)$, $b = \sigma(b, a)$, $c = (b, a)$.

We prove that a has infinite order by considering the forward orbit of 10^∞ under the action of a^2. We have

$$\begin{aligned} a^2 &= (ac, ca), & ac &= \sigma(cb, a^2), & ca &= \sigma(ac, ba) \\ cb &= \sigma(ab, ba), & ba &= (ac, ba), & ab &= (ab, ca). \end{aligned}$$

The equalities

$$\begin{aligned} &a^2(10^\infty) = ab(10^\infty) = 1110^\infty, \\ &ac(10^\infty) = ca(10^\infty) = cb(10^\infty) = 0010^\infty, \text{ and} \\ &ba(10^\infty) = 10110^\infty \end{aligned}$$

show that all members of the forward orbit of 10^∞ under the action of a^2 have only finitely many 1's and that the position of the rightmost 1 cannot decrease under the action of a^2. Since $a^2(10^\infty) = 1110^\infty$, the forward orbit of 10^∞ under the action of a^2 can never return to 10^∞ and a^2 has infinite order.

Since $a^2 = (ac, ca)$, the elements ca and $ab = (ab, ca)$ have infinite order, showing that G_{2280} is not contracting.

2283. Wreath recursion: $a = \sigma(c, b)$, $b = \sigma(b, a)$, $c = (b, a)$.

By Lemma 2 the element ac acts transitively on the levels of the tree and, hence, has infinite order. Since $ba = (ac, b^2)$ we obtain that ba also has infinite order. Finally, ba stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2283} is not contracting.

2284. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(b, a)$, $c = (b, a)$.

Define $u = b^{-1}a$, $v = a^{-1}c$ and $w = c^{-1}b$. Then $u = (u, v)$, $v = \sigma(w, 1)$ and $w = \sigma(u^{-1}, u)$. The group $\langle u, v, w\rangle$ is generated by the automaton symmetric to the one generating the subgroup $\langle x, y, z\rangle$ of G_{960} (see G_{960} for the definition). It is shown above that zxy has infinite order. Therefore wvu also has infinite order.

The element $(b^{-1}ac^{-1}a)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $(b^{-1}ac^{-1}a)^{a^{-1}b}$. Hence, $b^{-1}ac^{-1}a$ has infinite order. Let $t = b^{-1}ab^{-2}a^2$. Since $t|_{110} = b^{-1}ac^{-1}a$ and the vertex 110 is fixed under the action of t we see that t also has infinite order. Finally, t stabilizes the vertex 11101000 and has itself as a section at this vertex. Therefore G_{2284} is not contracting.

2285. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(b, a)$, $c = (b, a)$.

The element $ac^{-1} = \sigma(1, ca^{-1})$ is conjugate to the adding machine and has infinite order.

By Lemma 2 the element $abcb$ acts transitively on the levels of the tree and, hence, has infinite order. Since $(ba)^2\big|_{000} = (ac, b^2)$ and the vertex 000 is fixed under the action of $(ba)^2$ we obtain that ba also has infinite order. Finally, ba stabilizes the vertex 01 and has itself as a section at this vertex. Therefore G_{2285} is not contracting.

2286. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(b, a)$, $c = (b, a)$.

The element $(c^{-1}a)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $(c^{-1}a)^{a^{-1}b}$. Hence, $c^{-1}a$ has infinite order. Since $(c^{-2}a^2)\big|_{000} = (c^{-1}a)^{b^{-1}}$ and 000 is fixed under the action of $c^{-2}a^2$ we obtain that $c^{-2}a^2$ also has infinite order. Finally, $c^{-2}a^2$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2286} is not contracting.

2287. Wreath recursion: $a = \sigma(a, a)$, $b = \sigma(c, a)$, $c = (b, a)$.

The element $bc^{-1} = \sigma(cb^{-1}, 1)$ is conjugate to the adding machine and has infinite order.

Conjugating the generators by $g = (g, ag)$, we obtain the wreath recursion

$$a' = \sigma, \qquad b' = \sigma(a'c', 1), \qquad c' = (b', a'),$$

where $a' = a^g$, $b' = b^g$, and $c' = c^g$. Therefore

$$a' = \sigma, \qquad b' = \sigma(a'c', 1), \qquad a'c' = \sigma(b', a')$$

A direct computation shows that the iterated monodromy group of $\frac{z^2+2}{1-z^2}$ is generated by

$$\alpha = \sigma, \qquad \beta = \sigma(\gamma^{-1}\beta^{-1}, \alpha), \qquad \gamma = (\beta\gamma\beta^{-1}, \alpha),$$

where α, β, and γ are loops around the post-critical points 2, -1 and -2, respectively (recall the definition of iterated monodromy group in Section 5). We see that

$$\alpha = \sigma, \qquad \beta\gamma = \sigma(\beta^{-1}, 1), \qquad \beta = \sigma(\gamma^{-1}\beta^{-1}, \alpha)$$

satisfy the same recursions as a, b and ac, only composed with taking inverses. If we take second iteration of the wreath recursions, we obtain isomorphic self-similar groups.

It follows that the group G_{2287} is isomorphic to $IMG\left(\frac{z^2+2}{1-z^2}\right)$ and the limit space is homeomorphic to the Julia set of this rational function.

2293. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(c, a)$, $c = (b, a)$.

The element $(b^{-1}c)^2$ stabilizes the vertex 0 and its section at this vertex is equal to $(b^{-1}c)^{-1}$. Hence, $b^{-1}c$ has infinite order. Since $(c^{-1}bc^{-1}a)^2\big|_{000} = b^{-1}c$ and 000 is fixed under the action of $(c^{-1}bc^{-1}a)^2$ we obtain that $c^{-1}bc^{-1}a$ also has infinite order. Finally, $c^{-1}bc^{-1}a$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2293} is not contracting.

We have $b^{-1}c^2a^{-1} = (1, c^{-1}b^2c^{-1}), c^2a^{-1}b^{-1} = (b^2c^{-2}, 1)$, hence by Lemma 4 the group is not free.

2294. Baumslag-Solitar group $BS(1, -3)$. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(c, a)$, $c = (b, a)$.

The automaton satisfies the conditions of Lemma 1. Therefore cb has infinite order. Since $a^2 = (cb, bc)$, $c = (b, a)$ and $ba = (ab, c^2)$, the elements a, c and ba have infinite order. Finally, ba fixes the vertex 01 and has itself as a section at this vertex, showing that G_{2294} is not contracting.

Let $\mu = ca^{-1}$. We have $\mu = ca^{-1} = \sigma(ac^{-1}, 1) = \sigma(\mu^{-1}, 1)$, and therefore μ is conjugate of the adding machine and has infinite order. Further, we have $bc^{-1} = \sigma(cb^{-1}, 1) = \sigma((bc^{-1})^{-1}, 1)$, showing that $bc^{-1} = \mu = ca^{-1}$. Therefore $G_{2294} = \langle \mu, a\rangle$.

It can be shown that $a\mu a^{-1} = \mu^{-3}$ in G_{2294} (compare to G_{870}. Since both a and μ have infinite order $G_{2294} \cong BS(1, -3)$.

2295. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, a)$, $c = (b, a)$.

The element $cb^{-1} = \sigma(1, bc^{-1})$ is conjugate to the adding machine and has infinite order. Hence, its conjugate $a^{-1}cb^{-1}a$ also has infinite order. Since $c^{-1}ac^{-1}b = \left(c^{-1}ac^{-1}b, a^{-1}cb^{-1}a\right)$, the element $c^{-1}ac^{-1}b$ has infinite order and G_{2295} is not contracting.

We have $a^{-2}bab^{-2}ab = 1$, and a and b do not commute, hence the group is not free.

2307. Contains G_{933}. Wreath recursion: $a = \sigma(c, a)$, $b = \sigma(b, b)$, $c = (b, a)$.

We have $ba = (bc, ba)$, and $bc = \sigma(1, ba)$. Therefore G_{933} is a subgroup of G_{2307} (the wreath recursion for ba and bc defines an automaton that is symmetric to the one defining the automaton [993]).

The element $(a^{-1}b)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $a^{-1}b$. Hence, $a^{-1}b$ has infinite order. Furthermore, $a^{-1}b$ stabilizes the vertex 1 and has itself as a section at this vertex. Therefore G_{2307} is not contracting.

2313 $\cong G_{2277} \cong C_2 \ltimes (\mathbb{Z} \times \mathbb{Z})$. Wreath recursion: $a = \sigma(c,c)$, $b = \sigma(b,b)$, $c = (b,a)$.

Since all generators have order 2 the subgroup $H = \langle ba, bc\rangle$ is normal in G_{2313}. Furthermore, $ba = \sigma(bc,bc)$ and $bc = \sigma(1,ba)$. Hence, $H = G_{771} \cong \mathbb{Z}^2$.

Finally, $G_{2313} = \langle H, b\rangle = \langle b\rangle \ltimes H = C_2 \ltimes (\mathbb{Z} \times \mathbb{Z})$, where b inverts the generators of H. This action coincides with the one for G_{2277}, which proves that these groups are isomorphic.

2320 $\cong G_{2294}$. Baumslag-Solitar group $BS(1,-3)$. Wreath recursion: $a = \sigma(a,c)$, $b = \sigma(c,b)$, $c = (b,a)$.

It is proved in [BŠ06] that the automaton [2320] generates $BS(1,-3)$.

2322. Wreath recursion: $a = \sigma(c,c)$, $b = \sigma(c,b)$, $c = (b,a)$.

The element $(a^{-1}c)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $(a^{-1}c)^{b^{-1}}$. Hence, $a^{-1}c$ has infinite order. Since $(c^{-2}a^2)^2\big|_{000} = a^{-1}c$ and 000 is fixed under the action of $c^{-2}a^2$ we obtain that $c^{-2}a^2$ also has infinite order. Finally, $c^{-2}a^2$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2322} is not contracting.

We have $a^{-2}bab^{-2}ab = 1$, and a and b do not commute, hence the group is not free.

2352 $\cong G_{740}$. Wreath recursion: $a = \sigma(c,a)$, $b = \sigma(a,a)$, $c = (c,a)$.

We have $ac^{-1}b = (a,a)$. Therefore $G_{2352} = \langle a, ac^{-1}b, c\rangle = G_{740}$.

2355. Wreath recursion: $a = \sigma(c,b)$, $b = \sigma(a,a)$, $c = (c,a)$.

The element $(b^{-1}a)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $(b^{-1}a)^{a^{-1}c}$. Hence, $b^{-1}a$ has infinite order. Furthermore, $b^{-1}a$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2355} is not contracting.

We have $a^{-1}cb^{-1}c = (b^{-1}c, 1), cb^{-1}ca^{-1} = (1, cb^{-1})$, hence by Lemma 4 the group is not free.

2358 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(c,c)$, $b = \sigma(a,a)$, $c = (c,a)$.

The states a and c form a 2-state automaton generating D_∞ (see Theorem 7) and $b = aca$.

2361. Wreath recursion: $a = \sigma(c,a)$, $b = \sigma(b,a)$, $c = (c,a)$.

The element $bc^{-1} = \sigma(bc^{-1}, 1)$ is conjugate to the adding machine and has infinite order.

2364. Wreath recursion: $a = \sigma(c,b)$, $b = \sigma(b,a)$, $c = (c,a)$.

The element $cb^{-1} = \sigma(1, cb^{-1})$ is the adding machine and has infinite order. Therefore its conjugate $b^{-1}c$ also has infinite order. Since $(b^{-1}a)\big|_0 = b^{-1}c$ and 0 is fixed under the action of $b^{-1}a$ we obtain that $b^{-1}a$ also has infinite order. Finally, $b^{-1}a$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2364} is not contracting.

We have $c^{-1}ac^{-1}b = (1, a^{-1}bc^{-1}b), bc^{-1}ac^{-1} = (ba^{-1}bc^{-1}, 1)$, hence by Lemma 4 the group is not free.

2365. Wreath recursion: $a = \sigma(a,c)$, $b = \sigma(b,a)$, $c = (c,a)$.

By Lemma 2 the element cb acts transitively on the levels of the tree and, hence, has infinite order.

2366. Wreath recursion: $a = \sigma(b,c)$, $b = \sigma(b,a)$, $c = (c,a)$.

By Lemma 2 the element a acts transitively on the levels of the tree and, hence, has infinite order. Since $c = (c, a)$ we obtain that c also has infinite order and G_{2366} is not contracting.

We have $a^{-2}bab^{-2}ab = 1$, and a and b do not commute, hence the group is not free.

2367. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(b, a)$, $c = (c, a)$.

The states a and c form a 2-state automaton generating D_∞ (see Theorem 7).

Also we have $bc = \sigma(bc, 1)$ and $ca = \sigma(ac, 1)$. Therefore the elements bc and ca generate the Brunner-Sidki-Vierra group (see [BSV99]).

2368 $\cong G_{739} \cong C_2 \ltimes (C_2 \wr \mathbb{Z})$. Wreath recursion: $a = \sigma(a, a)$, $b = \sigma(c, a)$, $c = (c, a)$.

We have $bc^{-1}a = (a, a)$. Therefore $G_{2368} = \langle a, c, bc^{-1}a\rangle = G_{739}$.

2369. Wreath recursion: $a = \sigma(b, a)$, $b = \sigma(c, a)$, $c = (c, a)$.

By using the approach already used for G_{875}, we can show that the forward orbit of 10^∞ under the action of a is infinite, and therefore a has infinite order.

Since $a^2 = (ab, ba)$, the element ab also has infinite order. Furthermore, ab fixes 00 and has itself as a section at this vertex. Therefore G_{2369} is not contracting.

2371. Wreath recursion: $a = \sigma(a, b)$, $b = \sigma(c, a)$, $c = (c, a)$.

The element $(c^{-1}ab^{-1}a)^2$ stabilizes the vertex 01 and its section at this vertex is equal to $c^{-1}ab^{-1}a$, which is nontrivial. Hence, $c^{-1}ab^{-1}a$ has infinite order.

Let $t = b^{-1}c^{-1}a^2c^{-1}ba^{-1}ca^{-1}ca^{-2}cbc^{-1}ab^{-1}a$. Then t^2 stabilizes the vertex 00 and $t^2\big|_{00} = t^{a^{-1}ba^{-1}c}$. Hence, t has infinite order. Let $s = b^{-1}c^{-2}a^3$ Since $s^8|_{00100001} = t$ and s fixes the vertex 00100001 we see that s also has infinite order. Finally, s stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2371} is not contracting.

2372. Wreath recursion: $a = \sigma(b, b)$, $b = \sigma(c, a)$, $c = (c, a)$.

By Lemma 2 the elements b and ac act transitively on the levels of the tree and, hence, have infinite order. Since $(c^2)\big|_{100} = ac$ and the vertex 100 is fixed under the action of c^2 we obtain that c also has infinite order. Finally, c stabilizes the vertex 0 and has itself as a section at this vertex. Therefore G_{2372} is not contracting.

2374 $\cong G_{821}$. Lamplighter group $\mathbb{Z} \wr C_2$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(c, a)$, $c = (c, a)$.

The states a and c form a 2-state automaton that generates the Lamplighter group (see Theorem 7). Since $bc^{-1} = \sigma = c^{-1}a$, we have $b = a^c$ and $G = \langle a, c\rangle$.

2375. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(c, a)$, $c = (c, a)$.

The element $(a^{-1}c)^2$ stabilizes the vertex 01 and its section at this vertex is equal to $a^{-1}c$. Hence, $a^{-1}c$ and $c^{-1}a$ have infinite order. Since $c^{-1}b^{-1}ac^{-1}a^2\big|_{00} = c^{-1}a$ and the vertex 00 is fixed under the action of $c^{-1}b^{-1}ac^{-1}a^2$ we obtain that $c^{-1}b^{-1}ac^{-1}a^2$ also has infinite order. Finally, $c^{-1}b^{-1}ac^{-1}a^2$ stabilizes the vertex 11 and has itself as a section at this vertex. Therefore G_{2375} is not contracting.

2376 $\cong G_{739} \cong C_2 \ltimes (C_2 \wr \mathbb{Z})$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, a)$, $c = (c, a)$.

Since $\sigma = bc^{-1}$, we have $G_{2376} = \langle a, c, \sigma\rangle$. We already proved that $G_{972} = \langle a, c, \sigma\rangle$. Therefore $G_{2376} = G_{972} \cong G_{739}$.

2388 $\cong G_{821}$. Lamplighter group $\mathbb{Z} \wr C_2$. Wreath recursion: $a = \sigma(c, a)$, $b = \sigma(b, b)$, $c = (c, a)$.

The states a and c form a 2-state automaton generating the Lamplighter group (see Theorem 7) and $b = \sigma = ac^{-1}$.

2391. Wreath recursion: $a = \sigma(c, b)$, $b = \sigma(b, b)$, $c = (c, a)$.

The element $(c^{-1}ba^{-1}b)^2$ stabilizes the vertex 00 and its section at this vertex is equal to $c^{-1}ba^{-1}b$. Hence, $c^{-1}ba^{-1}b$ has infinite order. Since $(bc^{-2}b)^2\big|_{000}= c^{-1}ba^{-1}b$ and 000 is fixed under the action of $bc^{-2}b$ we obtain that $bc^{-2}b$ also has infinite order. Finally, $bc^{-2}b$ stabilizes the vertex 1 and has itself as a section at this vertex. Therefore G_{2391} is not contracting.

2394 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(b, b)$, $c = (c, a)$.

All generators have order 2, hence $H = \langle ba, bc\rangle$ is normal in G_{2394}. Furthermore, $ba = (bc, bc)$, $bc = \sigma(bc, ba)$, and therefore $H = G_{731} \cong \mathbb{Z}$. Thus $G_{2394} = \langle b\rangle \ltimes H \cong C_2 \ltimes \mathbb{Z} \cong D_\infty$ since $(bc)^b = (bc)^{-1}$.

2395. Wreath recursion: $a = \sigma(a, a)$, $b = \sigma(c, b)$, $c = (c, a)$.

By Lemma 2 the element ca acts transitively on the levels of the tree.

The element $(c^{-1}a)^2$ stabilizes the vertex 0 and its section at this vertex is equal to $c^{-1}a$. Hence, $c^{-1}a$ has infinite order. Since $(b^{-1}a)\big|_0= c^{-1}a$ and 0 is fixed under the action of $b^{-1}a$ we obtain that $b^{-1}a$ also has infinite order. Finally, $b^{-1}a$ stabilizes the vertex 1 and has itself as a section at this vertex. Therefore G_{2395} is not contracting.

Note that $ab = (ac, ab), ac = \sigma(ac, 1)$ and $ba = (ba, ca), ca = \sigma(1, ca)$, i.e., G_{2395} contains copies of G_{929}.

2396. Boltenkov group. Wreath recursion: $a = \sigma(b, a)$, $b = \sigma(c, b)$, $c = (c, a)$.

This group was studied by A. Boltenkov (under direction of R. Grigorchuk), who showed that the monoid generated by $\{a, b, c\}$ is free, and the group G_{2396} is torsion free.

Proposition 2. *The monoid generated by a, b, and c is free.*

Proof. By way of contradiction, assume that there are some relations and let $w = u$ be a relation for which $\max(|w|, |u|)$ minimal.

We first consider the case when neither w nor u is empty. Because of cancelation laws, the words w and u must end in different letters. We have $w = \sigma_w(w_0, w_1) = \sigma_u(u_0, u_1) = u$, where σ_w, and σ_u are permutations in $\{1, \sigma\}$. Clearly, $w_0 = u_0$ and $w_1 = u_1$ must also be relations.

Assume that w ends in b and u ends in c. Then w_0 and u_0 both end in c. Therefore, by minimality, $w_0 = u_0$ as words and $|u| = |w|$. Since $b \neq c$ in G_{2396} the length of w and u is at least 2. We can recover the second to last letter in w and u. Indeed, the second to last letter in u_0 can be only b or c (these are the possible sections at 0 of the three generators), while the second to last letter of w_0 can be only a or b (these are the possible sections at 1 of the three

generators). Therefore $w_0 = u_0 = \dots bc$, $w = \dots bb$, and $u = \dots ac$. Since $bb \neq ac$ in G_{2396} (look at the action at level 1), the length of w and u must be at least 3. Continuing in the same fashion we obtain that $w_0 = u_0 = b \dots bbc$, $w = \dots ababb$, and $u = \dots babac$. Since the lengths of w and u are equal, they have different action on level 1, which is a contradiction.

Assume that w ends in a and u ends in b or c. Then u_0 and w_0 end in b and c, respectively, and we may proceed as before.

It remains to show that, say, u cannot be empty word. If this is the case then $w_0 = 1 = w_1$, implying that $w_0 = w_1$ is also a minimal relation. But this is impossible since both w_0 and w_1 are nonempty. $\square$

For a group word w over $\{a, b, c\}$, define the exponent $\exp_a(w)$ of a in w as the sum of the exponents in all occurrences of a and a^{-1} in w. Define $\exp_b(w)$ and $\exp_c(w)$ in analogous way and let $\exp(w) = \exp_a(w) + \exp_b(w) + \exp_c(w)$.

Lemma 5. *If $w = 1$ in G_{2396} then $\exp(w) = 0$.*

Proof. By way of contradiction, assume otherwise and choose a freely reduced group word w over $\{a, b, c\}$ such that $w = 1$ in G_{2396}, $\exp(w) \neq 0$, and w has minimal length among such words. If $w = (w_0, w_1)$, w_0 and w_1 also represent 1 in G_{2396} and $\exp(w_0) = \exp(w_1) = \exp(w) \neq 0$. Since the exponents is nonzero, the words w_0 and w_1 are nonempty and, by minimality, their length must be equal to $|w|$. Note that $ac^{-1} = \sigma(bc^{-1}, 1)$ and $bc^{-1} = \sigma(1, ba^{-1})$. This implies that w cannot ac^{-1}, bc^{-1}, ca^{-1}, or cb^{-1} as a subword (otherwise the length of w_0 or w_1 would be shorter than the length of w). By the same reason, w_0 and w_1 cannot have the above 4 words as subwords, which implies that w does not have $ab^{-1} = (ab^{-1}, bc^{-1})$ or its inverse ba^{-1} as a subword. Therefore w has the form $w = W_1(a^{-1}, b^{-1}, c^{-1})W_2(a, b, c)$, and since $w = 1$ in G_{2396}, we obtain a relation between positive words over $\{a, b, c\}$, which contradicts Proposition 2. $\square$

Lemma 6. *If $w = 1$ in G_{2396} then $\exp_a(w)$, $\exp_b(w)$ and $\exp_c(w)$ are even.*

Proof. Indeed, $\exp_a(w) + \exp_b(w)$ must be even (since both a and b are active at the root). By Lemma 5, $\exp_c(w)$ must be even. If $w = (w_0, w_1)$, then $\exp_a(w_0) + \exp_b(w_0)$ and $\exp_a(w_1) + \exp_b(w_1)$ must be even. Since $\exp_a(w) + \exp_b(w) = \exp_b(w_0) + \exp_b(w_1)$, $\exp_a(w) + \exp_c(w) = \exp_a(w_0) + \exp_a(w_1)$ we obtain that $2\exp_a(w) + \exp_b(w) + \exp_c(w)$ is even, which then implies that $\exp_b(w)$ is even. Finally, since both $\exp_b(w)$ and $\exp_c(w)$ are even, $\exp_a(w)$ must be even as well (by Lemma 5). $\square$

Proposition 3. *The group G_{2396} is torsion free.*

Proof. By way of contradiction, assume otherwise. Let w be an element of order 2. We may assume that w does not belong to the stabilizer of the first level (otherwise we may pass to a section of w). Let $w = \sigma(w_0, w_1)$. Since $w^2 = (w_1w_0, w_0w_1) = 1$, we have the modulo 2 equalities $\exp_b(w_0w_1) = \exp_b(w_0) + \exp_b(w_1) = \exp_a(w) + \exp_b(w)$. Since $\exp_b(w_0w_1)$ is even, $\exp_a(w) + \exp_b(w)$ must be even, implying that w stabilizes level 1, a contradiction. $\square$

Since $b^{-1}a = (c^{-1}b, b^{-1}a)$, the group G_{2396} is not contracting (our considerations above show that $b^{-1}a$ is not trivial and therefore has infinite order).

We have $c^{-1}bc^{-1}a = (1, a^{-1}bc^{-1}b)$, $ac^{-1}bc^{-1} = (ba^{-1}bc^{-1}, 1)$, hence by Lemma 4 the group is not free.

2398. Dahmani group. Wreath recursion: $a = \sigma(a, b)$, $b = \sigma(c, b)$, $c = (c, a)$.

This group is self-replicating, not contracting, weakly regular branch group over its commutator subgroup. It was studied by Dahmani in [Dah05].

2399. Wreath recursion: $a = \sigma(b, b)$, $b = \sigma(c, b)$, $c = (c, a)$.

By Lemma 2 the elements ca and $c^4bc^2bc^2b^2cb^2cb^3acba^2$ act transitively on the levels of the tree and, hence, have infinite order. Since $(cba)^8\big|_{000010001} = c^4bc^2bc^2b^2cb^2cb^3acba^2$ and vertex 000010001 is fixed under the action of $(cba)^8$ we obtain that cba also has infinite order. Finally, cba stabilizes the vertex 01001 and has itself as a section at this vertex. Therefore G_{2399} is not contracting.

We have $a^{-2}bab^{-2}ab = 1$, and a and b do not commute, hence the group is not free.

2401. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(c, b)$ and $c = (c, a)$.

The states a and c form a 2-state automaton generating the Lamplighter group (see Theorem 7). Hence, G_{2401} is neither torsion, nor contracting and has exponential growth.

2402. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(c, b)$, $c = (c, a)$.

The element $(bc^{-1})^2$ stabilizes the vertex 00 and its section at this vertex is equal to bc^{-1}. Hence, bc^{-1} has infinite order.

We have $c^{-2}ba = (1, a^{-2}b^2)$, $ac^{-2}b = (ba^{-2}b, 1)$, hence by Lemma 4 the group is not free.

2403 $\cong G_{2287}$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, b)$, $c = (c, a)$.

The states a and c form a 2-state automaton generating D_∞ (see Theorem 7).

Also we have $bc = \sigma(1, ba)$ and $ba = (bc, 1)$. Therefore the elements bc and ba generate the Basilica group G_{852}.

By conjugating by $g = (cg, g)$, we obtain

$$a' = \sigma, \qquad b' = \sigma(1, c'b'), \qquad c' = (c', a'),$$

where $a' = a^g$, $b' = b^g$, and $c' = c^g$. Therefore

$$a' = \sigma, \qquad b' = \sigma(1, c'b'), \qquad c'b' = \sigma(a', b'),$$

and G_{2402} is isomorphic to G_{2287}, i.e., to $IMG(\frac{z^2+2}{1-z^2})$.

2422 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(a, a)$, $b = \sigma(c, c)$, $c = (c, a)$.

The states a and c form a 2-state automaton generating D_∞ (see Theorem 7) and $b = aca$.

2423. Wreath recursion: $a = \sigma(b, a)$, $b = \sigma(c, c)$, $c = (c, a)$.

Contains elements of infinite order by Lemma 1. In particular, ac has infinite order. Since $c^2\big|_{100} = ac$ and the vertex 100 is fixed under the action of c^2 we obtain that c also has infinite order. Since $c = (c, a)$ the group is not contracting.

We have $c^{-1}bc^{-1}a = (1, a^{-1}b)$, $ac^{-1}bc^{-1} = (ba^{-1}, 1)$, hence by Lemma 4 the group is not free.

2424 $\cong G_{966}$. Wreath recursion $a = \sigma(c, a)$, $b = \sigma(c, c)$, $c = (c, a)$.

We have $ac^{-1}b = (c, c)$. Therefore $G_{2424} = \langle a, ac^{-1}b, c\rangle = G_{966}$.

2426 $\cong G_{2277} \cong C_2 \ltimes (\mathbb{Z} \times \mathbb{Z})$. Wreath recursion: $a = \sigma(b, b)$, $b = \sigma(c, c)$, $c = (c, a)$.

Since all generators have order 2 the subgroup $H = \langle ab, cb\rangle$ is normal in G_{2426}. Furthermore, $ab = (bc, bc)$, $cb = \sigma(ac, 1) = \sigma(ab(cb)^{-1}, 1)$, so H is self-similar. Since $acb = bca$ in G_{2426} we obtain $ab \cdot cb = abcaab = aacbab = cb \cdot ab$, hence, H is an abelian self-similar 2-generated group.

Consider the $\frac{1}{2}$-endomorphism $\phi : \mathrm{Stab}_H(1) \to H$, given by $\phi(g) = h$, provided $g = (h, *)$ and consider the linear map $A : \mathbb{C}^2 \to \mathbb{C}^2$ induced by ϕ. It has the following matrix representation with respect to the basis corresponding to the generating set $\{ab, cb\}$:

$$A = \begin{pmatrix} 0 & \frac{1}{2} \\ -1 & -\frac{1}{2} \end{pmatrix}.$$

Its eigenvalues are not algebraic integers and, therefore, by [NS04], H is a free abelian group of rank 2.

Finally, $G_{2426} = \langle H, b\rangle = \langle b\rangle \ltimes H = C_2 \ltimes (\mathbb{Z} \times \mathbb{Z})$, where b inverts the generators of H. This action coincides with the one for G_{2277}, which proves that these groups are isomorphic.

2427. The element $(bc^{-1})^4$ stabilizes the vertex 000 and its section at this vertex is equal to bc^{-1}. Hence, bc^{-1} has infinite order.

We have $a^{-2}bab^{-2}ab = 1$, and a and b do not commute, hence the group is not free.

2838 $\cong G_{848} \cong C_2 \wr \mathbb{Z}$. Wreath recursion: $a = \sigma(c, a)$, $b = \sigma(a, a)$, $c = (c, c)$.

Since c is trivial, we have $G = \langle a, ba^{-1}\rangle$, where $a = \sigma(1, a)$ is the adding machine and $ba^{-1} = (1, a)$. Therefore $G_{2838} = G_{848}$.

2841. Wreath recursion: $a = \sigma(c, b)$, $b = \sigma(a, a)$, $c = (c, c)$.

The element c is trivial. Since $a^2 = (b, b)$, $b^2 = (a^2, a^2)$ and a^2 is nontrivial, the elements a and b have infinite order. Also we have $ba = (a, ab)$ and $ab = (ba, a)$, hence ba has infinite order and G_{2841} is not contracting.

We claim that the monoid generated by a and b is free. Hence, G_{2841} has exponential growth.

Proof. We can first prove (analogous to G_{2851}) that $w \neq 1$ for any nonempty word $w \in \{a, b\}^*$.

By way of contradiction, let w and v be two nonempty words in $\{a, b\}^*$ with minimal $|w| + |v|$ such that $w = v$ in G_{2841}. Assume that w ends with a and v ends with b. Consider the following cases.

1. If $w = a$ then $v|_0 = 1$ in G_{2841} and $v|_0$ is nontrivial word.

2. If w ends with a^2 then $w|_1 = v|_1$ in G_{2841}, $\big|w|_1\big| + \big|v|_1\big| < |w| + |v|$ and $w|_1$ ends with b, $v|_1$ with a.

3. If w ends with ba and v ends with ab, then $w|_1 = v|_1$ in G_{2841}, $\big|w|_1\big| + \big|v|_1\big| < |w| + |v|$ (because $\big|v|_1\big| < |v|$) and $w|_1$ ends with b, $v|_1$ with a.

4. If w ends with ba and v ends with b, then $w|_1 = v|_1$ in G_{2841}, $\big|w|_1\big|+\big|v|_1\big| \le |w| + |v|$ and $w|_1$ ends with ab, $|v_1|$ with a. Therefore, words $v|_1$ and $w|_1$ satisfy one of the first three cases.

In all cases we obtain either a shorter relation, which contradicts to our assumption, or a relation of the form $v = 1$, which is also impossible. □

There are non-trivial group relations, e.g. $a^{-1}b^{-1}a^{-2}ba^{-1}b^{-1}aba^2b^{-1}ab = 1$, while a and b do not commute, hence the group is not free.

2284 $\cong G_{730}$. Klein Group $C_2 \times C_2$.

Direct calculation.

2847 $\cong G_{929}$. Wreath recursion: $a = \sigma(c, a)$, $b = \sigma(b, a)$, $c = (c, c)$.

Since c is trivial, the generator $a = \sigma(1, a)$ is the adding machine and $b = \sigma(b, a)$. We have $ab = (ab, a)$. Therefore $G_{2847} = \langle a, ab\rangle = G_{929}$.

2850. Wreath recursion: $a = \sigma(c, b)$, $b = \sigma(b, a)$, $c = (c, c)$.

Since c is trivial, we have $a^2 = (b, b)$, $b^2 = (ab, ba)$, $ab = (b^2, a)$ and $ba = (a, b^2)$. Therefore the elements a, b, ab and ba have infinite order. Since ba fixes the vertex 11 and has itself as a section at that vertex, G_{2850} is not contracting.

The group is regular weakly branch over G'_{2850}, since it is self-replicating and $[b, a^2] = (1, [a, b])$.

Semigroup $\langle a, b\rangle$ is free. Hence, G_{2850} has exponential growth.

Proof. We can first prove (analogous G_{2851}) that $w \ne 1$ for any nonempty word $w \in \{a, b\}^*$.

By way of contradiction, let w and v be two nonempty words in $\{a, b\}^*$ with minimal $|w| + |v|$ such that $w = v$ in G_{2850}. Assume that w ends with a and v ends with b. Consider the following cases.

1. If $w = a$ then $v|_0 = 1$ in G and $v|_0$ is nontrivial word.

2. If w ends with a^2 then $w|_1 = v|_1$ in G, $\big|w|_1\big| + \big|v|_1\big| < |w| + |v|$ and $w|_1$ ends with b, $v|_1$ with a.

3. If w ends with ba then $w|_0 = v|_0$ in G, $\big|w|_0\big| + \big|v|_0\big| < |w| + |v|$ and $w|_0$ ends with a, $v|_0$ with b.

In all cases we obtain either a shorter relation, which contradicts to our assumption, or a relation of the form $v = 1$, which is also impossible. □

Since $a^{-4}bab^{-1}a^2b^{-1}ab = 1$ and a and b do not commute, the group is not free.

2851 $\cong G_{929}$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(b, a)$, $c = (c, c)$.

The automorphism c is trivial. Therefore $a = \sigma(a, 1)$ is the inverse of the adding machine. Since $ba^{-1} = (a, ba^{-1})$, the order of ba^{-1} is infinite and G_{2851} is not contracting.

Since G_{2851} is self-replicating and $[a^2, b] = ([a, b], 1)$, the group is a regular weakly branch group over its commutator.

The monoid $\langle a, b\rangle$ is free.

Proof. By way of contradiction, assume that w be a nonempty word over $\{a, b\}$ such that $w = 1$ in G_{2851} and w has the smallest length among all such words. The word w must contain both a and b (since they have infinite order). Therefore, one of the projections of w is be shorter than w, nonempty, and represents the identity in G_{2851}, a contradiction.

Assume now that w and v are two nonempty words over $\{a, b\}$ such that $w = v$ in G_{2851} and they are chosen so that the sum $|w| + |v|$ is minimal. Assume that w ends in a and v ends in b. Then

- if w ends in a^2, then w_0 is a nonempty word that is shorter than w ending in a, while v_0 is a nonempty word of length no greater than $|v|$ ending in b. Since $w_0 = v_0$ in G_{2851}, this contradicts the minimality assumption.

- if w ends in ba, then w_1 is a word that is shorter than w ending in b, while v_1 is a nonempty word of length no greater than $|v|$ ending in a. Since $w_1 = v_1$ in G_{2851}, this contradicts the minimality assumption.

- if $w = a$ then $v_1 = 1$ in G and v_1 is a nonempty word. Thus we obtain a relation $v_1 = 1$ in G_{2851}, a contradiction.

□

This shows that G has exponential growth, while the orbital Schreier graph $\Gamma(G, 000\ldots)$ has intermediate growth (see [BH05, BCSN]).

The groups G_{2851} and G_{929} coincide as subgroups of $\mathrm{Aut}(X^*)$. Indeed, $a^{-1} = \sigma(1, a^{-1})$ is the adding machine and $b^{-1}a = (b^{-1}a, a^{-1})$, showing that $G_{929} = \langle a^{-1}, b^{-1}a \rangle = G_{2851}$.

2852 $\cong G_{849}$. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(b, a)$, $c = (c, c)$.

The automorphism c is trivial. Therefore $a = \sigma(b, 1)$, $a^2 = (b, b)$ and $ab = (b, ba)$, which implies that G_{2852} is self-replicating and level transitive.

The group G_{2852} is a regular weakly branch group over its commutator. This follows from $[a^{-1}, b] \cdot [b, a] = ([a, b], 1)$, together with the self-replicating property and the level transitivity. Moreover, the commutator is not trivial, since G_{2852} is not abelian (note that $[b, a] = (b^{-1}ab, a^{-1}) \neq 1$).

We have $b^2 = (ab, ba)$, $ba = (ab, b)$, and $ab = (b, ba)$. Therefore b^2 fixes the vertex 00 and has b as a section at this vertex. Therefore b has infinite order (since it is nontrivial), and so do ab and a (since $a^2 = (b, b)$). Since ab fixes the vertex 10 and has itself as a section at that vertex, G_{2852} is not contracting.

The monoid generated by a and b is free (and therefore the group has exponential growth).

Proof. By way of contradiction assume that w is a word of minimal length over all nonempty words over $\{a, b\}$ such that $w = 1$ in G_{2851}. Then w must have occurrences of both a and b (since both have infinite order). This implies that one of the sections of w is shorter than w (since $a|_1$ is trivial), nonempty (since both $b|_0$ and $b|_1$ are nontrivial), and represents the identity in G_{2851}, a contradiction.

Assume now that there are two nonempty words $w, v \in \{a, b\}^*$ such that $w = u$ in G_{2852} and choose such words with minimal sum $|w| + |v|$. Let $w = \sigma_w(w_0, w_1)$ and $u = \sigma_u(u_0, u_1)$, where $\sigma_w, \sigma_w \in \{1, \sigma\}$. Assume that w ends in a and v ends in b (they must end in different letters because of the cancelation property and the minimality of the choice). Then $w_1 = v_1$ in G_{2851}, the word v_1 is nonempty, $|v_1| \leq |v|$, and $|w_1| < |w|$. Thus we either obtain a contradiction with the minimality of the choice of w and v or we obtain a relation of the type $v_1 = 1$, also a contradiction. $\square$

See G_{849} for an isomorphism between G_{2852} and G_{849}.

If we conjugate the generators of G_{2852} by the automorphism $\mu = \sigma(b\mu, \mu)$, we obtain the wrath recursion

$$x = \sigma(y, 1), \qquad y = \sigma(xy, 1),$$

where $x = a^\mu$ and $y = b^\mu$. Further,

$$y = \sigma(xy, 1), \qquad xy = (xy, y),$$

and the last recursion defines the automaton 933. Therefore $G_{2852} \cong G_{933}$.

2853 $\cong IMG\left(\left(\frac{z-1}{z+1}\right)^2\right)$. Wreath recursion $a = \sigma(c, c)$, $b = \sigma(b, a)$ and $c = (c, c)$.

The automorphism c is trivial and $a = \sigma$.

It is shown in [BN06] that $IMG\left(\left(\frac{z-1}{z+1}\right)^2\right)$ is generated by $\alpha = \sigma(1, \beta)$ and $\beta = (\alpha^{-1}\beta^{-1}, \alpha)$.

We have then $\beta\alpha = \sigma(\alpha, \alpha^{-1})$. If we conjugate by $\gamma = (\gamma, \alpha\gamma)$, we obtain the wreath recursion

$$A = \sigma, \qquad B = \sigma(B^{-1}, A)$$

where $A = (\beta\alpha)^\gamma$ and $B = \alpha^\gamma$. The group $\langle A, B\rangle$ is conjugate to G_{2853} by the element $\delta = (\delta_1, \delta_1)$, where $\delta_1 = \sigma(\delta, \delta)$ (this is the automorphism of the tree changing the letters on even positions).

Therefore $G_{2852} \cong IMG\left(\left(\frac{z-1}{z+1}\right)^2\right)$ and the limit space of G_{2852} is the Julia set of the rational map $z \mapsto \left(\frac{z-1}{z+1}\right)^2$.

Note that G_{2853} is contained in G_{775} as a subgroup of index 2. Therefore it is virtually torsion free (it contains the torsion free subgroup H mentioned in the discussion of G_{775} as a subgroup of index 2) and is a weakly branch group over H''.

The diameters of the Schreier graphs on the levels grow as $\sqrt{2}^n$ and have polynomial growth of degree 2 (see [BN, Bon07]).

2854 $\cong G_{847} \cong D_4$. Wreath recursion: $a = \sigma(a, a)$, $b = \sigma(c, a)$, $c = (c, c)$.

Direct calculation.

2860 $\cong G_{2212}$. Klein bottle group $\langle s, t \mid s^2 = t^2\rangle$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(c, a)$, $c = (c, c)\rangle$.

Note that c is trivial and therefore $a = \sigma(a, 1)$ and $b = \sigma(1, a)$. The element a has infinite order since a is inverse of the adding machine.

Let us prove that $G_{2860} \cong H = \langle s, t \mid s^2 = t^2 \rangle$. Indeed, the relation $a^2 = b^2$ is satisfied, so G_{2860} is a homomorphic image of H with respect to the homomorphism induced by $s \mapsto a$ and $t \mapsto b$. Each element of H can be written in the form $t^r(st)^l s^n$, $n \in \mathbb{Z}, l \geq 0, r \in \{0,1\}$. It suffices to prove that images of these words (except for the identity word, of course) represent nonidentity elements in G_{2860}.

We have $a^{2n} = (a^n, a^n)$, $a^{2n+1} = \sigma(a^{n+1}, a^n)$, $(ab)^l = (1, a^{2l})$. We only need to check words of even length (those of odd length act nontrivially on level 1). We have $(ab)^\ell a^{2n} = (a^n, a^{n+2\ell}) \neq 1$ in G if $n \neq 0$ or $\ell \neq 0$, since a has infinite order. On the other hand, $b(ab)^l a^{2n+1} = (a^{n+1+2l+1}, a^n) = 1$ if and only if $n = 0$ and $l = -1$, which is not the case, because l must be nonnegative. This finishes the proof.

2861 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(b, c)$, $b = \sigma(c, a)$, $c = (c, c)\rangle$.

Since c is trivial, $ba = (ab, 1)$, $ab = (1, ba)$, which yields $a = b^{-1}$. Also $a^{2n} = (b^n, b^n), b^{2n} = (a^n, a^n)$ and $a^{2n+1} \neq 1, b^{2n+1} \neq 1$. Thus a has infinite order and $G_{2861} \cong \mathbb{Z}$.

2862 $\cong G_{847} \cong D_4$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, a)$, $c = (c, c)\rangle$.

Direct calculation.

2874 $\cong G_{820} \cong D_\infty$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(b, b)$, $c = (c, c)$.

Since c is trivial, $G_{2874} = \langle b, ba \rangle$. Since $ba = (ba, b)$, the elements b and ba form a 2-state automaton generating D_∞ (see Theorem 7).

2880 $\cong G_{730}$. Klein Group $C_2 \times C_2$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(b, b)$, $c = (c, c)$.

Direct calculation.

2887 $\cong G_{731} \cong \mathbb{Z}$. Wreath recursion: $a = \sigma(a, c)$, $b = \sigma(c, b)$, $c = (c, c)$.

Note that c is trivial, b is the adding machine and $a = b^{-1}$.

2889 $\cong G_{848} \cong C_2 \wr \mathbb{Z}$. Wreath recursion: $a = \sigma(c, c)$, $b = \sigma(c, b)$, $c = (c, c)$.

Note that c is trivial. Since b is the adding machine and $ab = (1, b)$, we have $G_{2889} = \langle b, ab \rangle = G_{848}$.

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